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Concept Optimization and Folded Plate Theory

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Key Words : concept optimization, folded plate theory, composite materials.

ABSTRACT

Almost all buildings/infrastructures made of composite materials are fabricated without proper design. Unlike airplane or automobile parts, prototype test is impossible. One cannot destroy 10 story buildings or 100-meter long bridges. People try to build 100-story buildings or several thousand meter long bridges. In order to realize "composites in construction", the following subjects must be studied in detail, for his design. Concept optimization, Simple method of analysis, Folded plate theory, Size effects in failure, and Critical frequency. Unlike the design procedure with conventional materials, his design should include material design, selection of manufacturing methods, and quality control methods, in addition to the fabrication method. In this paper, concept optimization and folded plate theory are presented for practicing engineers.

1. Introduction

The educational background of the majority of the construction engineers is the bachelor's degree. Even the engineers with higher degrees have very much difficulty in design/ analysis, with acceptable accuracy, of buildings/infrastructures made of, even, conventional materials. Buildings/bridges by the reinforced concrete/steel are three-dimensional structures made of composite materials, such as cement, steel bars, etc.

However, the engineers can design/analyze such structures by considering them made of one-dimensional beams/columns. But, they are protected by codes and specifications. Almost all buildings/infrastructures made of composite materials are fabricated without proper design. Unlike airplane or automobile parts, prototype test is impossible. One cannot destroy 10 story buildings or 100-meter long bridges. People try to build 100-story building or several thousand meter long bridges. In this paper, two subjects out of several other subjects, namely, concept optimization and folded plate theory are briefly explained.

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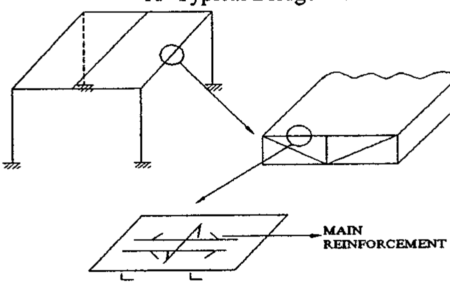
2. Concept Optimization

Modern materials engineering has produced numerous new structural materials, and the science of mathematical calculation and others related with structural analysis, and construction have reached near its zenith. It is necessary to develop or to redefine the new concept (or concepts) suitable for new materials. The senior author wished to call this the fifth basic concept of structures [1]. One calls it an advanced composite when controlled placement of reinforcements with optimum shape, amount, and direction, according to "accurate" analysis result, is made. This indicates that a composite material must be treated as a structure.

Very large portion of civil structures can be analyzed by considering them as frameworks of one-dimensional elements. Composite materials are, generally, strong in tension. When an element is designed based on tension load, it will have thin section, which is weak against any loading type other than in-plane on–axis tension load. This requires the section modulus increase by means of employing thin walled sections. The thin panels of such section are weak against the loads normal to these panels. The longitudinal stringers are added between transverse diaphragms to take care of such loads. The diaphragms transmit the loads from stringers to the walls of the beams by means of in-plane shear.

Even when the frames are analyzed as one-dimensional beams and columns, these one-dimensional elements are three dimensional structures made of thin walls, which are called as folded plates (shells). Thus, the analysis of structures made of composite materials, becomes that of folded plates, both prismatic or non-prismatic.

'A' Typical Bridge Frame



Total Section : Behaves like a Folded Plate Shell

Figure 1. Typical Bridge or Building Frame

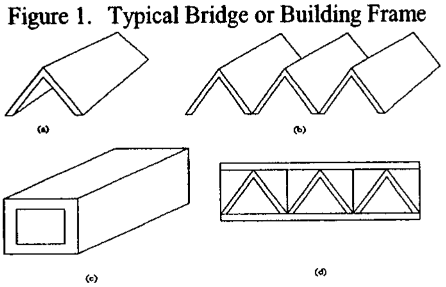


Figure 2. Prismatic Folded Plate Structures

3. Folded Plate Theory

Any curved surface can be considered as continuations of certain types of triangular plates. Therefore, the theory of non-prismatic folded plates can be applied to any type of shell structures [1,2,3,4]. Any three-dimensional structural configuration can be approximately represented with good accuracy by non-prismatic folded plates, which is composed of sectorial plates. Any sectorial plate problem in and normal to the plane forces can be solved by the finite difference method, finite element method and others. The problem then reduces to that of boundaries of two adjoining sectors. Each sector may be inclined. Fig. 3 and Fig. 4 show typical both upper and lower fold lines.

