

# A Modified MMSE Algorithm for Adaptive Antennas in OFDM/CDMA Systems

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## Abstract

This paper presents a semi-blind Minimum Mean Square Error (MMSE) beamforming adaptive algorithm used for OFDM/CDMA combined system. The proposed algorithm exploits the transmitting pilot signal in the initial period of the transmission to update the weight vector. Then it applies the blind adaptive period to update the weight vector, in which the pilot signal is no longer used. The derivation of the algorithm based on the Mean Square Error (MSE) criterion is also presented. Computer simulation is carried out to verify the performance of the proposed approach.

## Keywords

Beamforming algorithm, adaptive antenna, OFDM, MC-CDMA, MMSE

## I. Introduction

Recently, the combinations of Orthogonal Frequency Division Multiplexing (OFDM) and Code Division Multiple Access (CDMA) (so-called MC-CDMA) [1], [2], have been considered a leading candidate for the next generation high-speed wireless/mobile communication system. With the aid of OFDM technique, MC-CDMA has long symbol duration, *i.e.* narrow bandwidth. Consequently, the system gains a noticeable advantage of robustness in selective-fading environment [3], [4]. Moreover, by using OFDM, the baseband process of system can be efficiently implemented by using the Fast Fourier Transform (FFT) or Inverse Fast Fourier Transform (IFFT).

Adaptive antennas, another approach to improve system capacity and performance, have been well studied for wireless communication systems. An important aspect in adaptive antennas is the adaptive beamforming, which provides much enhanced coverage through range of extension, robustness to system perturbation [5]. Thus, the co-channel interference and multipath distortion of the systems can be eliminated.

The application of adaptive antenna in MC-CDMA is a very promising approach in mitigating the effects of multi user interference (MUI) and the multipath fading effect as well. However, for applying to MC-CDMA systems, the adaptive beamforming algorithms should be changed. Some works on this problem have been presented [6]-[8]. In [7], the authors calculate the error signals between pilot signals and the corresponding received signals of the

desired user, and then transfer the error signals into time domain to update the weight vector.

In this paper, we propose a semi-blind MMSE algorithm derived from MSE criterion in which the weight vector is updated in two periods: in the first period it is updated by the blocks of pilot signals which are known at both transmitter and receiver; then it is updated blindly by the transmitted information signal. This paper is organized as follows. In section II, the system and the signal models are presented. The proposed algorithm is introduced in section III. Section IV presents the numerical and simulations results. Finally, the conclusions are given in section V.

## II. System model

The transmitter scheme is described in Fig.1, and receiver scheme is presented in Fig.2. In the system, there are  $M$  signals impinging at an adaptive antenna of  $K$  elements ( $K \geq M$ ).

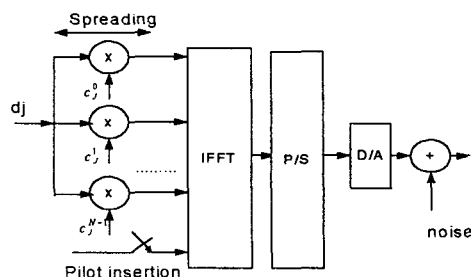


Fig.1: Scheme of MC-CDMA transmitter system

At the transmitter side, each user data symbol,  $d_j^n$ , of the  $j$ -th user,  $j=1..M$ , transmitted during the  $n$ -th symbol block, is multiplied by specific orthogonal spreading code of length  $N$  for each user. The  $i$ -th symbol,  $y_{j,i}(n)$ , (or called chip) in frequency domain, which is spread from the user data symbol of the  $j$ -th user, is then transformed into time domain signal by applying the Inverse Fast Fourier Transform (IFFT) as follows:

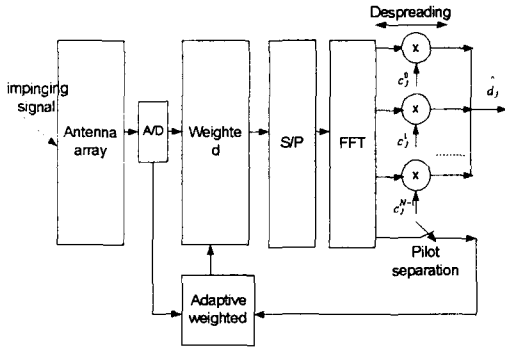


Fig. 2: Scheme of MC-CDMA receiver with adaptive antennas

$$x_{j,m}(n) = \sum_{k=0}^{N-1} y_{j,k}(n) e^{-j2\pi mk/N}$$

$$y_{j,k}(n) = d_j^n c_j^k \quad (k = 0, 1, \dots, N-1)$$

(1)

