A modified analytical model of proton Bragg curves

Ken-ichi Takizawa, Yoshihisa Takada, Takeharu Nakashima, Syunsuke Kohno, and Yuhsuke Kobayashi

Institute of Applied Physics, University of Tsukuba

ABSTRACT

An improved analytical model has been developed to calculate an accurate Bragg curve of proton beam with an arbitrary energy. The model takes the transport of the secondary protons produced by the nuclear inelastic reactions into account. By the model, measured Bragg curves of proton beams with ten energies between 250 and 70 MeV are reproduced well. It will serve to obtain fundamental data for treatment planning and for energy scanning.

Keywords: Bragg curve, energy scanning method, build-up effect, inelastic nuclear reaction

1. INTRODUCTION

Proton therapy takes advantage of the depth-dose curve known as the Bragg curve. Shape of the Bragg curve depends on energy of the proton beam and the energy spread. A Spread Out Bragg Peak (SOBP) is formed to obtain a flat dose distribution in depth direction covering the tumor by superposing multiple Bragg curves that have a series of peak positions. A ridge filter is usually used for the fixed modulation. Whereas a lateral beam scanning together with the energy scanning by a synchrotron is used to form a conformal dose distribution. In this case, many Bragg curve data are required for the treatment planning. Although a Monte Carlo method[1)–4)] is a good choice for calculation of the Bragg curves, a simpler method based on an analytical model is better for ease of use if it has an equivalent accuracy. We aimed at developing such an analytical calculation model by which we can obtain accurate Bragg curves for proton beams with arbitrary energies.

2. MATERIALS AND METHODS

2.1. Bortfeld model

Bortfeld [5)] proposed an analytical model based on the electromagnetic energy-loss model, fluence reduction due to inelastic nuclear reaction, and dose contribution of the secondary charged particles as a local energy deposit. Bortfeld gives an expression of the Bragg curve as follows.

$$D(z)=\Phi_0 \frac{e^{-\left(\frac{R_0-z}{\sigma}\right)^2/4}\sigma^{1/p}\Gamma(1/p)}{\sqrt{2\pi}\rho p\alpha^{1/p}(1+\beta R_0)}\times\left[\frac{1}{\sigma}D_{-1/p}\left(-\frac{R_0-z}{\sigma}\right)+\left(\frac{\beta}{p}+\gamma\beta+\frac{\varepsilon}{R_0}\right)D_{-1/p-1}\left(-\frac{R_0-z}{\sigma}\right)\right] \quad (1)$$

where D_q is the parabolic cylinder function. The first term in a bracket corresponds to dose by electromagnetic effect and second term corresponds to the dose by inelastic nuclear interaction. Since the first term is quite large compared with the second term, main contribution to the Bragg curve comes from the electromagnetic effect. The parameters and constants in this expression are summarized in Bortfeld paper. Although the model reproduces Bragg curves of proton beams with up to medium energy, noticeable discrepancies are observed for Bragg curves of proton beams with higher energies.

2.2. Consideration of the transportation of the secondaries

To improve the Bortfeld model, we consider the transport of the secondary protons, which is a major component of energetic charged secondaries. When inelastic nuclear reaction occurs, secondary particles are produced. Some of them have enough kinetic energy to transport it to the forward direction. This affects the dose distribution formed by the charged secondary particles especially in the plateau region. Thus shape of the calculated Bragg curve is affected. Contribution of charged secondaries is separated into two parts: the one contributes to the local energy deposit, the other contributes to energy transport to forward direction. These contributions are expressed by the parameters ε_1 (local energy deposit) and ε_2 (transport), respectively, instead of the ε parameter in equation (1). The energy transport process is considered as follows. The dose at a depth z is affected by secondary protons that are produced at z' before z and have a residual range longer than z-z'. It is obtained by convolution of the dose distribution and the range spectrum of secondary protons as follows

