A Fuzzy Variable Step Size LMS Algorithm for Adaptive Antennas in CDMA Systems

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Abstract

This paper proposes a new application of Fuzzy logic to Variable Step Size Least Mean Square (VS-LMS) adaptive beamforming algorithm in CDMA systems. The proposed algorithm adjusts the step size of the Least Mean Square (LMS) by using the application of Fuzzy logic in which the increase or decrease of step size depends on the fuzzy inference results of the Mean Square Error (MSE). Computer simulation results show that the proposed algorithm has a better capacity of tracking compared with the conventional LMS algorithms and other variable step size LMS algorithms.

Keywords

adaptive antennas, adaptive beamforming algorithm, LMS, variable step size

I. Introduction

Recent explosive wireless communications market has demanded increasingly larger capacity and higher speed in data communication than ever. Thus, future wireless communications systems must have ability to support higher data rate and faster media communication as well. However, the limitation of the available radio frequency spectrum poses a major challenge to the realization of the demanded systems. Thus, the approaches for enlarging the system capacity are of great interest. One promising approach to overcome the problem is to employ adaptive antennas (so-called smart antennas). Adaptive beamforming algorithms for adaptive antennas have been paid a lot of attention and many algorithms such as Least Mean Square (LMS), Recursive Least Square (RLS) have been exploited.

Least Mean Square (LMS) algorithm is one of the most frequently used beamforming algorithms in adaptive antennas mainly due to its low computational complexity. However, in conventional LMS algorithm, since the step size is kept constant, an appropriate value of step size is very important to speed up the convergence of algorithm. A small step size can result in low mis-adjustment in steady state, but the time for convergence is long. On the other hand, a large step size will lead to a fast convergence and therefore a better capacity of tracking, but it will result in large steady state error.

Variable step size methods have been reported to improve the convergence speed while preserving the steady state performance with a low increase of computational complexity [1]-[6]. Furthermore, variable step size methods have better tracking capacity in non-stationary environments than that of conventional LMS. In [1], the step size is controlled by exploiting the polarity of successive samples of the estimation error. In [2], the authors proposed a step size adjustment approach based on the fluctuation of predict square error. The approach proposed in [3] uses a profile step size function to control the step size.

All the above variable step size methods adjust the step size by exploiting some linguistic rules of step size adjustment translated into numerical formulae of mathematical model. Therefore, instead of interpreting these linguistic rules in a mathematic model, the step size can be adjusted by directly applying the fuzzy technique in the linguistic model. One form of the application has been introduced in [6] where the authors examined the variable set of three {*Small, Medium, Large*} in the two Fuzzy Inference System model of one and two input.

In this work, the proposed approach examined an alternative fuzzy variable definition in which the definition of step size depends on the mean square error (MSE). The paper is organized as follows. The problem formulation is presented in section II. The proposed approach is described in section III. In section IV, the computer simulation results are presented. Some conclusions are given in section V.

II. Problem formulation

Let us examine a CDMA base station with adaptive antennas as depicted in **Fig.1**. It is assumed that the number of array elements is K, the number of active users is M ($K > M$).

The received signal vector $\underline{x}(n)$ is given by:

$$\begin{aligned}\underline{x}(n) &= [x_1(n)\ \ x_2(n)\ \ \cdots\ \ x_K(n)]^T \\ &= A(\theta)\underline{s}(n) + \underline{n}(n)\end{aligned} \qquad (1)$$

where the impinging signal vector, $\underline{s}(n)$, the array response matrix, $A(\theta)$, and the Additive White Gaussian noise (AWGN) vector, $\underline{n}(n)$, are defined below.