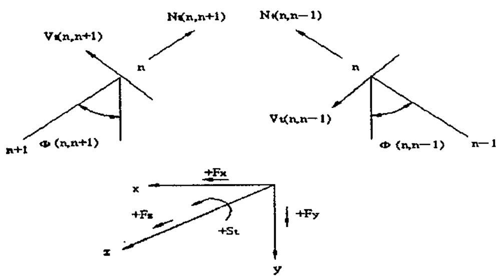
1. When n is at an upper fold line :

$$\begin{aligned}
F_{x(n,n+1)} &= -N_{t(n,n+1)}\sin\Phi_{(n,n+1)} + V_{t(n,n+1)}\cos\Phi_{(n,n+1)},\\
F_{y(n,n+1)} &= -N_{t(n,n+1)}\cos\Phi_{(n,n+1)} - V_{t(n,n+1)}\sin\Phi_{(n,n+1)},\\
F_{x(n,n-1)} &= N_{t(n,n-1)}\sin\Phi_{(n,n-1)} + V_{t(n,n-1)}\cos\Phi_{(n,n-1)},\\
F_{y(n,n-1)} &= -N_{t(n,n-1)}\cos\Phi_{(n,n-1)} + V_{t(n,n-1)}\sin\Phi_{(n,n-1)}, \qquad (1)\\
D_{x(n,n+1)} &= v_{(n,n+1)}\sin\Phi_{(n,n+1)} + w_{(n,n+1)}\cos\Phi_{(n,n+1)},\\
D_{y(n,n+1)} &= v_{(n,n+1)}\cos\Phi_{(n,n+1)} - w_{(n,n+1)}\sin\Phi_{(n,n+1)},\\
D_{x(n,n-1)} &= v_{(n,n-1)}\sin\Phi_{(n,n-1)} + w_{(n,n-1)}\cos\Phi_{(n,n-1)},\\
D_{y(n,n-1)} &= -v_{(n,n-1)}\cos\Phi_{(n,n-1)} + w_{(n,n+1)}\sin\Phi_{(n,n-1)}
\end{aligned}$$

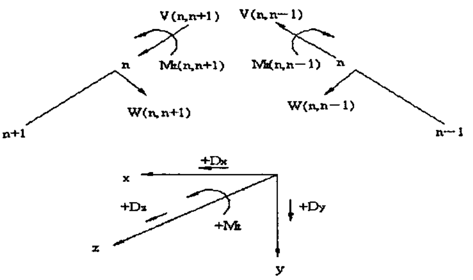
where. $N_t = \sigma_t h$

2. When n is at a lower fold line :

$$\begin{aligned}
F_{x(n,n+1)} &= -N_{t(n,n+1)}\sin\Phi_{(n,n+1)} - V_{t(n,n+1)}\cos\Phi_{(n,n+1)},\\
F_{y(n,n+1)} &= N_{t(n,n+1)}\cos\Phi_{(n,n+1)} - V_{t(n,n+1)}\sin\Phi_{(n,n+1)},\\
F_{x(n,n-1)} &= N_{t(n,n-1)}\sin\Phi_{(n,n-1)} - V_{t(n,n-1)}\cos\Phi_{(n,n-1)},\\
F_{y(n,n-1)} &= N_{t(n,n-1)}\cos\Phi_{(n,n-1)} + V_{t(n,n-1)}\sin\Phi_{(n,n-1)}, \qquad (2)\\
D_{x(n,n+1)} &= v_{(n,n+1)}\sin\Phi_{(n,n+1)} + w_{(n,n+1)}\cos\Phi_{(n,n+1)},\\
D_{y(n,n+1)} &= -v_{(n,n+1)}\cos\Phi_{(n,n+1)} + w_{(n,n+1)}\sin\Phi_{(n,n+1)},\\
D_{x(n,n-1)} &= v_{(n,n-1)}\sin\Phi_{(n,n-1)} - w_{(n,n-1)}\cos\Phi_{(n,n-1)},\\
D_{y(n,n-1)} &= v_{(n,n-1)}\cos\Phi_{(n,n-1)} + w_{(n,n+1)}\sin\Phi_{(n,n-1)}.
\end{aligned}$$