$$D_{tail}(z,z') = \int_{z-z'}^{R_0-z'} \Phi_R(R,R')\hat{D}_1(z-z',R')dR' \tag{2}$$

where R_0 is the range of protons that have initial energy, Φ_R the range (R') spectrum of secondary protons produced by the primary proton with a residual range R, $\hat{D}_1$(z-z', R') the dose distribution (electromagnetic contribution only) of a mono-energetic beam with the range R' at z-z'. The primary protons have already lost part of the initial energy at the nuclear interaction depth z' and the residual range is R_0-z'. Therefore the maximum range of secondary particles is R_0-z' that is the upper limit of integration (2). Φ_R is assumed to be the same as the tail spectrum in the Bortfeld model. The total dose at z by the transportation is obtained by the integration of equation(2) from 0 to z with consideration of the energy spread. Figure 1 shows the dose distribution by the transportation and the dose distribution of the local energy deposit. The solid line in Fig 1 is calculated using equation(2), and shows the build-up effect. The final expression of the Bragg curve by the modified model is shown in equation (3). Note that difference between equation(1) and equation(3) is only the final term in equation(3) in bracket. This equation has three free parameters, ε_1 and ε_2 and σ. The σ is the range spread that is determined by the average energy and the spread of the incident proton beam. The optimized parameters were found to reproduce well a measured Bragg curve using a minimum chi-square method. We also find best parameters of equation (1) by the same method, and compare the accuracies of both methods.

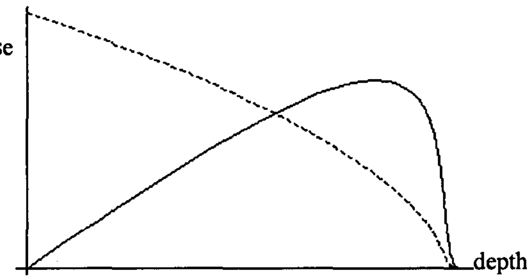

Fig.1 The dose distribution by inelastic nuclea interactions. The solid line represents the dose b the transportation, and the dashed line represents th dose by the local energy deposit. Both x and y axe are arbitrary units.

$$D(z) = \Phi_0 \frac{e^{-\left(-\frac{R_0-z}{\sigma}\right)^2/4}\sigma^{1/p}\Gamma(1/p)}{\sqrt{2\pi}\rho p\alpha^{1/p}(1+\beta R_0)} \times \left[\begin{array}{l} \frac{1}{\sigma}D_{-1/p}\left(-\frac{R_0-z}{\sigma}\right) + \left(\frac{\beta}{p}+\gamma\beta\right)D_{-1/p-1}\left(-\frac{R_0-z}{\sigma}\right) \\ + \frac{\varepsilon_1}{R_0}D_{-1/p-1}\left(-\frac{R_0-z}{\sigma}\right) \\ + \int_{-\infty}^{R_0} D_{tail-total}(\bar{z};\varepsilon_2)\frac{\exp(-(z-\bar{z})^2/2\sigma^2)}{\sqrt{2\pi}\sigma}d\bar{z} \end{array}\right] \tag{3}$$

3.COMPARISONS WITH MEASUREMENTS AND NUMERICALLY CALCULATED DATA

Measurements of the depth-dose curves were carried out at the gantry beam course where a lateral uniform field is formed by the dual-ring double scattering method. Ten Bragg curves were measured in water for proton beams with energies between 70MeV and 250MeV. Measurements were done using a p-type silicon detector in Proton Medical Research Center, University of Tsukuba. The sampling pitch is taken at 10 mm or 5 mm in the plateau region and at 0.2 mm in the region around the Bragg peak. The calculated relative dose by the Bortfeld's original model is lower in the mid-plateau region than that of measured data for proton beams with higher energies. It was improved by using the modified method. The most probable parameters found by optimization are shown in Table 1 with the chi-square data. The δ in the Table 1 is the standard deviation of momentum spread of the incident proton beam and parameter σ depends on δ.