$$\underline{s}(n) = [s_1(n)\ \ s_2(n)\ \ \cdots\ \ s_M(n)]^T$$

$$A(\theta) = [\underline{a}_1(\theta)\ \ \underline{a}_2(\theta)\ \ \cdots\ \ \underline{a}_K(\theta)]^T$$

$$\underline{a}_l(\theta) = [a_l(\theta_1)\ \ a_l(\theta_2)\ \ \cdots\ a_l(\theta_M)]\ (l = 1, 2, \ldots, K)$$

$$\underline{n}(n) = [n_1(n)\ \ n_2(n)\ \ \cdots\ \ n_K(n)]^T$$

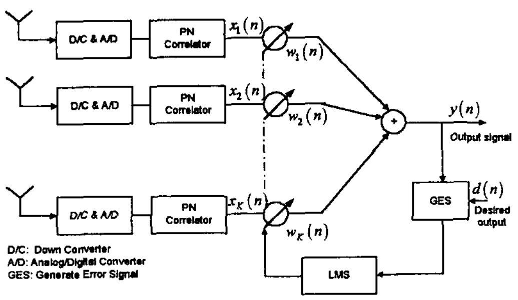

Fig.1: A schematic of a CDMA base station with adaptive antennas

The array output signal, $y(n)$, is achieved by multiplying the received signal with weight vector $\underline{w}(n)$ as represented in the following equation:

$$y(n) = \underline{w}^H(n)\underline{x}(n) \qquad (2)$$

where the weight vector is defined as:

$$\underline{w}(n) = [w_1(n)\ \ w_2(n)\ \ \cdots\ \ w_K(n)]^T$$

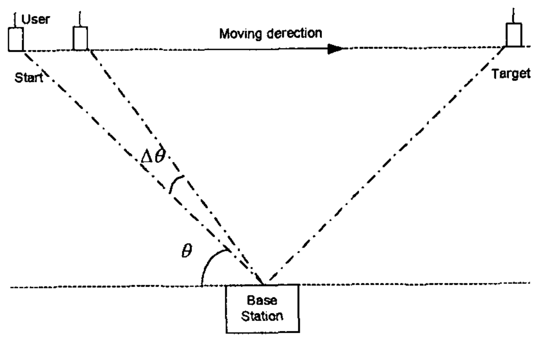

Fig.2: Model for examining tracking capacity of the proposed algorithm

In order to verify the ability of tracking of proposed approach and to compare with other conventional algorithms, the model for tracking in **Fig.2** is applied. The point of this model is to estimate the difference between the algorithm-computed angle of arrival (AOA) and the real AOA of the moving desired user. Let us assume that at the beginning of the communication session the desired user is at the start position and moves in the direction to the target position, as illustrated in **Fig.2**

III. Proposed algorithm

The error signal, $e(n)$, the difference between the array output and the desired signal, is given as:

$$e(n) = d(n) - y(n) \qquad (3)$$

By applying the LMS adaptive algorithm [7], the weight vector is updated by the following equation:

$$\underline{w}(n+1) = \underline{w}(n) - \mu(n)e^*(n)\underline{x}(n) \qquad (4)$$

where the step size $\mu(n)$ is defined as a fuzzy variable whose value depends on the mean square error, $MSE(e(n))$, where $MSE(e(n))$ is given by:

$$MSE(e(n)) = E[e^2(n)] \qquad (5)$$

The $MSE(e(n))$ is taken as the input of the Fuzzy Inference System (FIS). The linguistic concept of the relation between the step size value and the $MSE(e(n))$ parameter is denoted as follows:

$$\mu(n) = FIS(MSE(e(n))) \qquad (6)$$

where $FIS(\ )$ is the Fuzzy Inference Function whose operation based on the fuzzy *IF-THEN* rules presented later. The membership function and the fuzzy variable of mean square error and step size are defined and illustrated in **Fig.3** and **Fig.4**, where MBF_1 and MBF_2 are membership functions of mean square error and step size, respectively. The fuzzy variable of mean square error and the step size are denoted by the linguistic set of {*Zero (ZE), Small (S), Medium (M), Large (L), Huge (H)*}. The centroids of the membership function MBF_2 are the same form of the maximum step size (μ_{max}), which satisfies the step size bound [8] as the follows:

$$\mu = \frac{M}{3tr(R)} \tag{7}$$

$$\mu_{max} = \frac{2}{3tr(R)}$$

where $tr(R)$ represents the trace of the autocorrelation matrix of received signal, the factor M is set to 0.0005; 0.001; 0.005; 0.01; 0.05 for μ_{ZE} ; μ_S ; μ_M ; μ_L ; ;μ_H , respectively.

