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반복 학습법에 의한 비선형 계의 입력신호 재현

Input signal reconstruction for nonlinear systems using iterative learning procedures

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Key words : Coherence, iterative learning, nonlinear damper, power spectral density, signal reconstruction, stability

ABSTRACT

This paper demonstrates the reconstruction of input signals from only the measured signal for the simulation and endurance test of automobiles. The aim of this research is concerned with input signal reconstruction using various iterative learning algorithm under the condition of system characteristics. From a linear to nonlinear systems which provides the output signals are estimated in this algorithm which is based on the frequency domain. Our concerns are that the algorithm can assure an acceptable stability and convergence compared to the ordinary iterative learning algorithm. As a practical application, a ¼ car model with nonlinear damper system is used to verify the restoration of input signal especially with a modified iterative learning algorithm.

1. INTRODUCTION

When applying laboratory simulation testing, the test engineer is presented with the problem of reconstructing the service environment in the test laboratory from a set of field measurements (road test data). Since the true configuration of the dynamic behaviour of structure under test (to say 'the system') is not fully identifiable, the reconstruction relies on certain assumptions which must be made about the system. Using a separable set of signals fed to the system, the system is tested to yield an estimate of its linear characteristics, i.e., its frequency response function [1]. This linearly assumed system's frequency response function (hereafter 'estimate of system') is to be used to reconstruct the true excitation in the service environment by comparing the signals measured in the test of laboratory with those from the field measurements [2], [3].

When the system under consideration is purely linear, the iterative learning procedure is only affected by the accuracy of the estimated frequency response function of the system. However, in practical situations, several types of nonlinearity exist in any mechanical structures, which are usually poorly known. Also these nonlinearities increase with wear and tear, and change from component to component [4], [5], [6], [7].

The aim of this research is concerned with input signal reconstruction using various iterative learning algorithm under the condition of nonlinear system characteristics.

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Our concerns are that the inverse can exist and stable in applying to the reconstruction.

The underlying theoretical concerns are the adequacy of various models of the nonlinear system, particularly by local linear models, with respect to the convergence of these iterative learning algorithms.

2. Iterative learning algorithm for input estimation

In order to understand how the iterative learning algorithm can be applied to simulator control signal generation, a brief review of linear system analysis and its stability condition for the iterative input estimation will help. The information that is needed to construct the true input signal, which is directly related to the control signal, is the relationship between the input and output of a linear system. Suppose an unknown signal $x_p(t)$ is acting on any linear system $h(\tau)$ producing an output signal $y_d(t)$ (desired output signal) as shown in the following figure;

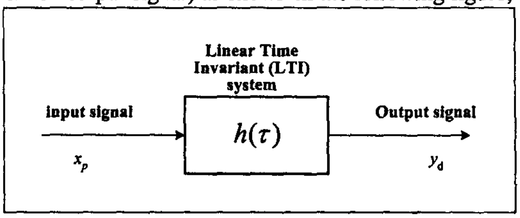


Figure 2.1 Acquisition of desired signal

The problem addressed here is to estimate the unknown input signal from the output and the estimate of the system is linear response. This input reconstruction is achieved in an iterative manner that reducing the error between the desired output and test output obtained from arbitrarily selected input signal excitation.

To be more specific, in the first stage, the system is driven by a test input x_c to give test output y_c, then the system's frequency response is calculated as,

$$\hat{G}(f) = \frac{S_{x_c y_c}(f)}{S_{x_c x_c}(f)} \tag{2.1}$$

Using the frequency response obtained from equation (2.1), the first input x_1, is calculated in the frequency domain as

$$X_1 = \frac{\alpha}{\hat{G}} Y_d \tag{2.2}$$

where Y_d is the desired output signal in frequency domain and α is a gain factor (constant).

Hence, the above relationship may be expressed in general form at the i-th iteration step

$$X_{i+1} = X_i + \frac{\alpha}{\hat{G}} E_i \tag{2.3}$$

and the error in each frequency at every iteration step will be,

$$E_i(f) = Y_d(f) + G(f) \cdot X_i(f) \tag{2.4}$$

where $G(f)$ is the true frequency response of the system.

