# A New Adaptive Beamforming Algorithm for Smart Antennas Applied to an OFDM System

Le Minh Tuan\*, Pham Van Su\*, Jewoo Kim\*\* and Giwan Yoon\*
\*School of Eng., Information & Communications University (ICU) 58-4, Hwaam-dong, Yusong-gu, Taejon
305-348, Korea

letuan@icu.ac.kr
Tel.: +82-42-866-6201 Fax.: +82-42-866-6227
\*\*TeleCIS, Inc., 418 South Abbott Avenue, Milpitas, CA 95129, U.S.A.

#### Abstract

In this paper, we consider an OFDM system with cochannel interference and the use of adaptive antenna arrays to suppress such interference. Based on the conventional RLS criterion, we derive a new Recursive Least Square (RLS) adaptive beamforming algorithm for antenna arrays applied in an OFDM system. Computer simulation shows that, when applied to the OFDM system, the proposed algorithm is capable of combating cochannel interference in both AWGN channel and multipath Rayleigh fading channel with AWGN.

#### I. Introduction

Future wireless communication systems must be able to accommodate a huge number of subscribers. In other words, they must meet the demands for high data rate and large capacity. Nonetheless, multipath fading effects and cochannel interference are the main limitations facing such systems. Therefore, finding approaches to overcome these limitations is of great importance. One possible solution for these systems to be realized is to use Orthogonal Frequency Division Multiplexing (OFDM) combined with adaptive antenna arrays.

OFDM [1]-[2] has proven to be a potential candidate for increasing bit rate and capacity in wireless communications. By using OFDM, bandwidth efficiency can be achieved. Furthermore, it is one of the most effective approaches for combating multipath delay spread in wireless communication systems. Another key advantage of using OFDM is that the modulation and demodulation can be achieved in the discrete-domain by using a discrete Fourier transform (DFT) [3], which is efficiently implemented by using the FFT.

It is also well known that adaptive antenna arrays have been introduced in the literature for TDMA and CDMA systems to improve the performances of such systems [4]-[5]. By using antenna arrays, rapid dispersive fading can be mitigated and co-channel interference can be suppressed, therefore communication capacity can be improved. More recently, smart antennas have

been applied to OFDM systems [6]-[7]. In [6], the authors explore the combination of adaptive antennas and OFDM for operation in a faded delay-spread channel. Antenna array is also used to suppress delay signal and Doppler-shifted signal in [7]. However, due to the use of the inverse FFT (IFFT) and FFT processing in transmitter and receiver, respectively, new adaptive beamforming techniques are required for the OFDM system.

In this paper, we propose a new RLS adaptive algorithm for an OFDM system. In the proposed algorithm, a short training process first updates the weight vector. Then a decision-directed technique is used for updating the weight vector. Both processes are performed in the time domain. Simulation results show that the proposed algorithm is able to extract the desired signal while it compresses other undesirable cochannel interferers. Thus, the resulting system has a very good performance.

# II. OFDM System with adaptive antenna array

First, let us consider the transmitter side of the OFDM system. N multiplexed symbols in the frequency-domain are transformed in the time domain signal by the IFFT as follows:

$$x_{m,l}(n) = \sum_{k=0}^{N-1} y_{m,k}(n) e^{j2\pi l k/N} \qquad l = 0,1,...,N-1$$
(1)

where  $y_{m,k}(n)$  is the symbol of the  $m^{th}$  user carried by the  $k^{th}$  subcarrier of the  $n^{th}$  block.  $x_{m,l}(n)$  is the  $l^{th}$  time-domain sample of the  $n^{th}$  block. Equation (1) can be rewritten in the vector form as follows:

$$\underline{x}_{m}(n) = \underline{y}_{m}(n)\mathbf{F}^{H}(n) \tag{2}$$

where

 $\underline{x}_m(n) = [x_{m,0}(n) \quad x_{m,1}(n) \quad \dots \quad x_{m,N-1}(n)]$  is the signal vector of the  $m^{th}$  user in the time domain.

$$\mathbf{F}(n) = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{-j2\pi(1)(1)/N} & \dots & e^{-j2\pi(1)(N-1)/N} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi(N-1)(1)/N} & \dots & e^{-j2\pi(N-1)(N-1)/N} \end{bmatrix}$$

represents the FFT operation matrix.