where  $x_{j,m}(n)$  is the  $m$ -th sample in time domain of the  $j$ -th user of the  $n$ -th block, and  $y_{j,k}(n)$  is symbol of the  $j$ -th user modulated by the  $k$ -th subcarrier,  $c_j^k$  is chip spreading sequence of the  $j$ -th user. The time domain signal  $x_{j,m}(n)$  is parallel-to-serial (S/P) converted, and then Digital/Analog (D/A) converted to transmit through wireless channel. The formula (1) can be written in vector form as:

$$\underline{x}_j(n) = \underline{y}_j(n) F^H(n)$$

(2)

where  $\underline{x}_j(n) = [x_{j,0}(n) \ x_{j,1}(n) \ \dots \ x_{j,N-1}(n)]$  is the signal vector of the  $j$ -th user in the time domain, corresponding to the  $n$ -th OFDM symbol block, and

$$\underline{y}_j(n) = [y_{j,0}(n) \ \dots \ y_{j,N-1}(n)]$$

is the signal vector of the  $j$ -th user in the frequency domain, corresponding to symbols sequence  $y_{j,i}(n)$ , and

$$F(n) = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{-j2\pi(1)0/N} & \dots & e^{-j2\pi(1)(N-1)/N} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi(N-1)0/N} & \dots & e^{-j2\pi(N-1)(N-1)/N} \end{bmatrix}$$

is the FFT transformation matrix representation for the FFT operation. So the received signal matrix at the antenna array can be written as:

$$V(n) = A(\theta) X(n) + N(n)$$

(3)

where  $V(n)$  is the received signal matrix of system at antenna arrays,  $X(n)$  is the impinging signal matrix of  $M$  users,  $A(\theta)$  is the array response matrix of array antennas,  $N(n)$  is the matrix of additive white Gaussian noise. They are defined as follows:

$$V(n) = [\underline{v}_0(n) \ \underline{v}_1(n) \ \dots \ \underline{v}_{K-1}(n)]^T$$

$$\underline{v}_i(n) = [v_{i,0}(n) \ v_{i,1}(n) \ \dots \ v_{i,N-1}(n)] \quad (i=0,1,\dots,K-1)$$

$$X(n) = [\underline{x}_0(n) \ \underline{x}_1(n) \ \dots \ \underline{x}_{M-1}(n)]^T$$

$$\underline{x}_i(n) = [x_{i,0}(n) \ x_{i,1}(n) \ \dots \ x_{i,N-1}(n)] \quad (i=0,1,\dots,M-1)$$

$$A(\theta) = [\underline{a}_0(\theta) \ \underline{a}_1(\theta) \ \dots \ \underline{a}_{K-1}(\theta)]^T$$

$$\underline{a}_i(\theta) = [a_i^0(\theta) \ a_i^1(\theta) \ \dots \ a_i^{M-1}(\theta)] \quad (i=0,1,\dots,K-1)$$

$$N(n) = [\underline{n}_0(n) \ \underline{n}_1(n) \ \dots \ \underline{n}_{K-1}(n)]^T$$

$$\underline{n}_i(n) = [n_{i,0}(n) \ n_{i,1}(n) \ \dots \ n_{i,N-1}(n)] \quad (i=0,1,\dots,K-1)$$

For convenience, we assume that the desired user is the 0-th user. So the signal vector of the desired user  $\underline{s}_0(n)$  is obtained by multiplying  $V(n)$  with the adaptive weight vector  $\underline{w}_0(n)$  as:

$$\begin{aligned}\underline{s}_0(n) &= \underline{w}_0^H(n)V(n) \\ &= [s_{0,0}(n) \ s_{0,1}(n) \ \cdots \ s_{0,N-1}(n)]\end{aligned}\quad (4)$$

and the weight vector is defined as follows:

$$\underline{w}_0(n) = [w_{0,0}(n) \ w_{0,1}(n) \ \cdots \ w_{0,K-1}(n)]^T$$

The weighted signal vector  $\underline{s}_0(n)$  is then converted into the frequency domain, thus yielding:

$$\begin{aligned}\tilde{\underline{y}}_0(n) &= \underline{s}_0(n)F(n) \\ &= [\tilde{y}_{0,0} \ \tilde{y}_{0,1} \ \cdots \ \tilde{y}_{0,N-1}]\end{aligned}\quad (5)$$

After that, the received signal vector in the frequency domain  $\tilde{\underline{y}}_0(n)$  is despread so as to retrieve the original signal as the following:

$$\hat{d}_0^n = \sum_{k=0}^{N-1} \tilde{y}_{0,k} c_0^k \quad (6)$$

### III. The proposed algorithms

#### 1. Training process

First, we use pilot signal as training sequence. The weight vector is updated by the blocks of pilot signal, which are known at both transmitter and receiver.