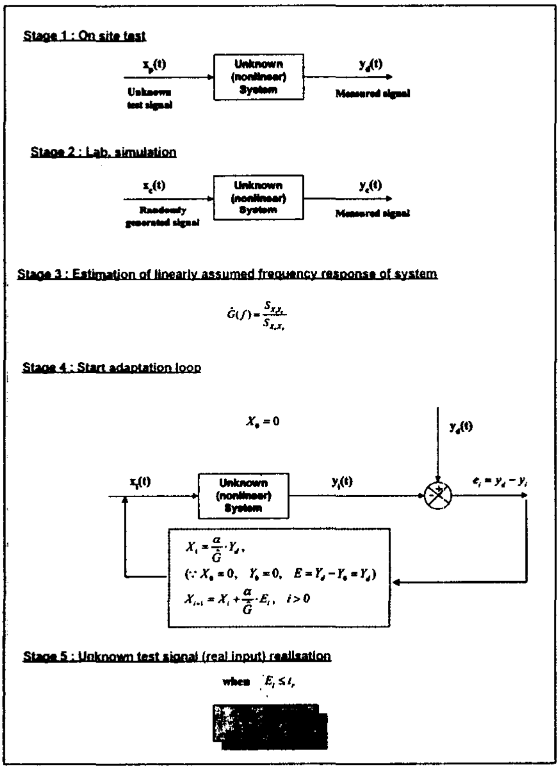


Figure 2.2 Iterative learning process for input signal reconstruction

3. Input estimation for nonlinear system

Actuator and sensor nonlinearities are among the key factors limiting the performance of input signal reconstruction. In this section, we applied the iterative learning algorithm to the systems which have nonlinearities in their nature.

3.1 Stability of the iterative learning algorithm considering the nonlinearity of a system

This section considers the stability of iterative learning algorithm when the system is non-linear. By representing the degree of non-linearity by a single parameter, the region of stability relating the degree of non-linearity and gain factor can be investigated.

The following figure illustrates the relationship between the signals for an iterative learning algorithm in which x denotes the input signal, y is the output of a certain non-linear system, d is the desired signal (field measured output of the non-linear system), and e represents the error signal between the desired and output signals. As a general non-linear expression, we assume the non-linear behaviour of the unknown system depends on the cubed term of input signal by which the degree of non-linearity can be expressed by a constant multiplier (denoted by δ) of the cubed term.

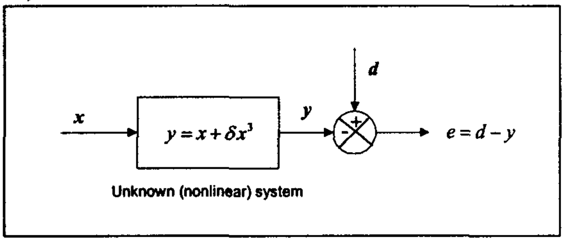


Figure 3.1 Signals and non-linear system used in the iterative learning procedure

Using the relationship of Figure 3.1, the adaptation equation takes the form of

$$x_{i+1}(n) = x_i(n) + \alpha e_i(n), \quad \text{i.e. } \hat{G} = 1 \tag{3.1}$$

in which the error is written as,

$$e_i(n) = d(n) - x_i(n) - \delta x_i^3(n) \tag{3.2}$$

Hence,

$$x_{i+1}(n) = x_i(n) - \alpha\left(x_i(n) + \delta x_i^3(n) - d(n)\right) \tag{3.3}$$

Let $x_{opt} + \delta x_{opt}{}^3 = d$, and substituting x_{opt} from both side of (3.3) becomes

$$\left(x_{i+1}(n) - x_{opt}\right) = \left(x_i(n) - x_{opt}\right) - \alpha\left(x_i(n) + \delta x_i^3(n) - x_{opt} - \delta x_{opt}{}^3\right) \tag{3.4}$$