 $\boldsymbol{H}$  denotes the Hermitian transpose.

$$\underline{y}_m(n) = [y_{m,0}(n) \quad y_{m,1}(n) \quad \dots \quad y_{m,N-1}(n)]$$
 is the corresponding signal vector in the frequency domain.

Now we consider the receiver side. Assume that there are M signals impinging at an array of K elements and that  $K \ge M$ . The signal matrix, V(n), received at the array antenna is represented by:

$$V(n) = A(\theta)X(n) + G(n)$$
where

where,
$$V(n) = \begin{bmatrix} v_{0,0}(n) & v_{0,1}(n) & \dots & v_{0,N-1}(n) \\ v_{1,0}(n) & v_{1,1}(n) & \dots & v_{1,N-1}(n) \\ \vdots & \vdots & \ddots & \vdots \\ v_{K-1,0}(n) & v_{K-1,1}(n) & \dots & v_{K-1,N-1}(n) \end{bmatrix}$$

$$X(n) = \begin{bmatrix} x_{0,0}(n) & x_{0,1}(n) & \dots & x_{0,N-1}(n) \\ x_{1,0}(n) & x_{1,1}(n) & \dots & x_{1,N-1}(n) \\ \vdots & \vdots & \ddots & \vdots \\ x_{M-1,0}(n) & x_{M-1,1}(n) & \dots & x_{M-1,N-1}(n) \end{bmatrix}$$

$$A(\theta) = \begin{bmatrix} a_{0}(\theta_{0}) & a_{0}(\theta_{1}) & \dots & a_{0}(\theta_{M-1}) \\ a_{1}(\theta_{0}) & a_{1}(\theta_{1}) & \dots & a_{1}(\theta_{M-1}) \\ \vdots & \vdots & \ddots & \vdots \\ a_{K-1}(\theta_{0}) & a_{K-1}(\theta_{1}) & \dots & a_{K-1}(\theta_{M-1}) \end{bmatrix}$$

Here,  $A(\theta)$  is the array response matrix for M users and G(n) is the K by N matrix of additive white Gaussian noises.

The desired signal vector of the  $m^{th}$  user,  $\underline{s}_{m}(n)$ , is given by:

$$\underline{s}_{m}(n) = \underline{w}_{m}^{H}(n)V(n)$$
where,
$$\underline{s}_{m}(n) = \begin{bmatrix} s_{m,0}(n) & s_{m,1}(n) & \dots & s_{m,N-1}(n) \end{bmatrix}$$

$$\underline{w}_{m}(n) = \begin{bmatrix} w_{m,0}(n) & w_{m,1}(n) & \dots & w_{m,K-1}(n) \end{bmatrix}^{T}$$

and T denotes the transpose.

The weighted signal vector  $\underline{s}_m(n)$  is then converted into the frequency-domain, thus yielding:

$$\underbrace{\widetilde{y}}_{m}(n) = \underline{s}_{m}(n)\mathbf{F}(n) = \underline{w}_{m}^{H}(n)V(n)\mathbf{F}(n)$$
(5)

This signal vector will be decoded to recover the original data from the transmitter. Let us denote the signal vector at the output of the decoder as  $y_{dm}(n)$ .