At the receiver, let us denote the received pilot signal and the reference pilot signal in the frequency domain as  $\tilde{\underline{y}}_{0,p}(n)$ ,  $\underline{y}_{0,p}(n)$ , respectively. Here the received pilot signal  $\tilde{\underline{y}}_{0,p}(n)$  is achieved after FFT demodulation as in (5). Thus the received pilot signal vector and the reference pilot signal vector corresponding in time domain  $\tilde{\underline{x}}_{0,p}(n)$ ,  $\underline{x}_{0,p}(n)$  can be expressed as:

$$\tilde{\underline{x}}_{0,p}(n) = \tilde{\underline{y}}_{0,p}(n)F^H(n) \quad (7)$$

$$\underline{x}_{0,p}(n) = \underline{y}_{0,p}(n)F^H(n) \quad (8)$$

where  $F^H(n)$  is IFFT transformation matrix. From the equations (4), (5) and (7), we define:

$$\begin{aligned}U_p(n) &= [\underline{u}_{0,p}(n) \ \underline{u}_{1,p}(n) \ \cdots \ \underline{u}_{N-1,p}(n)] \\ &= V(n)F(n)F^H(n)\end{aligned}\quad (9)$$

$\underline{u}_{i,p}(n) = [u_{i,p}^0(n) \ u_{i,p}^1(n) \ \cdots \ u_{i,p}^{K-1}(n)]^T$  ( $i=0,1,\dots,N-1$ ), and received pilot signal matrix  $V(n)$  is defined by (3). So the received pilot signal  $\tilde{\underline{x}}_{0,p}(n)$  can be rewritten as:

$$\begin{aligned}\tilde{\underline{x}}_{0,p}(n) &= \underline{w}_0^H(n)U_p(n) \\ &= [x_{0,p}^0(n) \ x_{0,p}^1(n) \ \cdots \ x_{0,p}^{N-1}(n)]\end{aligned}\quad (10)$$

The adaptive beamforming algorithm, which is based on MSE criterion, is to find out the optimum solution to minimize the cost function in time-domain defined by:

$$J(n) = \sum_{i=0}^{N-1} E \left[ \left| \underline{w}_0^H(n) \underline{u}_{i,p}(n) - x_{0,p}^i(n) \right|^2 \right]$$

And the gradient of  $J(n)$  with respect to the weight vector  $\underline{w}_0(n)$  is given by:

$$\nabla J(n) = \sum_{i=0}^{N-1} [2R_{i,p}(n)\underline{w}_0(n) - 2\underline{p}_{i,p}(n)] \quad (11)$$

where  $R_{i,p}(n) = E[\underline{u}_{i,p}(n)\underline{u}_{i,p}^H(n)]$  is the correlation matrix of the received pilot signal, and  $\underline{p}_{i,p}(n) = E[\underline{u}_{i,p}(n)x_{0,p}^i(n)^*]$  is the cross-correlation vector between the received pilot signal and the pilot signal at the receiver.

An adaptive solution, which minimizes the cost function  $J(n)$ , is [5]:

$$\underline{w}_0(n+1) = \underline{w}_0(n) - \frac{1}{2}\mu\nabla J(n) \quad (12)$$

From equations (12), (13) the weight vector is updated as follows:

$$\begin{aligned}\underline{w}_0(n+1) &= \underline{w}_0(n) \\ &\quad - \mu \sum_{i=0}^{N-1} [R_{i,p}(n)\underline{w}_0(n) - \underline{p}_{i,p}(n)]\end{aligned}\quad (13)$$

In this algorithm, the correlation matrix and cross-correlation vector are estimated as follows [9]:

$$R_{i,p}(n) = f * R_{i,p}(n-1) + \underline{u}_{i,p}(n) \underline{u}_{i,p}^H(n)$$

$$\underline{p}_{i,p}(n) = f * \underline{p}_{i,p}(n-1) + \underline{u}_{i,p}(n) x_{0,p}^{i*}(n)$$
(14)

where  $f, 0 \leq f < 1$ , is forgetting factor.