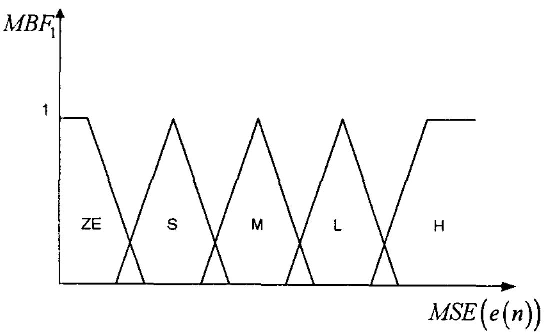

Fig.3: Definition of fuzzy mean square error variable

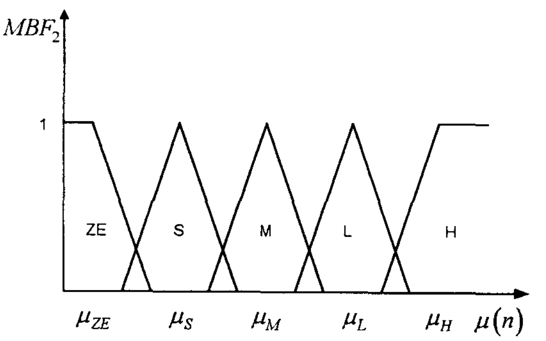

Fig.4: Definition of fuzzy step size variable

Here, the fuzzy variable step size $\mu(n)$ is converted or defuzzified by applying Centroid of Area (COA) defuzzification method [9] as follows:

$$\mu(n) = \frac{\sum_{k=1}^{N} \mu_k MBF_2(\mu_k)}{\sum_{k=1}^{N} MBF(\mu_k)} \tag{9}$$

where N is the number of discrete samples of membership function, μ_k is the value at the location used in approximating the area under the aggregated membership function, and $MBF(\mu_k)$ is the value of membership function at μ_k.

The fuzzy *IF-THEN* rules is constructed as follows (10):

IF $MSE(e(n))$ is *Zero THEN* step size is *Huge*
IF $MSE(e(n))$ is *Small THEN* step size is *Large*
IF $MSE(e(n))$ is *Medium THEN* step size is *Medium*
IF $MSE(e(n))$ is *Large THEN* step size is *Small*
IF $MSE(e(n))$ is *Huge THEN* step size is *Zero*

The above rules were selected mainly due to the following reasons. When $MSE(e(n))$ is *Zero, i.e.*, the mis-adjustment is low, therefore the step size is assigned *Huge* for achieving the more rapid speed of convergence. Whereas, $MSE(e(n))$ is *Huge*, this means that the mis-adjustment is high, thus an appropriate step size adjustment for stability is required, in order to gain stability the step size is assigned to *Zero*.

The proposed algorithm can be summarized as follows:

1. Initialize:
   $\underline{w}(0) = 0, n = 0$
2. Update the weight vector at time n:
   -Array output signal: $y(n) = \underline{w}^H(n)\underline{x}(n)$
   -Generate error signal: $e(n) = d(n) - y(n)$
   -Defuzzify the step size follows the rules (10):
   $$\mu(n) = FIS(MSE(e(n)))$$
   -Update weight vector:
   $$\underline{w}(n) = \underline{w}(n-1) - \mu(n)e\ (n)\underline{x}(n)$$

3. Iterate until the weight vector converges.

IV. Computer Simulations

In this section, the convergence behavior and the tracking capacity of the proposed algorithm for the base station with adaptive antennas in CDMA systems are presented. In the simulations, the distance between adjacent elements of antenna array is set to a half of wave-length ($\lambda/2$), the Gold codes spreading sequences with the processing gain of 64 were used. The modulation scheme used for the simulations is assumed to be BPSK. In addition, the number of active users is 3, the number of elements of antenna array is 6. The incident angle of the desired user is initialized -40^0, the undesired users incident angle are randomized. The desired user is assumed to move at velocity that results in the equivalent Doppler frequency is 66.67 (Hz). The multipath Rayleigh fading environment is taken into consideration and the number of multipaths is assumed to be 30.