# III. The proposed RLS algorithm

In order to update the weight vector, in this paper we propose a new RLS algorithm. The main principle of this criterion is that before transmitting the information bits (or information signal), the weight vector of the array is updated by a number of blocks of pilot signal, which are known by both transmitter and receiver. After training the weight vector of the array, information signal is transmitted. In the receiver side, the weight vector now is updated by blind method. That is, pilot signal is no longer used. The reference signal used for updating the weight vector is the received signal vector,  $y_{clm}(n)$ , at the output of the decoder.

## 1. Training period:

First, let us consider the training period in the receiver side. In this period only training signal is transmitted so as to train the weight vector of the array antenna. Note that, in the frequency-domain, both the received pilot signal and the received information signal are given by equation (5). In the time-domain, the pilot signal vector  $\underline{x}_{pm}(n)$  and the received pilot signal vector  $\underline{x}_{pm}(n)$  can be found from the corresponding frequency-domain pilot signal vector  $\underline{y}_{pm}(n)$  and the received pilot signal vector  $\underline{y}_{pm}(n)$  and the received pilot signal vector  $\underline{y}_{pm}(n)$  as follows:

$$\underline{x}_{pm}(n) = \left[x_{pm,0}(n) \ x_{pm,1}(n) \ \dots \ x_{pm,N-1}(n)\right]$$

$$= \underline{y}_{pm}(n) \mathbf{F}^{H}(n)$$
(6)

and

$$\widetilde{\underline{x}}_{\rho m}(n) = \widetilde{\underline{y}}_{\rho m}(n) \mathbf{F}^{H}(n) = \underline{w}_{m}^{H} V(n) \mathbf{F}(n) \mathbf{F}^{H}(n) \quad (7)$$

where, F(n) is the FFT operation matrix.

Now let
$$U_{pm}(n) = \left[\underline{u}_{pm,0}(n) \cdots \underline{u}_{pm,N-1}(n)\right]$$

$$= V(n)\mathbf{F}(n)\mathbf{F}^{H}(n)$$
(8)

$$\underline{u}_{pm,i}(n) = \begin{bmatrix} u_{pm,i}^{0}(n) & u_{pm,i}^{1}(n) & \cdots & u_{pm,i}^{K-1}(n) \end{bmatrix}^{T}$$

$$i = 0,1,\ldots,N-1$$

Then the received pilot signal vector in equation (7) can be rewritten as:

$$\widetilde{\underline{x}}_{pm}(n) = \underline{w}_{m}^{H} U_{pm}(n) 
= \left[ \underline{w}_{m}^{H} \underline{u}_{pm,0}(n) \quad \underline{w}_{m}^{H} \underline{u}_{pm,1}(n) \quad \dots \quad \underline{w}_{m}^{H} \underline{u}_{pm,N-1}(n) \right]$$

Based on the conventional cost function for RLS algorithm [8], we propose a new cost function for the RLS beamforming algorithm applied in an OFDM system as follows:

$$J(n) = \sum_{k=0}^{N-1} \left[ \sum_{k=1}^{n} \lambda^{n-k} \left| e_i(k) \right|^2 \right]$$
 (10)

Here, N is the number of symbols per OFDM block or number of sub-carriers in a block.

 $e_{i}(k)$  is the difference between the time-domain reference signal  $x_{pm,i}(k)$  and the time-domain array output  $\widetilde{x}_{pm,i}(k)$  at time k. That is,  $e_i(k)$  is defined

$$e_{i}(k) = x_{pm,i}(k) - \widetilde{x}_{pm,i}(k)$$

$$= x_{pm,i}(k) - \underline{w}_{m}^{H}(n)\underline{u}_{pm,i}(k)$$
(11)