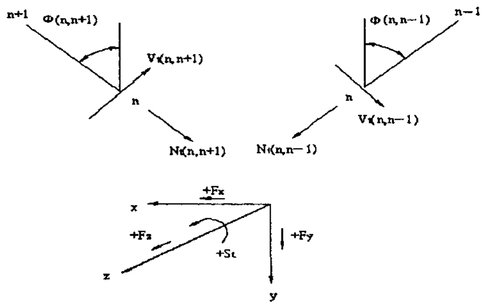


Forces

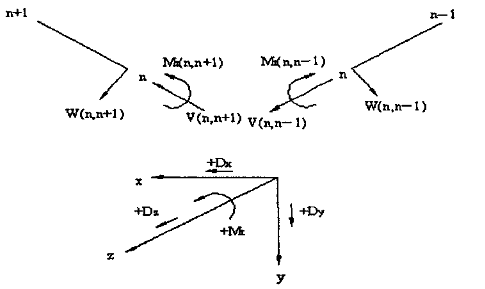


Displacements

Figure 3. Notations and Sign Conventions at the Upper Fold Lines



Forces



Displacements

Figure 4. Notations and Sign Conventions at the Lower Fold Line

The types of the joint compatibility and joint equilibrium conditions depend on which dependent variables are chosen. If the transverse bending moment, M_t, and the three displacement components, u, v, and w, are taken as unknowns, it is necessary to satisfy the slope compatibility condition and the three force equilibrium conditions at each fold as follows.

$$S_{t(n,n+1)} - S_{t(n,n+1)} = 0,$$
$$F_{xy(n,n+1)} + F_{xy(n,n-1)} = 0,$$
$$F_{x(n,n+1)} + F_{x(n,n-1)} = 0,$$
$$F_{y(n,n+1)} + F_{y(n,n-1)} = 0, \qquad (3)$$

where $F_{xy} = \tau_{r\theta} h = N_{rt}$.

Since these 'force' expressions are to be written in terms of displacements, the compatibility conditions are automatically satisfied. At each fold line, these conditions must be satisfied when the governing differential equations are integrated. For anisotropic materials three force equilibrium equations in terms of three displacement components, u, v, w, when the transverse sheer deformations are negligible, are as follows.

$$A_{11}\frac{\partial^2 u}{\partial x^2} + 2A_{16}\frac{\partial^2 u}{\partial x \partial y} + A_{66}\frac{\partial^2 u}{\partial y^2} + A_{16}\frac{\partial^2 v}{\partial x^2} + (A_{12} + A_{66})\frac{\partial^2 v}{\partial x \partial y} + A_{26}\frac{\partial^2 v}{\partial y^2}$$
$$- B_{11}\frac{\partial^3 w}{\partial x^3} - 3B_{16}\frac{\partial^3 w}{\partial x^2 \partial y} - (B_{12} + 2B_{66})\frac{\partial^3 w}{\partial x \partial y^2} - B_{26}\frac{\partial^3 w}{\partial y^3} = 0 \qquad (4)$$

$$A_{16}\frac{\partial^2 u}{\partial x^2} + (A_{12} + A_{66})\frac{\partial^2 u}{\partial x \partial y} + A_{26}\frac{\partial^2 u}{\partial y^2} + A_{66}\frac{\partial^2 v}{\partial x^2} + 2A_{26}\frac{\partial^2 v}{\partial x \partial y} + A_{22}\frac{\partial^2 v}{\partial y^2}$$
$$- B_{16}\frac{\partial^3 w}{\partial x^3} - (B_{12} + 2B_{66})\frac{\partial^3 w}{\partial x^2 \partial y} - 3B_{26}\frac{\partial^3 w}{\partial x \partial y^2} - B_{22}\frac{\partial^3 w}{\partial y^3} = 0 \qquad (5)$$

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} - B_{11}\frac{\partial^3 u}{\partial x^3} - 3B_{16}\frac{\partial^3 u}{\partial x^2 \partial y}$$
$$- (B_{12} + 2B_{66})\frac{\partial^3 u}{\partial x \partial y^2} - B_{26}\frac{\partial^3 u}{\partial y^3} - B_{16}\frac{\partial^3 v}{\partial x^3} - (B_{12} + 2B_{66})\frac{\partial^3 v}{\partial x^2 \partial y} - 3B_{26}\frac{\partial^3 v}{\partial x \partial y^2} - B_{22}\frac{\partial^3 v}{\partial y^3} = q(x, y) \qquad (6)$$

where A_{ij}, B_{ij} and D_{ij} are stiffnesses.

