Block Designs For Comparisons Within Two Groups in Diallel Crosses

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ABSTRACT. A class of block designs for general combining ability comparisons within two groups of inbred lines in diallel crosses is given. Use is made of balanced block designs obtained by cyclically developing a single initial block.

1. Introduction

Genetic properties of inbred lines in plant breeding experiments are investigated by carrying out diallel crosses. Let p denote the number of inbred lines and let a cross between lines i and j be denoted by (i, j) with $i < j = 1, 2, \cdots, p$. Let n_c denote the total number of distinct crosses in the experiment. Our interest lies in comparing the lines with respect to their general combining ability (gca) parameters. The complete diallel corss (CDC) involves al possible crosses among p parental lines with $n_c = p(p-1)/2$.

We consider the case where there are two groups of inbred lines, such as those coming from two different regions or laboratories. When the total number of lines in the two groups is large, sometimes it becomes impractical to carry out a complete diallel cross. In such situations, only a subset of all possible crosses is used in the experiment, which is called a partial diallel cross (PDC). Block designs for partial diallel crosses often require each cross to be replicated several times. In theis paper we consider the situation where the experiment is carried out in two phases in order to have fewer replications of the crosses. The object in the first phase is to select a few best lines from each group on the basis of their gca effects. The selected lines in one group are then compared with those of the other group in the second phase of the experiment. The purpose of this paper is to provide a class of block designs for the first phase of the experiment. In the literature PDC designs have been discussed for $n_c = ps/2$, $s < p-1$, distinct crosses where s is the constant number of other lines each line is crossed with. The PDC designs of this pager involve two distinct values of s. The designs given are especially useful as they require each cross to be replicated only once. A table of designs is also provided.

2. Method of construction

We consider the case where there are two groups of lines of sizes p_1 and p_2, with $p=p_1+p_2$. Let the lines in the ith group be denoted by $(i-1)p_1+j$, $j=1,2,\cdots,p_i$, $i=1,2$. Consider a blcok design D_b involving $n_c=p_1p_2$ distinct crosses laid out in b blocks of k crosses each, with each line from group i contributing to n_c/p_i crosses, $i=1,2$.
Following e.g. Gupta and Kageyama [7], the model for the data is assumed to be

$$Y=\mu 1_n+\Delta_1 g+\Delta_2\beta+\varepsilon \tag{2.1}$$

where Y is the $n\times 1$ vector of responses, μ is the overall mean, 1_t is the $t\times 1$ vector of 1's, and $g=(g_1,g_2,\cdots,g_p)'$ and $\beta=(\beta_1,\beta_2,\cdots,\beta_b)'$ are the vectors of p gca effects and b block effects respectively; the rectangular matrices Δ_1, Δ_2 are the corresponding design matrices, and ε is the $n\times 1$ vector of independent random errors with zero expectations and constant variance σ^2. The information matric C for estimating pairewise comparisons among the gca parameters is then given by

$$C=G-\frac{1}{k}N_bN_b' \tag{2.2}$$

where $G=(g_{ij})$ is a symmetric matrix, g_{ii} denotes the number of other lines line i is crossed with, and g_{ij} denotes the number of replications of the cross (i,j) for $i<j=1,2,\cdots,p$. Consider two lines in each of the n crosses as the block contents of a design D_c with block size $k=2$, and let N_c denote th $p\times n$ incidence matrix of the block design thus obtained. Then $G=N_cN_c'$. The matrix N_b is the $p\times b$ line versus block incidence matrix of the design. Thus N_b is the usual incidence matrix; in the present context, it is obtained by ignoring the crosses, and thus by considering $2k$ lines as the contents of a block. Note that $N_b1_b=r1_p$, $N_b'1_p=2k1_b$.

For $l=1,2$, our interest lies in the gca comparisons g_i-g_j, $i<j=(l-1)p_1+1$, $(l-1)p_1+2,\cdots$, $(l-1)p_1+p_l$. That is, we are interested in comparing the lines within each group with respect to their gca effects. A class of designs for estimating these comparisons is given in the next section. For the designs of the next section, $g_{ii}=1$ and $g_{ij}=n_c/p_l$, $i<j=(l-1)p_1+1$, $(l-1)p_1+2,\cdots$, $(l-1)p_1+p_l$, $l=1,2$.

Definition 2.1 A block design with parameters $p_1=b_1, r_1=k_1, \lambda$ is called a balanced block design (BB) design if (a) $\sum_{j=1}^{b_1} n_{ij}n_{lj}=\lambda$ for $i\neq j=1,2,\cdots,p_1$, and (b) $|n_{ij}-k_1/p_1|<1$ for all i,j, where $N=(n_{ij})$ denotes the incidence matrix of the design.

