관 구조물 파괴에서의 크기효과

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Size Effect in Failure of Tube Structure

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Key Words : concept optimization, size/scale effects, failure analysis.

ABSTRACT

Almost all buildings/infrastructures made of composite materials are fabricated without proper design. Unlike airplane or automobile parts, prototype test is impossible. One cannot destroy 10 story buildings or 100-meter long bridges. People try to build 100-story buildings or several thousand meter long bridges. In order to realize "composites in construction", the following subjects must be studied in detail, for his design. Concept optimization, Simple method of analysis, Folded plate theory, Size effects in failure, and Critical frequency. Unlike the design procedure with conventional materials, his design should include material design, selection of manufacturing methods, and quality control methods, in addition to the fabrication method. In this paper size/scale effects in failure criteria is briefly explained for practicing engineers.

1. Introduction

The educational background of the majority of the construction engineers is the bachelor's degree. Even the engineers with higher degrees have very much difficulty in design/ analysis, with acceptable accuracy, of buildings/infrastructures made of, even, conventional materials. Buildings/bridges by the reinforced concrete/steel are three-dimensional structures made of composite materials, such as cement, steel bars, etc. However, the engineers can design/analyze such structures by considering them made of one-dimensional beams/columns. But, they are protected by codes and specifications. Almost all buildings/infrastructures made of composite materials are fabricated without proper design. Unlike airplane or automobile parts, prototype test is impossible. One cannot destroy 10 story buildings or 100-meter long bridges. People try to build 100-story buildings or several thousand meter long bridges.

In this paper, size/scale effects in failure of composite material structures out of several other subjects are briefly explained.

2. Size/Scale Effects in the Failure of Composite Structures

Size effects influence the material properties of quasi-brittle materials (e.g. concrete and rocks). In case of any material, the larger the volume the greater is the probability of larger flaws. More recently, the mechanics of materials were studied at various scales ranging from atomic scale to microns to large macro or structural behavior. It has been known that linear elastic fracture mechanics (LEFM) applied to laboratory size quasi-brittle materials underestimates fracture toughness. Classical LEFM technique may underestimate the true

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toughness of certain quasi-brittle materials such as geo-materials by as much as an order of magnitude, especially for those with large scale heterogeneities, and using typical laboratory size specimens. The question remains as to how laboratory tests could produce a toughness value closer to the in-situ true fracture toughness. We can either build a huge laboratory and test huge specimens: or we can abandon the concept of LEFM. In composite structures reasonable theory of size/scale effects on the failure mechanism is still lacking. Reduction in fiber strength is experienced when the size of the structures fiber bundle increases.

An efficient method to characterize the relationship between strength distribution and size in composites is not complete yet. It has been known that large composites are generally weaker than small composites. There could be several reasons for such phenomenon. One of the most important causes is the scale effect in brittle reinforcing fibers. Brittle fibers are generally strong and uniform in diameter but have the possibility of containing flaws with different strength. A longer fiber may have more of such possibility than a short fiber.

Based on the experience of a composite manufacturing specialist, the rate of decrease of tensile strength of glass fibers used for filament wound tubes as the mass of fibers increases is as shown in Fig. 1. From the test result reported by Crasto and Kim [8], an approximate relation between 90° tensile strength reduction rate, y, and the volume (proportional to the mass), for the unidirectional composites of AS4/3501-6, can be expressed as Fig.2.

Unless there is the test result for the same matrix to be used, this result for epoxy can be used to estimate the rate of the decrease of 90° tensile strength. For each of the constituent materials, both fibers and matrices, the rates of decrease of strengths, X, X', Y, Y', and S, as the mass increases, must be obtained in the future. The manufacturing method and other possible factors also have to be considered.

Any strength theory can be used with "reduced" strength as given above.

2.1 Maximum Strength Theory

Jenkins extended the concept of the maximum normal or principal stress theory to predict the strength of planar orthotropic materials such as wood. According to this theory, failure will occur when one or more than one of the stresses acting into the directions of material symmetry, σ_1, σ_2 and τ_{12}, reaches a respective maximum value, X, Y, and S. Mathematically stated, failure will not occur as long as

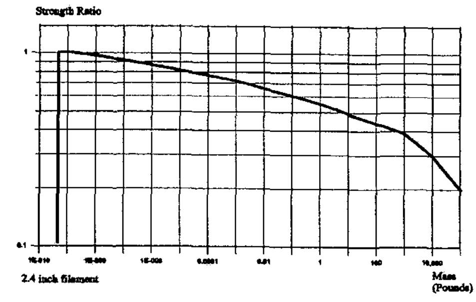

Figure 1. Rate of Decrease of Glass Fiber Tensile Strength Based on Mass (Courtesy of Mr J. Lowrie Mclarty)