### 2. Blind adaptive process

In this period, the received pilot signal and the reference pilot signal are now replaced by the received information signal. The equation (10) is now given:

$$U(n) = [\underline{u}_0(n) \quad \underline{u}_1(n) \quad \dots \quad \underline{u}_{N-1}(n)]$$

$$= V(n) F(n) F^H(n)$$
(15)

$$\underline{u}_i(n) = [u_i^0(n) \quad u_i^1(n) \quad \dots \quad u_i^{K-1}(n)]^T$$
(16)

where  $V(n)$  is received signal matrix, which is defined by (3). The received signal in frequency domain  $\underline{y}_0(n)$ , which is defined in (5), is now projected onto discrete value to yield estimation vector  $\underline{y}_0(n)$ .

The estimation vector  $\underline{y}_0(n)$  of received signal is transformed into time domain as:

$$\underline{x}_0(n) = \underline{y}_0(n) F^H(n)$$

$$= [x_0^0(n) \quad x_0^1(n) \quad \dots \quad x_0^{N-1}(n)]$$
(17)

This signal is applied directly for the process of updating the weight vector.

The correlation matrix and the cross-correlation vector given in (14) can be rewritten as follows:

$$R_i(n) = f * R_i(n-1) + \underline{u}_i(n) \underline{u}_i^H(n)$$

$$\underline{p}_i(n) = f * \underline{p}_i(n-1) + \underline{u}_i(n) x_0^{i*}(n)$$
(18)

where  $x_0^i(n)$  is the reference signal at the receiver given in (17),  $\underline{u}_i(n)$  is defined in (15), (16). Then, the weight vector is updated as (13).

### IV. Simulation results

In the simulations, we consider the MC-CDMA system mentioned in Section II. The spacing of element antenna arrays was set to  $\lambda/2$ , and the Gold codes spreading sequences with

the processing gain of 32 were used. The number of symbols per OFDM block was also set to 32. The synchronization of system is assumed to be proper so that the effect of frequency offset can be disregarded. In addition, user data symbol is assumed to be BPSK modulated. The number of active users is set to 5. The incident angle of the desired user signal was fixed  $20^\circ$ , the incident angle of undesired users is randomized. The number of blocks for training is 500.

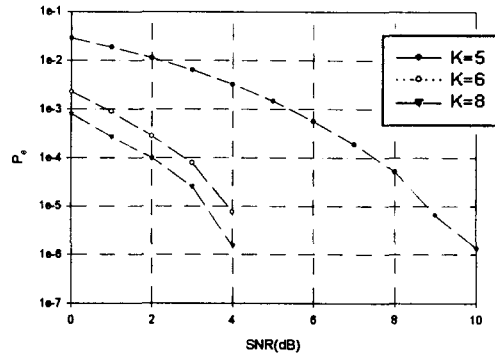


Fig.3: Probability of bit error of the proposed algorithm on MC-CDMA system with adaptive antennas in AWGN channel for different value of array elements (K)

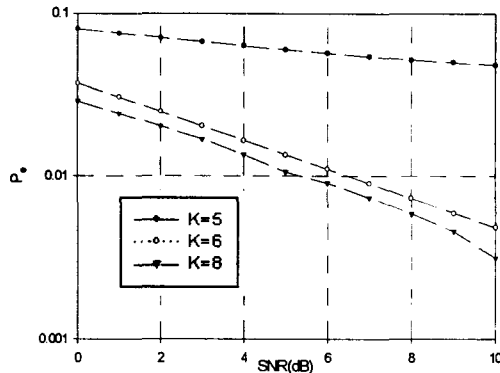


Fig.4: Probability of bit error of the proposed algorithm on MC-CDMA system with adaptive antennas in Rayleigh fading channel with AWGN for different value of array elements (K)

The probability of bit error ( $P_e$ ) versus the SNR performance of the proposed algorithm in MC-CDMA system in AWGN channel is illustrated in Fig.3. It can be seen that the more elements of array result in the lower  $P_e$  values. Similar results are also observed in multipath Rayleigh fading channel with AWGN, as shown

in Fig.4 where the number of multipaths is 30 and the velocity of user is assumed so that the Doppler frequency equivalent is 66.67Hz.

### V. Conclusions

In this paper, the semi-blind MMSE approach for updating the weight vector of the antenna array is presented. By using the proposed semi-blind adaptive beamforming algorithm in MC-CDMA system, we can improve the system performance not only in synchronization, but also in robustness in selective fading. Furthermore, the simulations also show the significant improvement in  $P_e$  performance. Thus, the proposed algorithm for adaptive antenna arrays applied to the MC-CDMA system is promising technique for efficiently high data rate transmission over mobile wireless channel.

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