 $\underline{w}_{m}(n)$  is the weight vector for the  $m^{th}$  user at time

From the cost function given by (10) and by using the approach similar to that presented in [8], the weight vector is updated by the following equation:

$$\underline{\hat{w}}_{m}(n) = \underline{\hat{w}}_{m}(n-1) + \frac{1}{N} \sum_{i=0}^{N-1} \underline{q}_{i}(n) \eta_{i}^{*}(n)$$
 (12)

where 
$$\underline{q}_{i}(n) = \frac{\lambda^{-1}R_{i}(n-1)\underline{u}_{pm,i}(n)}{1+\lambda^{-1}\underline{u}_{pm,i}^{H}(n)R_{i}(n-1)\underline{u}_{pm,i}(n)}$$

$$R_i(n) = \lambda^{-1} R_i(n-1) - \lambda^{-1} q_i(n) \underline{u}_{pm,i}^H(n) R_i(n-1)$$

and 
$$n_i(n) = x_{pm,i}(n) - \underline{\hat{w}}_m^H(n-1)\underline{u}_{pm,i}(n)$$

Thus, during training period, the proposed RLS algorithm can be summarized as follows:

#### 1. Initialize:

$$R_i(0) = \delta^{-1}I$$
,  $i = 0, 1, \dots, N-1$ ,  $\delta$  is a small constant.

$$\hat{w}_{m}(0) = 0.$$

2. Update the weight vector, n = n + 1Receive a new signal matrix V(n)

Calculate:

$$U_{nm}(n) = V(n)\mathbf{F}(n)\mathbf{F}^{H}(n)$$

$$\underline{q}_{i}(n) = \frac{\lambda^{-1}R_{i}(n-1)\underline{u}_{pm,i}(n)}{1+\lambda^{-1}\underline{u}_{pm,i}^{H}(n)R_{i}(n-1)\underline{u}_{pm,i}(n)}$$

$$R_{i}(n) = \lambda^{-1}R_{i}(n-1)-\lambda^{-1}q_{i}(n)\underline{u}_{pm,i}^{H}(n)R_{i}(n-1)$$

$$i=0,1,\cdots,N-1$$

$$\underline{\hat{w}}_{m}(n) = \underline{\hat{w}}_{m}(n-1) + \frac{1}{N} \sum_{i=1}^{N-1} \underline{q}_{i}(n) \eta_{i}^{*}(n)$$

3. Iterate step 2) until the weight vector converges

### 2. Communication period:

In this period a decision-directed approach is applied for updating the weight vector. The process for updating the weight vector is almost the same as that in the training period, except for the followings:

The received information signal matrix  $U_m(n)$  now replaces the received pilot signal matrix  $U_{pm}(n)$  and is still calculated by using equation (8).

-The time-domain reference signal is now created from the decoder output  $y_{dec}(n)$  and is

$$\underline{x}_{dm}(n) = [x_{dm,0}(n) \quad \cdots \quad x_{dm,N-1}(n)] = \underline{y}_{dm}(n)\mathbf{F}^{H}(n)$$

-The received signal in the time domain before decoding is now given by:

$$\underbrace{\widetilde{\mathbf{x}}_{m}(n)} = \underbrace{\widetilde{\mathbf{y}}_{m}(n)} \mathbf{F}^{H}(n) = \underbrace{\mathbf{w}^{H}_{m}} V(n) \mathbf{F}(n) \mathbf{F}^{H}(n)$$
(14)

Or, equivalently

$$\widetilde{\underline{x}}_{m}(n) = \underline{w}_{m}^{H} U_{m}(n) 
= \left[\underline{w}_{m}^{H} \underline{u}_{m,0}(n) \cdots \underline{w}_{m}^{H} \underline{u}_{m,N-1}(n)\right]$$
(15)

During this period, the proposed algorithm

works as follows:

4. Receive a new information signal matrix V(n), n=n+1

- Calculate the output of the array:

$$\underline{s}_{m}(n) = \hat{\underline{w}}_{m}^{H}(n-1)V(n)$$

- Convert into the frequency domain

$$\widetilde{y}_{m}(n) = \underline{s}_{m}(n)F(n) = \underline{w}_{m}^{H}(n)V(n)F(n)$$