Normailsing the input signal and convergence gain factor $\tilde{x}_i(n) = x_i(n) - x_{opt}$ and

$$\tilde{\alpha}_i(n) = \alpha\left[\frac{x_i(n) + \delta x_i^3(n) - x_{opt} - \delta x_{opt}{}^3}{x_i(n) - x_{opt}}\right]$$ to make

equation (3.4) as

$$\tilde{x}_{i+1}(n) = \left(1 - \tilde{\alpha}_i(n)\right)\tilde{x}_i(n) \tag{3.5}$$

Note that the normalised adaptation can be written as $\tilde{\alpha}_i(n) = \alpha\left[1+\delta\left(\frac{x_i^3(n)-x_{opt}^{\ 3}}{x_i(n)-x_{opt}}\right)\right]$.

From this, if $x_i(n)=0$, then

$$\tilde{\alpha}_i(n) = \alpha\left[1+\delta x_{opt}^{\ 2}\right] \quad (3.6)$$

and if $x_i(n) = x_{opt}+\Delta,\ \Delta \ll x_{opt}$, then $x_i^3(n) = \left(x_{opt}+\Delta\right)^3 \approx x_{opt}^{\ 3}+3\Delta x_{opt}^{\ 2}$, so that the normalised convergence coefficient becomes

$$\tilde{\alpha}_i(n) = \alpha\left[1+3\delta x_{opt}^{\ 2}\right] \quad (3.7)$$

From this, the algorithm becomes unstable when

$$\alpha\left[1+3\delta x_{opt}^{\ 2}\right] > 1 \quad (3.8)$$

This equation is exact if x and d are slow-varying dc levels and is true on average if x and d are randomly varying and the mean square value of x_{opt} is used in equation (3.8). Thus, the upper limit of the gain factor for stable convergence becomes

$$\alpha < \frac{1}{1+3\delta x_{opt}^{\ 2}} \quad (3.9)$$

As an example, if we choose δ to be 1.0 and $x_{opt}^{\ 2}$ is taken to be the variance of the signal and assumed to be unity, then the algorithm retains stable for $\alpha \le 0.25$ but when α is taken as 0.5 and $\delta = 1.0$, then the algorithm is driven into the instable condition.
We also notice that the coherence is degraded for a fixed value of δ ($\delta = 1$), as the variance of the input signal, $\sigma_x^{\ 2}$, increases.

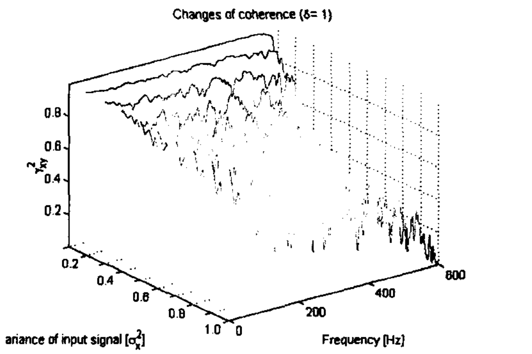


Figure 3.2 Change of coherence varying the variance of input with fixed δ ($\delta = 1$)

It is thus intuitive that by monitoring the coherence function we can monitor the degree of non-linearity in the system experienced by the iterative learning algorithm [2]. In the next Section the form of the coherence function is derived for a general memoryless nonlinearity.

3.2 Analytical form of coherence function

Following figure illustrates an input-output process with parallel liner and nonlinear co-existent system.

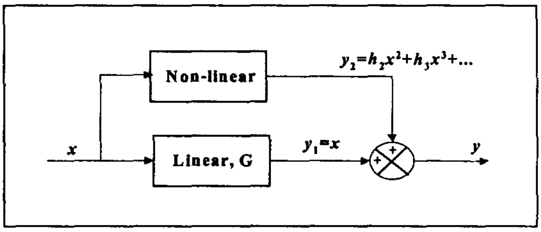


Figure 3.3 A simple parallel non-linear system

The input output relationship can be expressed as

$$y(n) = y_1(n) + y_2(n) \quad (3.10)$$

where $y_1(n) = \sum_{\tau=-\infty}^{\tau=+\infty} h_1(\tau)x(n-\tau)$

and

$$y_2(n) = \sum_{i=2}^{\infty}\sum_{\tau_1=-\infty}^{\tau_1=+\infty}\sum_{\tau_2=-\infty}^{\tau_2=+\infty}\cdots\sum_{\tau_n=-\infty}^{\tau_n=-\infty} h_i(\tau_1,\tau_2,\ldots,\tau_n)x(n-\tau_1)x(n-\tau_2)\cdots x(n-\tau_n)$$