Fig.5 illustrates the learning curve of the proposed algorithm and comparison with fixed step size LMS (μ=0.0001), the variable step size LMS (μ_c=0.0001) [3] and the fuzzy step size FSSE_LMS algorithm [6]. In this simulation, the SNR is kept at 7dB. From the figure, the convergence speed of the proposed approach is superior to the fixed step size conventional LMS algorithm and the other variable step size algorithms compared here.

Fig.6 presents the tracking ability of the proposed approach and its comparison with the fixed step size LMS (μ=0.0001), the variable step size LMS (μ_c=0.0001) [3] and the FSSE-LMS [6] in term of the difference between the algorithm-computed AOA and the AOA of the desired moving user versus angle step at a snapshot. In this simulation, the angle step at a snapshot is varied from 0.01^0 to 0.1^0, whereas the SNR is kept still at 7 dB. The figure shows that when the value of angle step at a snapshot increases, the AOA deviation resulted from the fixed step size LMS (μ=0.0001) grows much more steeply than those of the other variable step size LMS algorithms. For instance, when the angle step is 0.08^0, these differences of the fixed step size LMS, FSSE-LMS [6], the proposed variable step size LMS, and variable step size LMS (μ_c=0.0001) [3] are 76^0, 7^0, 0^0, 3^0, respectively. Moreover, when the angle step increases, the deviation of algorithm-computed AOA of the FSSE-LMS [6] and variable step size LMS [3] increase slightly, while the deviation of AOA resulted from the proposed algorithm is still around zero.

V. Conclusions

In this paper, a new application of fuzzy logic to variable step size LMS algorithms for adaptive antennas has been presented. In particular, the step size adjustment is based on the fuzzy *IF-THEN* rules that are interpreted from linguistic meaning of the mean square error (MSE). The proposed algorithm has a better performance as compared to the fixed step size LMS algorithm and other variable step size in term of the convergence speed and the reduction of steady-state error as well.

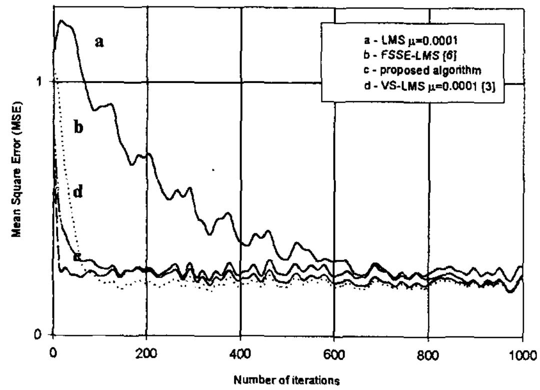

Fig.5: Learning curve comparison of the proposed algorithm and other conventional LMS algorithms

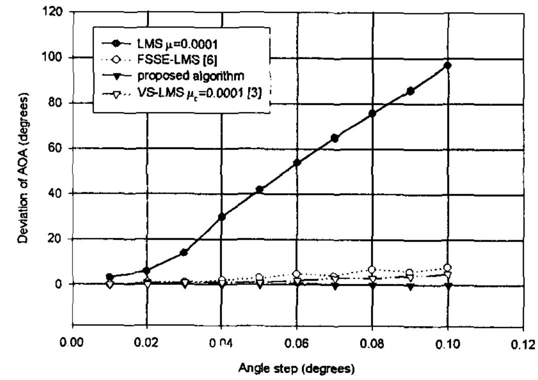

Fig.6: Tracking capacity comparison of the proposed algorithm and other conventional LMS algorithms in term of the deviation of AOA's

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