Availability Analysis of a Warm Standby Redundant System

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Introduction

❑ Redundant Structure

- *The redundant structure of a system has been adopted by system designers in order to improve the system reliability.*
- *The types of redundancies*
  - *Active redundancy*
  - *Inactive redundancy*
- *The types of standby redundancies*
  - *Hot standby redundancy*
  - *Cold standby redundancy*
  - *Warm standby redundancy*

Introduction

- Availability [MIL-STD-721C(1981)]
  - *A measure of the degree to which an item is in the operable and committable state at the start of the mission, when the mission is called for at an unknown (random) time.*
  - $$A = \frac{MTBF}{MTBF + MTTR}$$

    where MTBF is interpreted as the expected value of each of the (i.i.d.) uptimes $X_1, X_2, \ldots$ and correspondingly, MTTR is the expected value of each of the (i.i.d.) downtimes $Y_1, Y_2, \ldots$. MTBF stands for mean time between failures and MTTR stands for mean time to repair.

Introduction

- Unavailability
  - *Unavailability(U) = 1 – Availability*
- Annual Downtime
  - *Annual Downtime (minutes/year) = Unavailability* x *525600*

Introduction

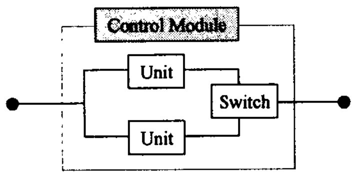
- Some Works on Redundant Structure
  - The redundant systems having imperfect switchover device have been extensively studied by many authors as follows.
  - Osaki (1982)
  - Srinivasan (1983)
  - Gupta, Jaiswal and Goel (1983)
  - Veklerov (1987)
  - Shen and Xie (1991)
  - Rander, Kumar and Tuteja (1992)
  - Lim (1996), Lim and Koh (1997)

Introduction

- The Purpose of the Paper
  - In this paper, we consider the following two systems:
    - (i) A single component structure (SCS), and
    - (ii) A simple standby redundant structure with the function of switchover processing (SRS).
  - The type of standby is warm standby. That is, the failure rate of standby unit ranges from 0 to the failure rate of the active unit.
  - The goal of this paper is
    - to propose the availability model which includes the SRS
    - calculate the steady state availability of SRS
    - to compare it with a single component system in terms of availability.

A Reference Model of Redundant Structure

- A Reference Model of Standby Redundant Structure with a Function of Switchover Processing



- The switchover device switches to the standby unit when the active one fails to perform its required mission.
- The control module monitors the state of an active unit and informs the switchover device the state of the active component.
- The switchover device is not 100% perfect.

A Reference Model of Redundant Structure

- Throughout this paper, we assume the followings:
  - All units are independent and have exponential life distributions with a mean of $1/\lambda$ for the active unit and a mean of $1/(\lambda\beta)$ for the standby unit, where $0 \leq \beta \leq 1$.
  - Repairs occur one at a time (sequential repair) and the repair time is exponentially distributed with a mean of $1/\mu$.
  - The repair time of the switchover device is approximately exponentially distributed with the mean of R.
  - The probability of successful switchover operation is given by p.
  - The type of standby unit in the redundant system is a warm standby unit.

Notations

- Notations
  - A_S Availability of SCS
  - A_W Availability of warm standby redundant system(WSRS)
  - A_C Availability of cold standby redundant system(CSRS)
  - A_H Availability of hot standby redundant system(HSRS)
  - p The probability of successful switchover operation
  - λ Failure rate of the active unit
  - $\lambda\beta$ Failure rate of warm standby unit, where $0 \leq \beta \leq 1$.
  - λ_α Increment of failure rate caused by the control module
  - μ Repair rate of the failed unit
  - R Mean repair time of the switchover device

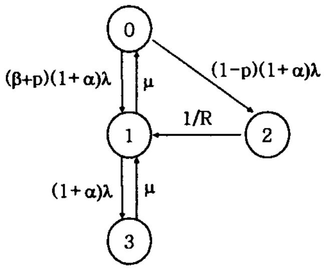
Availability Modeling

- Basic ideas for availability modeling are as follows:
  - In the SRS, the failure of control module does not have an effect on the operation of a system as far as the active unit is working.
  - However, it has an effect on the switchover processing if the active unit fails while the control module is in failure state.
  - Hence, it is natural to assume that the switchover processing causes the increase of the failure rate of the system. (That is, the failure of the control module can occur in our availability model.)
  - We assume that this increment of the failure rate is distributed to each string of the system in such a way that the failure rate of each unit increases by $\lambda_\alpha = \alpha\lambda$, where λ_α is relatively smaller than the failure rate of a unit λ, i.e. $0 \leq \alpha \leq 1$.