- Decode to retrieve the original signal, denoted as  $\underline{y}_{dm}(n)$ .
- 5. Update the weight vector,
- Find the reference signal by converting  $y_{m}(n)$  into the time domain as in (34)

$$\underline{x}_{dm}(n) = [x_{dm,0}(n) \quad x_{dm,1}(n) \quad \dots \quad x_{dm,N-1}(n)]$$
  
=  $y_{\perp}(n)\mathbf{F}^{H}(n)$ 

Calculate

$$U_{m}(n) = \left[\underline{u}_{m,0}(n) \cdots \underline{u}_{m,N-1}(n)\right] = V(n)\mathbf{F}(n)\mathbf{F}^{H}(n)$$

$$\underline{q}_{i}(n) = \frac{\lambda^{-1}R_{i}(n-1)\underline{u}_{m,i}(n)}{1 + \lambda^{-1}\underline{u}_{m,i}^{H}(n)R_{i}(n-1)\underline{u}_{m,i}(n)}$$

$$R_{i}(n) = \lambda^{-1}R_{i}(n-1) - \lambda^{-1}\underline{q}_{i}(n)\underline{u}_{m,i}^{H}(n)R_{i}(n-1)$$
  

$$i = 0,1,\dots,N-1$$

$$\underline{\hat{w}}_{m}(n) = \underline{\hat{w}}_{m}(n-1) + \frac{1}{N} \sum_{i=0}^{N-1} \underline{q}_{i}(n) \eta_{i}^{*}(n)$$

6. Repeat from step 4) until the end of the communication process.

#### III. Simulation results

In the simulation, the channel is assumed to be AWGN and multipath Rayleigh fading with AWGN. The data symbols at the output of the encoder is binary; that is,  $y_{m,k} = 1$  for bit 1 and  $y_{m,k} = -1$  for bit 0. The number of sub-carriers for one block is N = 32. The antenna array is linear and the distance between adjacent elements is half of the wavelength,  $\lambda/2$ . The number of users is 6.

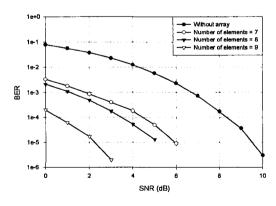


Fig. 1: BER performance versus SNR for the OFDM system in AWGN channel with the proposed

beamforming algorithm when the number of elements is varied.

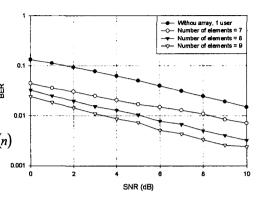


Fig. 2: BER performance versus SNR for the OFDM system in multipath Rayleigh fading channel with AWGN applying the proposed RLS algorithm when the number of elements is varied.

Fig. 1 and Fig. 2, respectively, show the Bit Error Rate (BER) performances for the OFDM system in AWGN channel and in multipath Rayleigh fading channel with AWGN when an adaptive antenna array is applied. The number of elements is changed. As can be seen from two figures, when the proposed algorithm is applied, BER performances of the OFDM system are greatly improved in comparison with that of the system applying no array. In addition, the improvement in BER performances of the system is proportional to the number of elements exploited. For instant, in AWGN channel, at SNR = 2 dB, without array, BER is 0.0377, while for the number of elements K = 7, K = 8, and K = 9 the corresponding values of BER are 0.00087, 0.000494, and 0.000017.

For the simulation of Rayleigh fading channel, the number of multipath is assumed to be 30. The mobile users move at a speed that results in the maximum Doppler shift  $f_m = 66.67Hz$ .

#### IV. Conclusions

In this paper, we proposed a new RLS algrithm for the OFDM system with an antenna array. The proposed algorithm has capability of combating cochannel interference. Moreover, the weight vector is updated in a similar manner as the conventional beamforming algorithms for TDMA and CDMA systems. Consequently, the proposed algorithm is highly applicable to the real OFDM system.

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