For simple example, however, we have selected the nonlinear system which is represented as

$$y_2(n) = \delta_1\{x(n)\}^2 + \delta_2\{x(n)\}^3 + \varepsilon\left(x(n)\right)^k,\quad k>3 \quad (3.11)$$

as such the nonlinear equation given in (3.11) limits the higher order nonlinearity up to 3rd order and the contributions beyond which are considered to be trivial.
The coherence between the input and output is defined to be

$$\gamma_{xy}^{\ 2} = \frac{\left|S_{xy}\right|^2}{S_{xx}S_{yy}} \quad (3.12)$$

where

$$\begin{aligned} S_{xy} &= E\{X^*Y\} \\ &= E\{X^*(Y_1+Y_2)\} \\ &= E\{X^*Y_1\}+E\{X^*Y_2\} \end{aligned} \quad (3.13)$$

and

$$\begin{aligned} S_{yy} &= E\{(Y_1+Y_2)^*(Y_1+Y_2)\} \\ &= E\{Y_1^*Y_1\}+E\{Y_2^*Y_1\}+E\{Y_1^*Y_2\}+E\{Y_2^*Y_2\} \end{aligned} \quad (3.14)$$

Since we have assumed that the general non-linear system can be expressed by the relationship given in equation (3.11), the cross spectral density can be expressed using the relationship by the Bussgang process as [9], [10]

$$S_{xy} = S_{xy_1} + 3\delta_2\sigma_x^2 S_{xx} \quad (3.15)$$

The equation (3.12) becomes

$$\gamma_{xy}^2 = \frac{|S_{xy}|^2}{S_{xx}S_{yy}}$$

$$= \frac{|S_{xy_1} + S_{xy_2}|^2}{S_{xx}(S_{y_1y_1} + S_{y_1y_2} + S_{y_2y_1} + S_{y_2y_2})} \quad (3.16)$$

$$\leq \frac{|S_{xy_1}|^2}{S_{xx}S_{y_1y_1}}$$

in which the equality holds when $\delta_2 = 0$. The inequality relationship given in (3.16) implies that the coherence of the nonlinear system is always less than that of the linear system.

For further detailed non-linear system identification, there can be various ways such as the Higher Order Spectra (e.g. Volterra series expansion) [11], [12], [13] that can deal with the stability of the iterative learning algorithm. In this study, however, the degree of non-linearity is monitored by simple parameter as the cost of iterative learning procedure escalates by implementing further complicated non-linear system identification.

3.3 Further modification of iterative learning process

Apart from the ordinary iterative learning process considered so far, we have introduced a modified iterative learning (IL) process, which takes into account the error signal and system characteristics at each iteration step, to overcome problems encountered in simulating the control of nonlinear systems. This modified IL process has three components.

3.3.1 Estimation using adaptive gain factor application

In the ordinary IL process, the selection of gain factor becomes crucial as the magnitude effects on the stability of the IL process. However, the gain factor is only selected empirically, which may cause unexpected problems in the algorithm if the system is nonlinear as described above. To avoid this, one could let the gain factor change its values in accordance of the coherence function in each frequency. The relationship given equation (3.16) implies that the gain factor can be selected following the change of coherence function in each iteration of the algorithm. To be more specific, we modify the gain factor to tackle the stability of the algorithm in view of the coherence as

$$\alpha_{\text{mod}} = \alpha_0 \cdot \gamma_{xy}^2, \quad 0 < \text{fixed value } \alpha_0 \leq 1 \quad (3.17)$$

where $\gamma_{xy}^2 = \frac{|S_{xy}|^2}{S_{xx} \cdot S_{yy}}$ denotes the coherence function.

