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Evaluation of the Equivalent First Modal Damping Ratio of a Structure with Additional Damping Devices

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ABSTRACT

The purpose of this study is to propose a new method for evaluating equivalent damping ratios of a structure with supplemental damping devices to assess their control effect quantitatively. A MDOF system is transformed to an equivalent SDOF system based on the assumption that the first mode dominates structural response. Approximate closed-form formulas for the evaluation of the first damping ratio are presented for various damping devices. Through numerical analysis of a ten-story building equipped with damping devices, the effectiveness of equivalent SDOF model and closed form formulas are verified.

1. Introduction

This study proposes a new and general approach to evaluate the equivalent damping ratios of a structure with any supplemental damping devices, whose control forces are linearly or nonlinearly dependent on the structural responses. Also, this study presents an approximate, but accurate closed-form formulas of the first modal equivalent damping ratios for MDOF structures with damping devices. Lyapunov function, of which derivative can be expressed in autoregressive form, was defined and then, the equivalent damping ratios by using Lyapunov function and its derivative was evaluated[1]. It is assumed that the response of a structure is stationary random process and control devices do not affect the modal shape of structure, and the structure has proportional damping. This assumption can be justified by the fact that supplemental damping devices are minor elements in building structures and proportional damping can describe the mechanism of energy dissipation with little error.

A MDOF system is transformed to an equivalent SDOF system, since general building structures are governed by their fundamental modes. The proposed method is also applied to the equivalent SDOF system and closed form formulas for equivalent damping ratios are derived with probabilistic concept.

To show the effectiveness of the proposed approach, we evaluate the equivalent damping ratios of a structure with viscous dampers(VD), active mass damper(AMD) and friction dampers(FD). They, which are simply obtained from the formulas, are compared to ones obtained by conventional eigenvalue analysis for linear damping devices. RMS and peak responses of top floor, which are simply obtained by equivalent SDOF system, are compared to ones by MDOF system.

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2. Equivalent SDOF Model for MDOF Systems

Generally, the dynamic responses of typical civil or building structures are represented well by the first few natural vibration modes. Especially for a structure of which responses are dominated by a fundamental mode, the effect of damping devices on the fundamental mode is enough to estimate the effect of damping devices on the whole system responses. This fact also means that the effects of damping devices on a fundamental mode should be identified more accurately than on higher modes. In FEMA 273, for the case that structural members except damping devices remain in elastic region, linear static procedure and linear dynamic procedure are proposed for the evaluation of responses of a structure with damping devices[2]. These procedures adopt the concept of equivalent damping ratio for representing response reduction effect due to damping devices. For linear static procedure, assuming that a structure vibrates in one deflection shape which is obtained not from eigenvalue analysis but from static analysis, equivalent damping ratio for this deflection shape is determined and used for the evaluation of structural responses. While only the fundamental mode is considered for linear static procedure, higher modes are considered for linear dynamic procedure. However, response of a building structure is usually dominated by the first mode, so the benefit of damping devices in reducing response related to the higher modes can be ignored and only the first mode damping ratio to reflect the effect of damping devices is applied for linear dynamic procedure.

Assuming that responses of a structure are dominated by a fundamental mode, which is normalized to the top floor displacement, a MDOF system is transformed to an equivalent SDOF system and the effect of damping device on this fundamental mode is estimated by using the proposed method. Since this procedure requires seismic analysis only for SDOF system, time effort for the estimation of structural responses are much reduced and the effect of damping devices can be easily predicted. Comparisons are performed between the results given in the previous section, where the proposed method deals with MDOF system with estimated equivalent damping ratios and ones obtained from the analysis of equivalent SDOF system.

Also, based on the probabilistic properties of structural responses, closed-form formulas for equivalent damping ratios are proposed for MDOF system with damping devices adding stiffness or viscosity to a structure, with nonlinear FD, and with active damping devices using linear feedback law or nonlinear bang-bang law. Since these formulas are explicit functions expressed with parameters of damping devices, they can be used for the evaluation of equivalent damping ratios for fundamental mode without any seismic analysis. So, they are helpful for the preliminary design of damping devices to predict control effect of a structure subjected to earthquake loading.