Availability Modeling

- State Transition Diagram

State	Description
0	All Components are Normal
1	Simplex
2	Uncoverage Outage
3	System Failure



Availability Modeling

- A Flow Rate Equation

$$(1+\beta)(1+\alpha)\lambda P_0 = \mu P_1,$$
$$[(1+\alpha)\lambda+\mu]P_1 = (\beta+p)(1+\alpha)\lambda P_0 + P_2/R + \mu P_3,$$
$$P_2/R = (1-p)(1+\alpha)\lambda P_0,$$
$$\mu P_3 = (1+\alpha)\lambda P_1,$$
$$P_0 + P_1 + P_2 + P_3 = 1.$$

- Steady State Availability

$$A_W = P_0 + P_1 = \frac{\mu^2 + \mu(1+\beta)(1+\alpha)\lambda}{\mu^2 + \mu(1+\beta)(1+\alpha)\lambda + (1-p)R\mu^2(1+\alpha)\lambda + (1+\beta)(1+\alpha)^2\lambda^2}$$

Availability Modeling

- Steady State Availability
  - Note that if $\beta=0$, then the equation of A_w is reduced to

$$A_C = \frac{\mu^2 + \mu(1+\alpha)\lambda}{\mu^2 + \mu(1+\alpha)\lambda + (1-p)R\mu^2(1+\alpha)\lambda + (1+\alpha)^2\lambda^2} \quad \text{(Cold Standby Redundancy)}$$

  - Note that if $\beta=1$, then the equation of A_w is reduced to

$$A_H = \frac{\mu^2 + 2\mu(1+\alpha)\lambda}{\mu^2 + 2\mu(1+\alpha)\lambda + (1-p)R\mu^2(1+\alpha)\lambda + 2(1+\alpha)^2\lambda^2} \quad \text{(Hot Standby Redundancy)}$$

Availability Comparison

- Availability Comparison of SRS and SCS
  - Steady state availability of SCS is $A_S = \mu/(\lambda+\mu)$.

Theorem

Let $\gamma = (1+\beta)\lambda + \mu^2 R$. Suppose that

$$\frac{-\gamma + \sqrt{\gamma^2 - 4(1+\beta)\lambda\mu(\mu R - 1)}}{2(1+\beta)\lambda} \leq \alpha \leq \frac{-(1+\beta)\lambda + \sqrt{[(1+\beta)\lambda]^2 + 4(1+\beta)\lambda\mu}}{2(1+\beta)\lambda}$$

Then there exists a $p^* \in [0,1]$ such that $A_S \geq A_W$ for $0 \leq p \leq p^*$ and $A_S \leq A_W$ for $p^* \leq p \leq 1$, where $p^* = 1 - \frac{\mu - (1+\beta)\alpha(1+\alpha)\lambda}{\mu^2 R(1+\alpha)}$.

Availability Comparison

- Availability Comparison of SRS and SCS
  - **Sketch of the proof**

Note that $A_W = \frac{\mu^2 + \mu(1+\beta)(1+\alpha)\lambda}{\mu^2 + \mu(1+\beta)(1+\alpha)\lambda + (1-p)R\mu^2(1+\alpha)\lambda + (1+\beta)(1+\alpha)^2\lambda^2}$

and $A_S = \frac{\mu}{\mu+\lambda}$. A_W is non-decreasing in p and A_S is constant.

Hence it is sufficient to show that $A_S \geq A_W$ for p=0 and $A_S \leq A_W$ for p=1.

It is somewhat tedious but straightforward to show that $A_S \geq A_W$ when p=0,

as far as $\alpha \geq \frac{-\gamma + \sqrt{\gamma^2 - 4(1+\beta)\lambda\mu(\mu R - 1)}}{2(1+\beta)\lambda}$.

Similarly, we can show that when p=1, $A_S \leq A_W$ as far as

$\alpha \leq \frac{-(1+\beta)\lambda + \sqrt{[(1+\beta)\lambda]^2 + 4(1+\beta)\lambda\mu}}{2(1+\beta)\lambda}$.