By considering the status of input and output signal in each iteration (using the coherence function), this equation then suppresses the instability caused by the improperly selected gain factor automatically.

3.3.2 Instantaneous frequency response function

Instead of using the identified frequency response of system (denoted $\hat{G}$) under test expressed in equation (2.4), the input signal reconstruction is given

$$X_{i+1} = X_i + \frac{\alpha}{\hat{G}_i}(Y_d - Y_i) \quad (3.18)$$

where the estimate of the frequency response function is estimated from the current input and output signals - depending on the status of the phase response of the system in each iteration- using coherence function in each iteration such that

$$\hat{G}_i = \frac{S_{x_iy_i}}{S_{x_ix_i}} \quad (3.19)$$

For a nonlinear system, the global estimation of frequency response is not always valid as the estimation is based on the linear assumption. Thus, it is recommended to employ the modified frequency response estimation for the input signal reconstruction, which is called 'instantaneous frequency response' estimator.

This frequency response application is used to ensure the rapid convergence of error function and stability in its convergence. Firstly, the merit of this estimation can be said that since the gain factor is varying with the status of the input and output signals in each iteration, γ_{xy}^2 provides an additional gain factor to reduce the error between $(Y_d - Y_i)$. Secondly, the instantaneous linearity can be achieved when we select the local gradient for nonlinear case.

3.3.3 Application of regularisation for inversion of frequency response function

Regularisation is used to ensure numerical stability in the inverse of the frequency response function, i.e.

$$\hat{G} = \frac{S_{xy} + \beta}{S_{xx}} \quad (3.20)$$

in which the small constant β is introduced to ensure the numerical instability of $\hat{G}$ when it is inverted.

Simulations on the inversion of frequency response function on input signal reconstruction have been performed with;

Sample length (N) : 8192 samples
Input signal : pink noise (with $\sigma_x^2 = 1.0$)
Sampling frequency (f_s) : 200 Hz
Window : Hanning 256 segments
Averaging : 50 % overlapping
Number of iterations : 10
System : 8th order Butterworth filter (Cut-off 50 Hz) (memoryless)

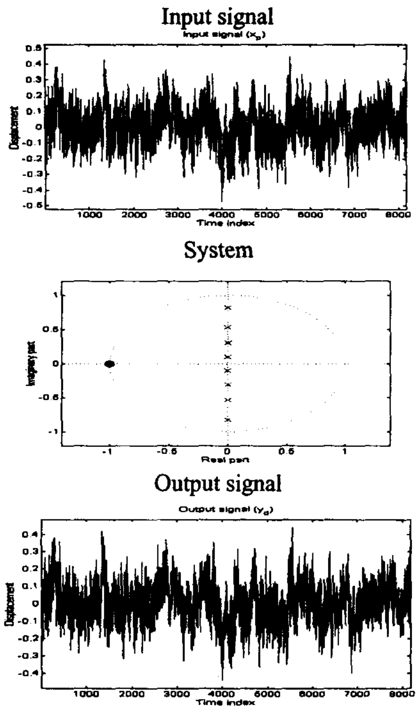


Figure 3.4 Input signal, low pass filter system (memoryless) and output signal

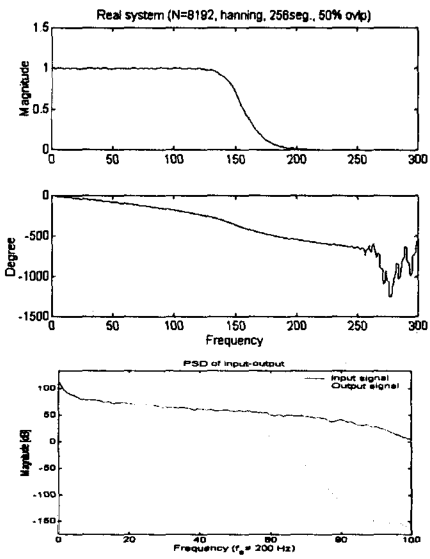


Figure 3.5 Magnitude and phase of the system and spectral density of input and out signal

As shown in Figure 3.5, the system is a kind of low pass filter which results the output of the system loses its spectral characteristics in high frequency region. In this case, when we invert the frequency response function, the numerical instability occurs.