Displacement vector $x(t)$ can be expressed approximately by

$$x(t) = \phi d(t) \tag{1}$$

where ϕ is a vector of deflection shape of fundamental mode and normalized to a top story displacement $d(t)$.

The equation of motion for the equivalent SDOF system is obtained as

$$M^{*}\ddot{d}(t) + C^{*}\dot{d}(t) + K^{*}d(t) = \phi^{T} L_{u} u(t) + \phi^{T} L_{e} f(t) \tag{2}$$

where $M^* = \phi^T M\phi$, $C^* = \phi^T C\phi$, and $K^* = \phi^T K\phi$.
Dividing both sides of the equation by M^*, we obtain

$$\ddot{d}(t) + 2\xi_o\omega_o\dot{d}(t) + \omega_o^2 d(t) = \frac{\phi^T L_u}{M^*}u(t) + \frac{\phi^T L_e}{M^*}f(t) \tag{3}$$

where $\omega_o^2 = K^*/M^*$ and $\xi_o = C^*/2\omega_o M^*$

Equation (3) is exactly same as one we already applied in the work by Lee, et.al. [1] The method presented in their work can be used and thus, increased damping ratio ξ_{eq} by damping device for equivalent SDOF system is derived as follows

$$\xi_{eq} = -\frac{E\left[z(t)^T PBu(t)\right]}{\omega_o E\left[z(t)^T Pz(t)\right]} \tag{4}$$

where $z(t)^T = \left[d(t) \quad \dot{d}(t)\right]^T$, $P = \begin{bmatrix} \omega_o^2 & \xi_o\omega_o \\ \xi_o\omega_o & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \frac{\phi^T L_u}{M^*} \end{bmatrix}$

For analysis of the SDOF system equivalent to MDOF system, the determination of shape vector is essential for the evaluation of damping ratio. In this study, ϕ is assumed to be the shape vector corresponding to the deflected shape under the action of statically applied lateral loads with an inverted triangular distribution pattern [3].

3. Closed Form Formulas for Equivalent Damping Ratios

Structural responses subjected to dynamic loads such earthquake or wind loads have probabilistic characteristics. It means that responses can be described by mean and standard deviation values. Considering that forces of damping devices are dependent on the structural responses, equivalent damping ratios increased by these damping forces has probabilistic characteristics. The method proposed in this study begins with taking expectation of the responses for their mean values, based on the assumption that responses are stationary and Gaussian. Then, probabilistic properties of responses are obtained and closed form formulas for equivalent damping ratios can be derived.

It is assumed that top floor displacement d and velocity $\dot{d}$ have following probabilistic characteristics.

$$E\left[d^2(t)\right] = \sigma_d^2 \tag{5}$$

$$E\left[\dot{d}^2(t)\right] = \sigma_{\dot{d}}^2 = \omega_o^2\sigma_d^2 \tag{6}$$

$$E\left[d(t)\cdot\dot{d}(t)\right] = 0 \tag{7}$$

where σ_d and $\sigma_{\dot{d}}$ are mean values of displacement and velocity, respectively.
Using the above equations, the denominator of Eq.(4) becomes

$$\omega_o E\left[z(t)^T Pz(t)\right] = \omega_o\left[\omega_o^2 d^2(t) + 2\xi_o\omega_o d(t)\dot{d}(t) + \dot{d}^2(t)\right] = 2\omega_o^3\sigma_d^2 \tag{8}$$

In order to evaluate the value of the nominator in Eq.(4), the control force $u(t)$ should be known. As $u(t)$ is generally a function of the displacement and velocity responses, equivalent damping ratio is evaluated for the following cases for $u(t)$;

1) $u(t)$is linearly proportional to the relative displacement between the ends of a device installed between adjacent stories along the device axis of device
2) $u(t)$is linearly proportional to the relative velocity between the ends of a device installed between adjacent stories along the device axis of device
3) $u(t)$is constant multiplied by the sign of the relative velocity between the ends of a device installed between adjacent stories along the device axis of device.
4) $u(t)$ is a linear function of states.