Availability Comparison

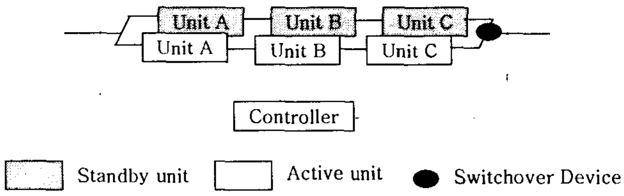
- Availability Comparison of SRS and SCS
  - **Sketch of the proof**

Finally, p^* can be obtained by solving the following equation with respect to p.

$$\frac{\mu^2 + \mu(1+\beta)(1+\alpha)\lambda}{\mu^2 + \mu(1+\beta)(1+\alpha)\lambda + (1-p)R\mu^2(1+\alpha)\lambda + (1+\beta)(1+\alpha)^2\lambda^2} = \frac{\mu}{\lambda+\mu} \quad \blacksquare$$

EXAMPLE

- The Structure of an Optical Transportation System



PBA	Unit A	Unit B	Unit C	Control Module
Failure Rate	9,000	7,500	9,500	5,800

EXAMPLE

- Annual Downtime Comparison of SCS and WSRS when β=0.0 (Us=2.59993E-05, Annual Downtime of SCS = 13.66525)

p	α=0.0	α=0.3	α=0.6	α=0.9
	Downtime	Downtime	Downtime	Downtime
0.0	13.6652	17.7647	21.8641	25.9634
0.1	12.2988	15.9883	19.6778	23.3673
0.2	10.9323	14.2120	17.4916	20.7711
0.3	9.5659	12.4356	15.3053	18.1750
0.4	8.1994	10.6592	13.1190	15.5788
0.5	6.8329	8.8828	10.9327	12.9826
0.6	5.4664	7.1064	8.7464	10.3864
0.7	4.0999	5.3300	6.5600	7.7902
0.8	2.7334	3.5535	4.3737	5.1939
0.9	1.3669	1.7771	2.1873	2.5976
1.0	0.0004	0.0006	0.0009	0.0013

EXAMPLE

- Annual Downtime Comparison of SCS and WSRS when β=0.3
  (Us=2.59993E-05, Annual Downtime of SCS = 13.66525)

p	α=0.0	α=0.3	α=0.6	α=0.9
	Downtime	Downtime	Downtime	Downtime
0.0	13.6652	17.7647	21.8641	25.9634
0.1	12.2988	15.9883	19.6778	23.3673
0.2	10.9323	14.2120	17.4916	20.7712
0.3	9.5659	12.4356	15.3054	18.1751
0.4	8.1994	10.6593	13.1191	15.5790
0.5	6.8329	8.8829	10.9328	12.9828
0.6	5.4665	7.1065	8.7465	10.3867
0.7	4.1000	5.3301	6.5602	7.7904
0.8	2.7335	3.5537	4.3739	5.1942
0.9	1.3670	1.7772	2.1876	2.5980
1.0	0.0005	0.0008	0.0012	0.0017

EXAMPLE

- Annual Downtime Comparison of SCS and WSRS when β=0.6
  (Us=2.59993E-05, Annual Downtime of SCS = 13.66525)

p	α=0.0	α=0.3	α=0.6	α=0.9
	Downtime	Downtime	Downtime	Downtime
0.0	13.6652	17.7647	21.8641	25.9634
0.1	12.2988	15.9884	19.6779	23.3674
0.2	10.9324	14.2120	17.4917	20.7713
0.3	9.5659	12.4357	15.3055	18.1753
0.4	8.1995	10.6594	13.1193	15.5792
0.5	6.8330	8.8830	10.9330	12.9831
0.6	5.4665	7.1066	8.7468	10.3870
0.7	4.1001	5.3302	6.5605	7.7908
0.8	2.7336	3.5538	4.3742	5.1946
0.9	1.3671	1.7774	2.1879	2.5984
1.0	0.0006	0.0010	0.0015	0.0022

EXAMPLE

- Annual Downtime Comparison of SCS and WSRS when β=1.0 (Us=2.59993E-05, Annual Downtime of SCS = 13.66525)

p	α=0.0	α=0.3	α=0.6	α=0.9
	Downtime	Downtime	Downtime	Downtime
0.0	13.6652	17.7647	21.8641	25.9634
0.1	12.2988	15.9884	19.6779	23.3674
0.2	10.9324	14.2121	17.4917	20.7714
0.3	9.5660	12.4358	15.3056	18.1754
0.4	8.1995	10.6594	13.1194	15.5793
0.5	6.8331	8.8831	10.9332	12.9833
0.6	5.4666	7.1067	8.7469	10.3872
0.7	4.1001	5.3304	6.5607	7.7911
0.8	2.7337	3.5540	4.3744	5.1949
0.9	1.3672	1.7776	2.1881	2.5988
1.0	0.0007	0.0012	0.0018	0.0026

EXAMPLE

- Turning Point (p*)for some value of α and β

	Proportion of increment in failure of each string(α)		
	α=0.3	α=0.6	α=0.9
β=0.0	0.230779371	0.3750156	0.473707611
β=0.3	0.230779371	0.37502028	0.473714631
β=0.7	0.230782491	0.37502652	0.473723991
β=1.0	0.230784831	0.3750312	0.473731011