To compare the performance of the ordinary and modified method in IL process, the term "error" used in this study is represented by MSE, which is defined as

$$\mathrm{MSE} = \sum \left(y_d - y_{a_i} \right)^2 \tag{3.21}$$

where y_d is the measured signal in time domain and y_{a_i} is the i-th output signal respectively.

These are demonstrated in the following figures.

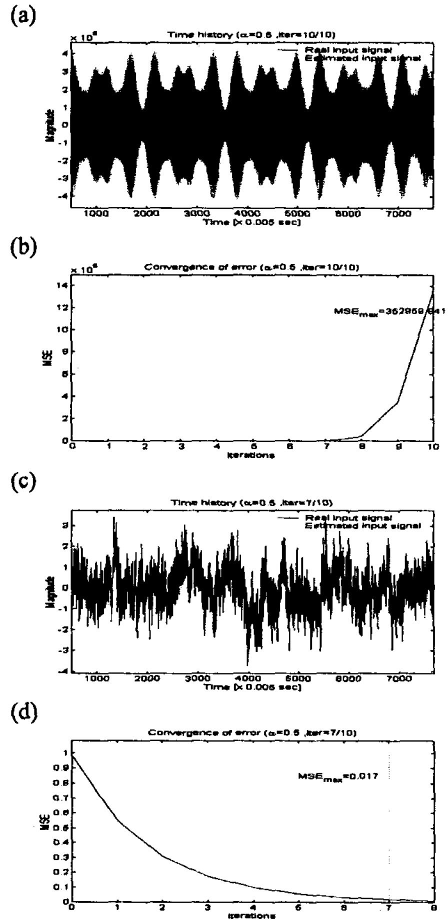


Figure 3.6 Results of input signal reconstruction from three different methods, (a) & (b) : restored signal and error from ordinary frequency response function inversion, (c) & (d) : restored signal and error from regularisation (from inversion of pre-whitened frequency response function)

The error (b) of Figure 3.6 clearly demonstrates the numerical instability of the IL process which use only the inversion of the estimated frequency response function as

$$X_{i+1} = X_i + \frac{\alpha}{\hat{G}} \cdot E_i \tag{3.22}$$

whereas the IL process with some constant addition (prewhitening)

$$X_{i+1} = X_i + \frac{\alpha}{\hat{G}(1+\sigma^2 I)} \cdot E_i \qquad (3.23)$$

The amount of constant addition is suggested by the Percentage of Pre-Whitening (PPW) and is given ($r_y(0)$ is the auto correlation of the output signal)

$$\text{PPW} = \frac{\sigma^2}{r_y(0)} \times 100 \qquad (3.24)$$

and normally, takes 0.5 □ 5% [14].

4. Input estimation for dynamic system with nonlinearity

This section deals with a practical problem of input signal reconstruction by taking a simplified car model with piece-wise nonlinear damper system. The input signal to be restored is the pink noise signal used in previous sections. A brief description of the nonlinear system and the comparison of the input signal construction by the ordinary IL process and modified process are described.

4.1 Input, system, output and analysis of dynamic system

For a practical application of 1/4 car model, nonlinear dynamic system configuration and input estimation processes are given as following;

Key parameters;

M=50kg, c=200N/m/sec, k=1800N/m

(f_n=0.955 Hz, ζ=0.333)

Nonlinear damping coefficients : c_1=300, c_2=100 (see right graph of Figure 4.1)