Table.1 Most probable parameters in equation (2)

Energy (MeV)	N (number of sampling points)	δ (%)	ε_1	ε_2	x^2	x^2/N
250	103	0.21	0.38	0.15	0.0158	0.000153
230	103	0.20	0.3	0.14	0.0049	0.000048
200	88	0.20	0.22	0.16	0.0061	0.000069
175	73	0.18	0.18	0.15	0.0073	0.0001
155	71	0.20	0.136	0.13	0.0051	0.000072
140	70	0.20	0.1	0.14	0.0023	0.000033
125	54	0.18	0.07	0.18	0.0057	0.00011
105	49	0.21	0.08	0.1	0.0068	0.00014
85	31	0.21	0	0.12	0.0055	0.00018
70	32	0.21	0.14	0	0.0032	0.0001

Figure 2 shows the Bragg curves of proton beams with energy of 250MeV and 70MeV together with calculations by the modified model. The differences between measurements and calculations are within one per cent of the peak dose at most points. Subtle discrepancies (within a few percent of the peak dose) remain in the Bragg peak region. These were also found in calculations by the Bortfeld original model. To calculate a Bragg curve of a proton beam with an arbitrary energy, we searched the set of parameters for the proton beam with the energy by interpolation using parameters in Table1. We also calculated a SOBP by superposing Bragg curves of proton beams with individual energies obtained by the method.

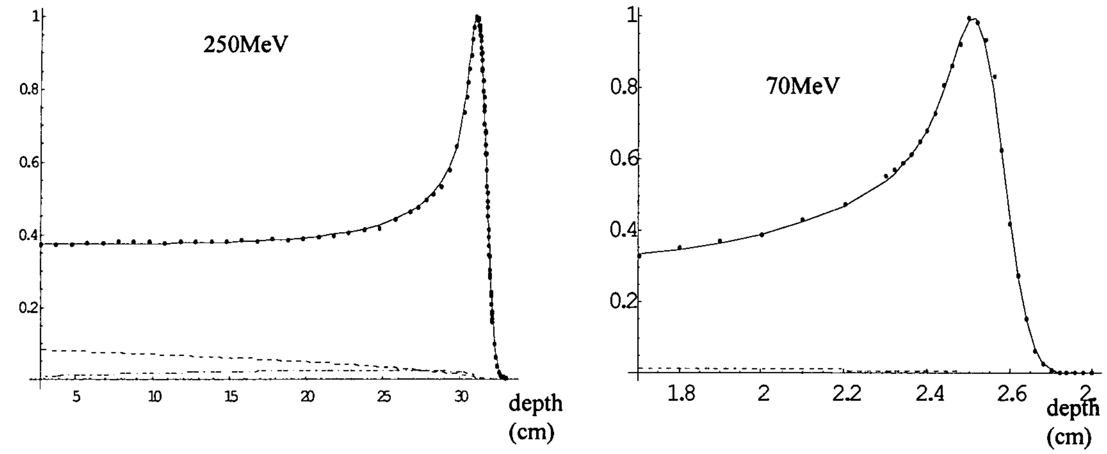

Fig.2 Comparison between the measured and calculated Bragg curves of proton with energies (a)250MeV, (b)70MeV The solid line represents calculated data, dots represent measured data, and the dotted and dashed lines represen contributions of charged secondaries. The dotted line is the local deposit and the dashed line is the energy transport.

4.DISCUSSIONS

To improve small discrepancies around the Bragg peak region, we tried an asymmetric Gaussin distribution for the initial momentum distribution. However, the noticeable improvement was not obtained.

5.CONCLUSION

We have improved the accuracy of calculated Bragg by using the modified Bortfeld model, which takes the energy transport of the secondary protons into account. Although within-one-percent accuracy is attained at most positions, small discrepancy remains around the peak region.

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