Case 1) and 2) mean that damping device adds stiffness or viscosity to a structure, respectively. Case 3) means that friction damper or bang-bang control law is adopted as damping device or active control law, respectively. Active control devices using linear feedback control law correspond to case 4).

The respective *j*th control force for each case, which is generated from a damping device installed at the *j*th inter-story or on the *j*th floor, is given by

$$u_j(t) = k_{Dj} d(t)\delta_{rj}\cos\theta_j\ ; \qquad \text{for Case 1)} \tag{9}$$

$$u_j(t) = c_{Dj}\dot{d}(t)\delta_{rj}\cos\theta_j\ ; \qquad \text{for Case 2)} \tag{10}$$

$$u_j(t) = u_{\max j}\,\mathrm{sgn}\left[\dot{d}(t)\delta_{rj}\right]\cos\theta_j; \qquad \text{for Case 3)} \tag{11}$$

$$u(t) = -Gz(t) = -G_1\phi d(t) - G_2\phi\dot{d}(t); \qquad \text{for Case 4)} \tag{12}$$

in which θ_i is the the angle between the jth device axis and the floor at which the device is installed.
k_{Di}, c_{Di} and $u_{\max i}$ is stiffness, viscosity and maximum control force of device j, respectively. δ_{ri}, relative displacement between the ends of device j along the device axis jth, is determined based on the deflection shape vector ϕ. G_1 and G_2 are gains for displacement feedback and velocity feedback, respectively.

Substituting the expressions of *P* and *B*, appeared below Eq.(4) into the denominator of Eq.(4) leads to

$$E\left[z(t)^T PBu(t)\right] = \frac{1}{M^*}E\left[\left(\xi_o\omega_o d(t) + \dot{d}(t)\right)\phi^T L_u u(t)\right] \tag{13}$$

The *j*th element of $\phi^T L_u$ for Cases 1), 2), and 3) is given by

$$\left[\phi^T L_u\right]_{j_{th}\_element} = -\delta_{rj}\cos\theta_j \tag{14}$$

Thus, corresponding denominators can be derived for each case

Now, using derived denominators and numerators gives formulas for equivalent damping ratios of a structure with damping devices for each case.

$$\xi_{eq} = \frac{\xi_o}{2M^* \omega_o^2} \sum_j k_{Dj} \delta_{rj}^2 \cos\theta_j^2 \qquad \text{for Case 1)} \qquad (15)$$

$$\xi_{eq} = \frac{1}{2M^* \omega_o} \sum_j c_{Dj} \delta_{rj}^2 \cos\theta_j^2 \qquad \text{for Case 2)} \qquad (16)$$

$$\xi_{eq} = \frac{1}{\sqrt{2\pi} M^* \omega_o^2 \sigma_d} \sum_j u_{\max j} \left|\delta_{rj}\right| \cos\theta_j^2 \qquad \text{for Case 3)} \qquad (17)$$

$$\xi_{eq} = \xi_o \frac{\phi^T L_u G_1 \phi}{2M^* \omega_o^2} + \frac{\phi^T L_u G_2 \phi}{2M^* \omega_o} \qquad \text{for Case 4)} \qquad (18)$$

While equivalent damping ratio ξ_{ea} for Case 1), Case 2) and Case 4) can be easily determined only with information of ϕ and the properties or the gain of damping devices, σ_d should be known for the evaluation of ξ_{ea} for Case 3). However, since σ_d is a mean of top floor displacement obtained by seismic analysis of the building-damping system, ξ_{ea} cannot be predicted in advance of seismic analysis for the control efficiency. So, iterative seismic analysis procedure can not be avoided for the evaluation of ξ_{eq} for Case 3).

The prediction of ξ_{ea} may be possible by using an estimation technique. Kasai suggested a simplified expression which represents the response variation due to change of damping ratio from ξ_{ea} to ξ [4].

$$\frac{S_d(\xi)}{S_d(\xi_o)} = \frac{\sqrt{1+25\xi_o}}{\sqrt{1+25\xi}} \qquad (19)$$

in which S_d denotes a spectral displacement of a structure.