Theorem 2.1 The existence of a BB design D_1 with parametes $p_1 = b_1, r_1 = k_1, \lambda$ obtained by developing a single initial block implies the existence of a PDC block design D_b with parameters $\{p_1, p_2 = k = k_1, r_c = 1, n_c = p_1 p_2, s_1 = p_2, s_2 = p_1\}$, and

$$\sigma_1^2 = var(\widehat{g}_i - \widehat{g}_j) = \frac{2p_2\sigma^2}{p_1\lambda}, \quad i < j = 1, 2, \cdots, p_1,$$

$$\sigma_2^2 = var(\widehat{g}_i - \widehat{g}_j) = \frac{2\sigma^2}{p_1}, \quad i < j = p_1 + 1, p_1 + 2, \cdots, p. \tag{2.3}$$

Example 2.1 The following design with parameters $\{p_1 = 4, p_2 = 5, b = 4, k = 5, r_c = 1, s_1 = 5, s_2 = 4, \lambda = 6\}$, $\sigma_1^2 = 5\sigma^2/12$, $\sigma_2^2 = 0.5\sigma^2$, belongs to the above theorem:

{(1,5),(2,6),(3,7),(4,8),(1,9)}
{(2,5),(3,6),(4,7),(1,8),(2,9)}
{(3,5),(4,6),(1,7),(2,8),(3,9)}
{(4,5),(1,6),(2,7),(3,8),(4,9)}

Corollary 2.1 There exists a PDC design D_b with parameters:

$$\{p_1,\ p_2 = k = tp_1,\ b = p_1,\ r_c = 1,\ n_c = p_1 p_2,\ s_1 = tp_1,\ s_2 = p_1,\ \lambda = t^2 p_1\}$$

where t is a positive integer. Also,

$$\sigma_i^2 = \frac{2p_i}{n_c}\sigma^2, \quad i = 1, 2.$$

Let a BB design D_1 with parameters $\{p_1 = b_1,\ r_1 = k_1 = tp_1,\ \lambda = t^2 p_1\}$ be obtained by cyclically developing the initial block

$$\{1, 2, \cdots, p_1, 1, 2, \cdots, p_1, \cdots, 1, 2, \cdots, p_1\}.$$

Let l_{ij} denote the ith element in block j of D. Then, the jth block of a PDC design D_b of Corollary 3.1 is obtained by replacing l_{ij} by the cross $(l_{ij}, p_1 + i)$, $i = 1, 2, \cdots, tp_1, j = 1, 2, \cdots, p_1$.

Example 2.2 The following design with parameters $\{p_1 = 3, p_2 = k = 6, b_1 = 3, r_c = 1, n_c = 18, s_1 = 6, s_2 = 3, \lambda = 12\}$, $\sigma_1^2 = \sigma^2/3$, $\sigma_2^2 = 2\sigma^2/3$, belongs to Corollary 3.1:

{(1,4),(2,5),(3,6),(1,7),(2,8),(3,9)}
{(2,4),(3,5),(1,6),(2,7),(3,8),(1,9)}
{(3,4),(1,5),(2,6),(3,7),(1,8),(2,9)}

Corollary 2.2 There exists a PDC design D_b with parameters $\{p_1, p_2 = k = tp_1 + 1,\ b = p_1, r_c = 1, n_c = p_1 p_2, s_1 = tp_1 + 1, s_2 = p_1, \lambda = t(tp_1 + 2)\}$, where t is a positive integer.

Let a BB design D_1 with parameters $\{p_1 = b_1, r_1 = k_1 = tp_1 + 1, \lambda = t(tp_1 + 2)\}$ be obtained by cyclically developing the initial block:

$$\{1, 2, \cdots, p_1, 1, 2, \cdots, p_1, \cdots, 1, 2, \cdots, p_1, 1\}$$

Then, the jth block of the PDC design D_b of Corollary 3.2 is obtained by replacing l_{ij} by the cross $(l_{ij},\ p_1+i)$ where l_{ij} is as defined after Corollary 3.1, $i=1,2,\cdots,tp_1+1$, $j=1,2,\cdots,p_1$.

Example 2.3 The following design with parameters $\{p_1=3, p_2=k=7, b_1=3, r_c=1, n_c=21, s_1=7, s_2=3, \lambda=16\}$, $\sigma_1^2=7\sigma^2/24$, $\sigma_2^2=2\sigma^2/3$ belongs to Corollary 3.2:

$\{(1,4),(2,5),(3,6),(1,7),(2,8),(3,9),(1,10)\}$

$\{(2,4),(3,5),(1,6),(2,7),(3,8),(1,9),(2,10)\}$

$\{(3,4),(1,5),(2,6),(3,7),(1,8),(2,9),(3,10)\}$