Sample length (*N*) : 8192

Sampling frequency (f_s) : 200 Hz

Window : Hanning 256 segments

Averaging : 50 % overlapping

Fixed gain factor (α_0) : 0.5

Number of iterations : 10

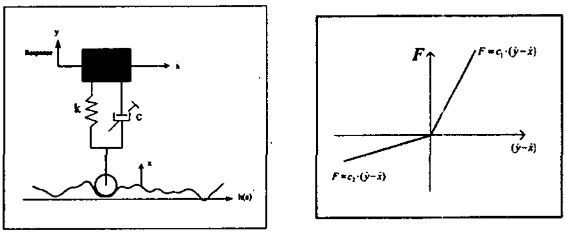


Figure 4.1 Car suspension system (non-linear damper)

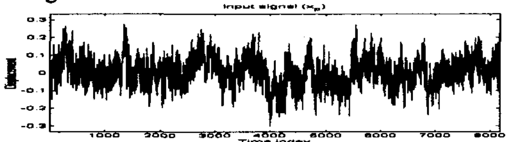
The dynamic behaviour of this system becomes

$$\begin{aligned} M\ddot{y} &= -k(y-x) - c(\dot{y}-\dot{x}) \\ &\text{when } \dot{y}-\dot{x} \ge 0, \quad c = c_1 \\ &\text{when } \dot{y}-\dot{x} < 0, \quad c = c_2 \end{aligned} \qquad (4.1)$$

In the simulation based on the above equation (4.1), the 4th order Runge-Kutta method has been used to solve the ordinary differential equation. The nonlinear damping coefficient takes either c_1 or c_2 depending on the sign of the relative velocity between the input (road profile) and that of the car body. In this manner, the damping term takes the partial linear behaviour depending on the relative velocity between the input and output.

Figure 4.2 depicts signals which represent simple 'vehicle with nonlinear damper system' running over spatially homogeneous rough ground.

Input signal



Output from non-linear damper

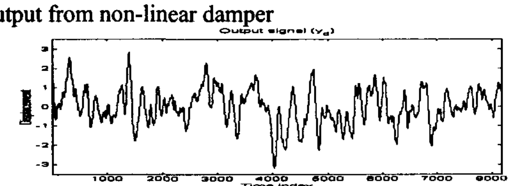


Figure 4.2 Input signal, output signals from non-linear damper car model

Simulation steps;

Step 1 : Any arbitrarily selected random white noise (x_c) is selected to excite the nonlinear car model given in (4.1) to obtain the output of the system (y_c)

Step 2 : The linearly assumed frequency response function of the system ($\hat{G}$) is estimated from the input and output.

Step 3 : The coherence function is estimated to use a modified gain factor for the iteration as such $\alpha_{mod} = \alpha_0 \cdot \gamma_{xy}^2$, $\alpha_0 = 1$.

Step 4 : The spectral differences between the output of the system and the desired output (y_d) as such $E_i = Y_d - Y_a$.

Step 5 : Estimation of frequency response function $\hat{G}$ is done for modified IL process.

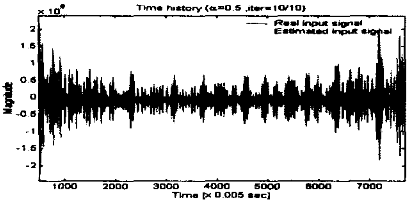
Step 6 : A new input signal is then calculated as such $X_{i+1} = X_i + \frac{\alpha}{\hat{G}} E_i$ (where α becomes α_0 or α_{mod} depending on the method selected).

Step 7 : Check the E_i with the threshold value.

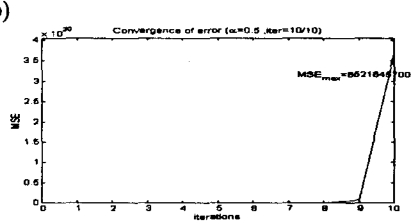
Step 8 : If the error E_i is not satisfactory, use the new input to excite the system and repeat from the step 3.