Many laminates with certain orientations have decreasing values of B_{ij}, D_{16} and D_{26} as the number of laminae increases. In such cases the following three equations can be used instead of the above three equations[8].

$$A_{11}\frac{\partial^2 u}{\partial x^2} + (A_{12} + A_{66})\frac{\partial^2 v}{\partial x \partial y} + A_{66}\frac{\partial^2 u}{\partial y^2} = -X \qquad (7)$$

$$A_{66}\frac{\partial^2 v}{\partial x^2} + (A_{12} + A_{66})\frac{\partial^2 u}{\partial x \partial y} + A_{22}\frac{\partial^2 v}{\partial y^2} = -Y \qquad (8)$$

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = q(x, y) \qquad (9)$$

Any coordinate system can be used depending on the geometry of the structure. For non-prismatic folded plate structures, one of the useful methods is to use the polar coordinates. In such case, above three equations are transformed to the following equations.

$$\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{r \partial r} - \frac{u}{r^2} + \frac{(1-\nu)\partial^2 u}{2r^2 \partial \theta^2} - \frac{(3-\nu)\partial v}{2r^2 \partial \theta} + \frac{(1+\nu)\partial^2 v}{2r \partial r \partial \theta} = -\frac{(1-\nu^2)}{E}K_r \qquad (10)$$

$$\frac{(1-\nu)}{2}\left(\frac{\partial^2 v}{\partial r^2} + \frac{\partial v}{r \partial r} - \frac{v}{r^2}\right) + \frac{\partial^2 v}{r^2 \partial \theta^2} + \frac{(3-\nu)\partial u}{2r^2 \partial \theta} + \frac{(1+\nu)\partial^2 u}{2r \partial r \partial \theta} = -\frac{(1-\nu^2)}{E}K_\theta \qquad (11)$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 w}{\partial r^2} + \frac{\partial w}{r \partial r} + \frac{\partial^2 w}{r^2 \partial \theta^2}\right) = \frac{q_p}{D} \qquad (12)$$

If the finite difference method is used to integrate the differential equations, some elaborate work is necessary. A very high degree of accuracy can be obtained by this method [3,4]. With the method of analysis as mentioned above available the problem is reduced to solving a plate, either prismatic or non-prismatic with arbitrary elastic boundary conditions.

4. Numerical Example

4.1 Non-Prismatic Folded Plates

As an example, the structure in Fig.5 is considered.

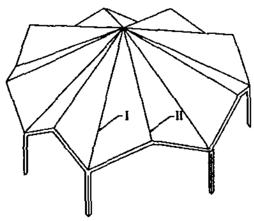


Figure 5. Perspective View of the Shell

This shell is under symmetric vertical load. Finite difference method is used for analysis. After very lengthy calculation, the stresses are obtained, and the general configuration of important stresses are as given in Fig.6 and Fig.7.

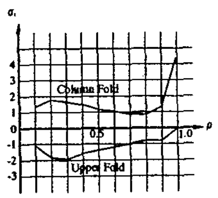


Figure 6. σ_r Stresses at the Fold Line

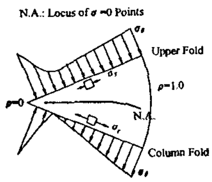


Figure 7. In-plane Stress Distribution

4.2 Prismatic Folded Plates

The structure considered is given in Fig. 8. The stiffnesses are as given in Table 1. Finite difference method is used. The results by folded plate theory and beam theory are compared in Table 2.

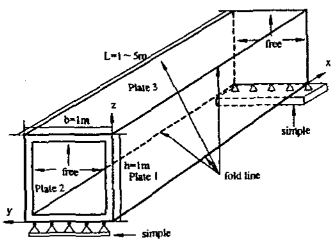


Figure 8. Composite Laminated Folded Plate Structure

Table 1. Stiffnesses of Laminated Plate

Extensional stiffness		Flexural stiffness	
(N/m)		(N · m)	
A_{11}	506091648.8697	D_{11}	56824.503261
A_{22}	710721888.5885	D_{22}	57250.822503
A_{12}	362768463.4065	D_{12}	42103.137633
A_{66}	376345429.8028	D_{66}	43489.119558

Table 2. Deflection at the Center of Lower Fold Line(x=L/2) by Beam Theory and Folded Plate Theory

Aspect Ratio	2	1	3	4	5
Folded Plate Theory	4.79 E-4	2.31E-5	2.84 E-3	9.50E-3	2.37 E-2
Beam Theory	6.18 E-4	3.86E-5	3.13E-3	9.88E-3	2.41E-2
R	0.78	0.599	0.907	0.961	0.983

Aspect Ratio = L/h

$$Ratio = \frac{Result\ of\ Folded\ Plate\ Theory}{Result\ of\ Beam\ Theory}$$

5. Conclusion

In this paper, concept optimization and folded plate theory are briefly explained in order to help engineers to design safe and sound, and yet, economical structures. Unlike airplane or automobile parts, prototype tests for buildings and bridges are impossible. Nevertheless, almost all buildings/infrastructures made of composite materials are fabricated without proper design. Design/analysis of such structure is simply too difficult for most of the engineers.

Reference

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