Since spectral response is approximately proportional to RMS response, with the help of Eq.(19). σ_{d_n} can be expressed as a function of σ_{d_o}. Here, σ_{d_o} is displacement of a structure without damping devices.

Therefore, ξ_{ea} for Case 3) can be obtained as

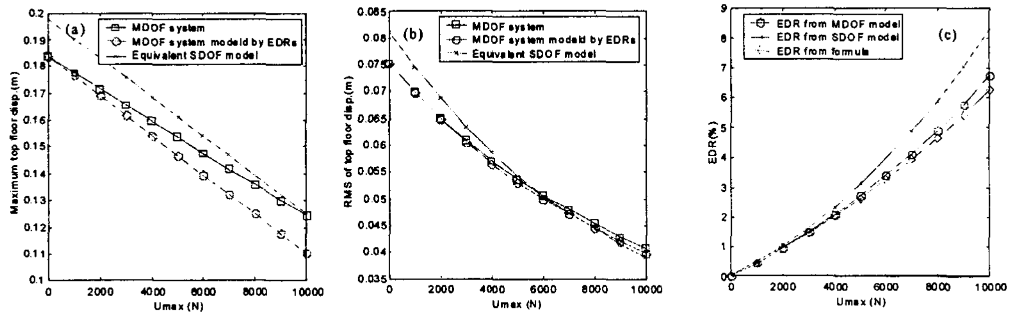
$$\xi_{eq} = \frac{25C_1^2 + C_1\sqrt{625C_1^2 + 4\left(1+25\xi_o\right)^2}}{2\left(1+25\xi_o\right)} \qquad (20)$$

in which $C_1 = \dfrac{\sum_j U_{\max j} \left|\delta_{rj}\right| \cos\theta_j^2}{\sqrt{2\pi} M^* \omega_o^2 \sigma_{d_o}}$

4. Numerical Analysis

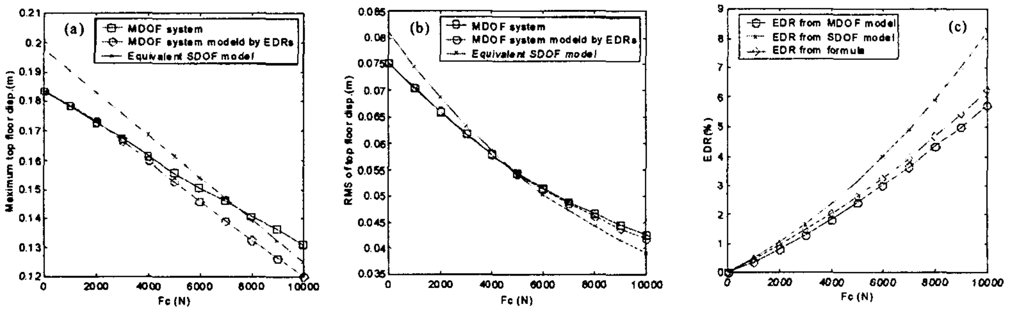
To verify the applicability of the proposed equivalent SDOF model for a MODF system with damping device and closed form formulas for equivalent damping ratio, analysis are performed on a ten-story shear building. LVD, FD, and AMD using LQR or bang-bang control law are used as damping devices. LVD and FD are installed at every inter story of the building and AMD is installed on top floor and the angles between LVDs or FDs and associated floors are zero. Each story unit of the building has identical lumped mass of 100ton; story stiffness of $k_{i=1,\ldots,4}=15000\,\mathrm{kN}$, $k_{i=5,6,7}=10500\,\mathrm{kN}$ and $k_{i=8,9,10}=7350\,\mathrm{kN}$. Damping matrix is composed from modal damping ratios of all modes to be 0.02. Similarly in the previous analysis, white noise is used input earthquake excitation.