4.2 Results -nonlinear damper car model-

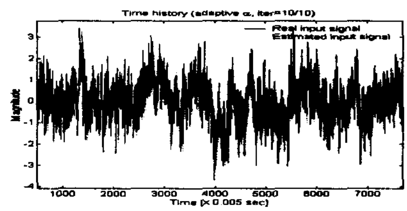
(a)



(b)



(c)



(d)

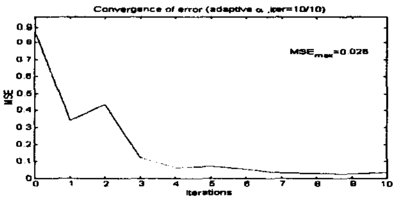


Figure 4.3 Results of input signal reconstruction from two different methods,(a) & (b) : restored input signal and errors in each iteration from standard iterative learning algorithm, (c) & (d) : from modified gain factor iterative learning algorithm.

As can be seen in Figure 4.3, for the nonlinear case, the input signal reconstruction by two different IL process clearly demonstrate the important role of unknown system's FRF estimation and gain factor selection.

5. Discussions and future development

The input signal reconstruction using the iterative learning algorithm has been investigated for linear and nonlinear systems. Some of conclusions and further development parts are listed below.

5.1 Discussions

- For linear case, the iterative learning algorithm yields satisfactory and robust results.
- For the simplified car model, using the instantaneous frequency response function, adaptive gain factor and the prewhitening for the inversion stability, the input signal reconstruction has been achieved more effectively, which leaves the following aspects to be further investigated.

5.2 Future development

- A full scale on-site experiment is requires to determine the validity of this process for the real system.
- There is a more general question as to whether an inverse for these nonlinear systems exist and whether it is unique.
- A nonlinear system identification could provide a more accurate input signal reconstruction, which may be achieved using higher order spectra, for example.

References

[1] J. F. Fletcher, *Sound and Vibration*, "Global Simulation: New Technique for Multiaxis Test Control", November, 1990.

[2] C. J. Dodds and A. R. Plummer, The Engineering Society For Advancing Mobility Land Sea Air and Space, "Laboratory Road Simulation for Full Vehicle Testing : A Review", SIAT 2001-SAE Conference, 2001-01-0047, Pune, INDIA, 10th-13th January 2001.

[3] S. J. Elliott, *IEEE* Transactions on Signal Processing, "Optimal controllers and adaptive controllers for multichannel feedforward control of stochastic disturbances",
Vol. 48, 1053-1060, 2000.

[4] Gang Tao and Petar V. Kokotovic, *Adaptive Control of Systems woth Actuator and Sensor Nonlinearities*, Wiley Inter-science, 1996.

[5] Darrel Recker and P.V. Kokotovic, *Proceedings of the 32nd IEEE Conference on Decision and Control*, "Indirect adaptive nonlinear control of discrete-time systems containing a dead-zone", pp 2647-2653, 1993.

[6] Darrel Recker, P.V. Kokotovic, D. Rhode and J. Winkleman, *Proceedings of the 30th IEEE Conference on Decision and Control*, "Adaptive nonlinear control of systems containing a dead-zone", pp 2111-2115, 1991.

[7] D. Recker, *Adaptive Control of Systems Containing piecewise Linear Nonlinearities*, Ph.D. Thesis, University of Illinois, Urbana, IL, USA, 1993.

[8] S. J. Elliott, *Signal Processing for Active Control*, Academic Press, 2001.

[9] S. A. Billings and S. Y. Fakhouri, "Identification of nonlinear systems using the Wiener model", *Electronics letters*, 18th August 1977, Vol. 13, No. 17.

[10] J. J. Bussgang, "Correlation Functions of Amplitude-Distorted Gaussian Signals", Tech. Rep. 216, MIT Research Laboratory of Electronics, Cambridge, 1952.

[11] M. Schetzen, *The Volterra and Wiener theories of nonlinear systems*, John Wiley & Sons, 1980.

[12] W. B. Collis, "Higher Order Spectra and their application to Nonlinear mechanical systems", PhD Thesis, ISVR, University of Southampton, UK, 1996.

[13] A. Isidori, *Nonlinear Control Systems, 3rd ed.*, Springer-Verlag, Berlin, 1995.

[14] P. M. Clarkson, *Optimal and Adaptive Signal Processing*, CRC Press Inc., 1993.