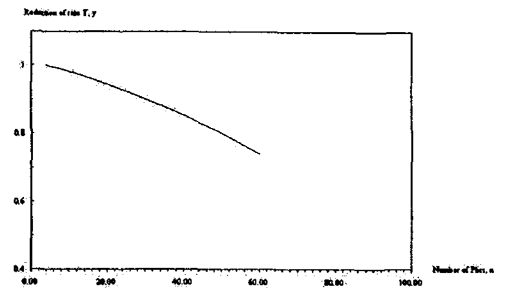

Figure 2. Tensile Strength Reduction Rate of Epoxy Matrix Based on Mass

$$-X' < \sigma_1 < X,\quad -Y' < \sigma_2 < Y,\quad -S' < \tau_{12} < S. \tag{1}$$

Because of orthotropic symmetry, shear strength is independent of the sign of τ_{12}. There are five independent modes of failure, and there is no interaction among the modes according to this theory. The reality is that failure processes are highly interacting and far more complex than the values of stress components. If stress ratio is used, this criteria can be expressed as

$$R_x = X/\sigma_x,\ \text{if}\ \sigma_x > 0,\ \text{or}\ R_x' = X'/|\sigma_x|,\ \text{if}\ \sigma_x < 0,$$
$$R_y = X/\sigma_y,\ \text{if}\ \sigma_y > 0,\ \text{or}\ R_y' = Y'/|\sigma_y|,\ \text{if}\ \sigma_y < 0, \tag{2}$$
$$R_s = S/|\sigma_s|,\ \text{if}\ \sigma_s < 0.$$

2.2 Maximum Strain Theory

The maximum strain theory is an extension of the maximum principal strain theory, promoted by Poncelet and Saint-Venant, to anisotropic media. The strain components for an orthotropic lamina are referred to the principal material axes, and there are three strain components in this criterion. Since linear elastic response is assumed to failure, this criterion can predict strength in terms of loads or stresses. A ply of a laminate is considered failed when one of ε_x, ε_y, and ε_s reaches the maximum value obtained from simple one-dimensional testing. This maximum strain from each test is either measured or computed from the measured strength divided by the tangent modulus :

$$\varepsilon_x^* = X / E_x, \text{ or } \varepsilon_x^* = X' / E_x,$$
$$\varepsilon_y^* = Y / E_y, \text{ or } \varepsilon_y^* = Y' / E_y, \quad (3)$$
$$\varepsilon_s^* = S / E_s.$$

The minimum common envelope of the superposition of the interaction failure diagrams, for either stress or strain, of all individual plies, becomes the failure diagram for the laminate. The strength ratio is expressed by the lowest of three ratios of the maximum strain to the applied strain. Note similar procedure taken for the maximum stress criteria,

$$R_x = \varepsilon_x^* / \varepsilon_x, \text{ if } \varepsilon_x > 0, \text{ or } R_x' = \varepsilon_x^* / |\varepsilon_x|, \text{ if } \varepsilon_x < 0,$$
$$R_y = \varepsilon_y^* / \varepsilon_y \text{ if } \varepsilon_y > 0, \text{ or } R_y' = \varepsilon_y^* / |\varepsilon_y|, \text{ if } \varepsilon_y < 0, \quad (4)$$
$$R_s = \varepsilon_s^* / |\varepsilon_s| .$$

2.3 Comments on Both Criteria

Both the maximum stress and maximum strain criteria assume no interactions among the possible five modes. Since the Poisson's ratio is not zero, there is always coupling between the normal components, and this leads to disagreement between these two criteria regarding the magnitude of the load and the mode for the failure. For example, consider a unidirectionally reinforced laminate acted upon by uniaxial tension, σ, at some angle θ to the reinforcements.

The result of two criteria agrees only on the shear plane and along the four lines of constant failures due to uniaxial stresses. Just as the deformation of a body is always coupled by the nonzero Poisson's ratio, failure of a body is also coupled. Because the micromechanics of failure is highly coupled, we should not extend the simple failure modes based on maximum stress or maximum strain components to fiber, matrix, and interfacial failure modes.

2.4 Recommended Strength- Failure Analysis Procedure

With available information at present, the following strength-failure analysis procedure is recommended for glass fiber reinforced composites with epoxy matrix.

1. Obtain reduced X by Fig. 1.
2. Assume the scale effect is the same for both tension and compression. (This assumption may be corrected when detailed research result is available).
3. Obtain Y=Y(Coupon) by Fig. 2.
4. Obtain Y'=Y'(Coupon) by Fig. 2.
   (Again, this may be corrected when accurate study result is available).
5. Assume S=S (Coupon).
6. Use Tsai-Wu failure criteria for stress space. Since the rates of decrease of the moduli are not known, use of the criteria for strain space is complicated.

The strength obtained by the above steps may not be "exact" for the composite with a given "increased" size. However, the result should not be too far off. Something is always better than nothing. Using strength theory with reduced tensile strength value alone is far better than designing the structure with the coupon test values. The recommended procedure will result in safer structures and will accelerate further studies for the exact failure-strength theories for composite structures with different scales/sizes, and with various constituent materials. When materials other than glass fibers and epoxy are used, only Fig. 1 and Fig. 2 may be modified. When detailed information on size effect for materials other than grass fiber and epoxy is not known, one can use Fig. 1 and Fig. 2 given above.