Maximum displacements and RMS of the top floor are obtained for comparison by the proposed equivalent SDOF model, the ten-story shear building model with damping devices(denoted as MDOF system), and the ten-story shear building model with equivalent damping ratios , (denoted as MDOF system by EDRs). Also, equivalent damping ratios are compared by eigenvalue analysis for the ten-story shear building model with damping devices, the proposed method for the ten-story shear building model, (denoted as EDR from MDOF model), the proposed equivalent SDOF model with the equivalent damping ratios, (denoted as EDR from SDOF model), and finally equivalent damping ratios by formulas shown in Eqs.(15), (16), (17) and (20)



(a)Maximum Top Floor Displ. (b)RMS of Top Floor Displ. (c) First Modal Damping Ratio

Figure 1. Ten-story Building with AMD using bang-bang control law



(a) Maximum Top Floor Displ. (b) RMS of Top Floor Displ. (c) First Modal Damping Ratio

Figure 2. Ten-story Building with FD

Figures 1 and 2 represent the results for the building with AMD using bang-bang control law and FDs, respectively. As these two damping devices generate maximum control forces, regardless of response magnitude, their control effects are similar. Figures 1(a) and (b) indicate that the equivalent damping ratios proposed in this study represent accurately the control effects of FDs or AMD using bang-bang on RMS and maximum responses. Especially, RMS response is closer to real value, which is obtained for integrated MDOF model, than maximum response. As control force limit u_{max} or F_c increases, the discrepancy between MDOF system modeled by equivalent damping ratios and MDOF system, which appears in Fig.1(a), becomes large. It means that the equivalent damping ratio for MDOF system overestimates the control effects of FD or AMD for large force limit. Compared with the first equivalent damping ratio by MDOF system, equivalent damping ratios given by equivalent SDOF model in Figure 1(c) and 2(c) are overestimated, which is obvious particularly for large u_{max} or F_c. This tendency is compensated by the closed form formula given by Eq.. The reason why Eq. compensates the equivalent damping ratio by equivalent SDOF model is that Eq. for response variation due to variation of damping ratio conservatively expresses the control effect of damping on response reduction. Therefore, for a conservative design of a structure with Coulomb friction-type damping devices, Eq. is recommended at the stage of preliminary design.

It is noted from Figs. 1 and 2 that the control effect of single AMD on top floor with the same maximum control force as one FD corresponds to that of ten FDs which are installed at every inter story. This is because single AMD generates control force of which sign is opposite to the relative velocity of top floor to the base floor while one FD makes control force with respect to the relative velocity between both ends of the device. AMD on top floor can be interpreted to have larger controllability than that of one FD installed at inter story and the proposed method can reflect this fact.

5. CONCLUSION

The objective of this study is to find simple method for evaluating equivalent damping ratios of a MDOF structure equipped with supplemental damping devices and also, propose closed-form formulas of equivalent damping ratios to assess the control effect of various damping devices. The findings of the study can be summarized as follows:

Modal-energy-formed Lyapunov function for a structure with damping devices subjected to earthquake loadings is defined and its derivative is expressed in autoregressive form. And then, equivalent damping ratios are proposed with derived Lyapunov function and its derivative to assess the control effect of various damping devices quantitatively.

A MDOF system is transformed to an equivalent SDOF model and the control effect of damping device on fundamental mode is estimated by using the proposed method. Also, closed form formulas for the equivalent damping ratios are presented for linear and nonlinear damping devices. Through the numerical analysis of a ten-story building equipped with aforementioned damping devices, the effectiveness of equivalent SDOF model and closed form formulas are verified.

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REFERNECE

1. Lee SH, Min KW, Kang KS, and Hwang JS, Evaluation of Equivalent Damping Ratios of a Structure with Supplemental Vibration Control Device, 3rd World Conference on Structural Control, 7-12 April 2002, Como

Italy.

2. Building Seismic Safety Council, NEHRP guidelines for the seismic rehabilitation of building. FEMA 273, Washing, D.C.1997.

3. Associate Committee on National Building Code, National Building Code of Canada 1955, National Research Council, Ottawa, 1995.

4. Fu Y, Kasai K, Comparative Study of Frames Using Viscoelastic and Viscous Dampers, *Journal of Structural Engineering*, ASCE 1998,124(5), 513-522.