3. Numerical Examples

The structure under consideration is the pressure pipe as shown in Fig.3.
Internal diameter : 4m, Thickness of the pipe : 0.031m,
Max. operating pressure : 2MPa,
Design tensile strength of circular ply material : 352MPa,
Design tensile strength of longitudinal ply material : 352MPa,
Wall thickness h=248ho, ho=0.000125m.

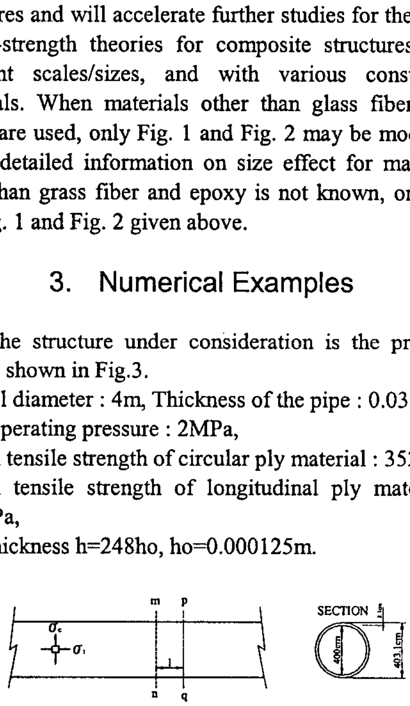

Figure 3. The structure under consideration

Stresses due to the internal pressure

$$\sigma_c = \frac{PD}{2t} = \frac{2\times4}{2\times0.031} = 129MPa$$
$$\sigma_l = \frac{PD}{4t} = \frac{2\times4}{4\times0.031} = 64.5MPa, \quad \sigma_s = 0$$

3.1 Safety factor, R, without Size Effect Considered.

1) Ordinary strength theory

$$\sigma_{\max} = R\sigma_{applied}$$
$$\sigma_{c(\max)} = R_c \sigma_{applied}$$
$$\sigma_{l(\max)} = R_l \sigma_{applied}$$
$$R_c = 2.7287 \ , \ R_l = 5.4574 .$$

1) Tsai-Wu failure criteria considering tensile and compression strengths only

	$F_{xy}^* = 0$	$F_{xy}^* = -1/2$
R	2.4340	3.1404

3.2 Safety Factor, with Size Effect Considered.

Assuming the filament diameter nomenclature as J, one ply of ho=0.000125m has about 10 fiber diameter thickness. With $V_f = 0.45$, one ply has about 5 fibers through its thickness.

Volume of fibers=

$$(5\times 248)\times(\frac{100}{2.54\times 2.4})\times(\frac{2\times 3.14\times 200}{0.125}) = 216{,}537{,}728$$

times of one fiber.

*. From Fig.1, the stress reduction ratio of fiber is 0.59, and that of matrix is 0.71. Thus

1) Ordinary strength theory

$$\sigma_{max} = R\sigma_{applied}$$

$$\sigma_{c(max)} = R_c\sigma_{applied}$$

$$\sigma_{t(max)} = R_t\sigma_{applied}$$

$$R_c = 1.6061 \quad R_t = 3.875$$

2) Tsai-Wu failure criteria considering tensile and compression strengths only

	$F_{xy}^* = 0$	$F_{xy}^* = -1/2$
R	1.5699	1.9713

3.3 Comparison

1) Ordinary strength theory

	S_c	S_t
Without size effect considered	2.720	5.459
Size effect considered	1.6061	3.875

2) Tsai-Wu failure criteria considering tensile and compression strengths only

	$F_{xy}^* = 0$	$F_{xy}^* = -1/2$
Without size effect considered	2.4340	3.1404
Size effect considered	1.5699	1.9713

The senior author, in his previous papers[2,3,4], proposed the strength-failure analysis procedure considering size effect, and concluded that the strength ratio depends on five factors : two cases of test coupon strengths, that is, A) reduction is made for both tensile and compression, B) reduction is made for tensile strength only, two failure criteria, Fxy*=0 and Fxy*=-1/2, and the status of applied stress. The proposed R/D direction on size/scale effect, then, can be summarized as follows.

A. Obtain the rate of decrease of fiber strength based on mass, for each of the possible candidate materials for large size structure.
B. Same as A for matrix.
C. For each of the laminate types to be used for design, perform tests, under all possible combination of applied stresses.
D. With the result of above A, B and C, find out which one of the failure criteria, Fxy*=0 and Fxy*=-1/2, is closer to the test result, for each combination of stresses.
E. With the result of A, B, and C, find out whether reduction of transverse strength is significant or not, for each state of stresses.
F. Find out whether reduction should be made for both tensile and compression strengths or tensile strength only, for each state of stresses.

4. Conclusion

Unlike airplane or automobile parts, prototype tests for buildings and bridges are impossible. Nevertheless, almost all buildings/infrastructures made of composite materials are fabricated without proper design. Design/analysis of such structure is simply too difficult for most of the engineers. In this paper, size/scale effects in failure of composite material structure are briefly explained. The effect of size/scale may be very serious. The numerical example in this paper shows that the safety factor is between 5.459 and 1.5699.

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