Buyer's EOQ model for deteriorating products under order-size-dependent delay in payments

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Abstract

This paper deals with the problem of determining the buyer's economic lot sizing policy for exponentially deteriorating products under trade credit. Assuming that the supplier's credit terms are already known and the length of delay is a function of the buyer's order size, we formulate the mathematical model and the solution algorithm is developed based on the properties of an optimal solution.

I. Introduction

In many real inventory situations, buyers(distributors or retailers) are permitted a certain fixed period(credit period) on settling the account for the products supplied without paying any interest. In this regard, many research papers analyzed the inventory model when the supplier permits delay in payments for an order of a product. Chung[2] and Goyal[3] evaluated the effects of trade credit on the economic lot sizing policy. The above research works implicitly assume that stock is depleted by customer's demand alone. This assumption is quite valid for products whose utility remains constant over time. However, there are numerous types of product whose utility does not remain constant over time. In this case, inventory is depleted not only by demand but also by deterioration. Jaggi and Aggarwal[5], and Hwang and Shinn[4] evaluated the effect of trade credit in determining the inventory policy of deteriorating products. Recently, Chu *et al.*[1] examined the economic lot sizing model for deteriorating products under the condition of permissible delay in payments. Note that a common assumption of the above researches concerning trade credit is the availability of a certain length of delay that is set by the supplier. However, the length of delay is considered as supplier's dominant strategy against the competitive suppliers in expectation of increasing the sales volume. In this sense, some pharmaceutical companies and agricultural machinery manufacturers in Korea offer a longer credit period for larger amount of purchase rather than giving some discount on unit selling price. Their policy tends to make the buyer's lot size large enough to qualify a certain credit period break.

In this paper, an attempt has been made to develop an inventory model for obtaining the buyer's economic lot sizing policy of deteriorating products in the presence of trade credit depending on the amount of purchase.

II. Assumptions and Notations

Assumptions:

(i) The demand for the items is constant with time.

(ii) Time period is infinite.

(iii) Shortages are not allowed.

(iv) Inventory is depleted not only by demand but also by deterioration with exponential distribution.

(v) The supplier permits delay in payments for the products supplied and the length of delay is a function of the amount purchased by the buyer.

(vi) The purchasing cost of the items sold during the credit period is deposited in an interest bearing account with rate i. At the end of the credit period, the account is settled and the buyer starts paying for the interest charges on items in stock with rate $r(r \geq i)$.

Notations:

D : annual demand

H : unit stock holding cost per item per year excluding interest charges

r : interest charges per \$ investment in stocks per year

i : interest rate which can be earned per \$ in a year

C : unit purchase cost in \$

S : cost of placing one order

Q : size of one order

T : time interval between successive orders

θ : positive number representing the stock deteriorating rate($0 \leq \theta \leq 1$)

$I(t)$: stock level at time t

tc_j : credit period for the amount purchased CQ, $v_{j-1}^0 \leq CQ < v_j^0$, where $tc_{j-1} < tc_j$, $j = 1, 2, \cdots, m$ and $v_0^0 < v_1^0 < \cdots < v_m^0$, $v_0^0 = 0$, $v_m^0 = \infty$

III. Development of the Mathematical Model

In the case of exponential deterioration, as stated by Hwang and Shinn[4] the quantity ordered each cycle becomes

$$Q = D(e^{\theta T} - 1)/\theta. \tag{1}$$

And the stock level at time t is

$$I(t) = D(e^{\theta(T-t)} - 1)/\theta, \quad 0 \leq t \leq T. \tag{2}$$

Also, for the formulation of the buyer's total cost with respect to T, we consider the supplier's credit plan, $v_{j-1}^0 \leq CQ < v_j^0$, $j = 1, 2, \cdots, m$. By equation (1), if we denote

$v_j = CD\frac{1}{\theta}\ln\left(\frac{\theta}{CD}v_j^0+1\right)$, then the inequality $v_{j-1}^0 \le CQ < v_j^0$ can be rewritten as

$$v_{j-1} \le CDT < v_j \text{ for } j=1,2,\cdots,m. \tag{3}$$

Therefore, depending on the relative size of tc_j to T, the buyer's total variable cost per year, $TC(T)$ has two different expressions as follows:

Case 1: $T \ge tc_j$

$$TC_{1,j}(T) = \frac{S}{T} + \frac{CD(e^{\theta T}-1)}{\theta T} + \frac{HD(e^{\theta T}-\theta T-1)}{\theta^2 T} + \left(\frac{rCD(e^{\theta(T-tc_j)}-\theta(T-tc_j)-1)}{\theta^2 T} - \frac{iCDtc_j^2}{2T}\right) \tag{4}$$

$$, \quad v_{j-1} \le CDT < v_j, \quad j=1,2,\cdots,m.$$

Case 2: $T < tc_j$

$$TC_{2,j}(T) = \frac{S}{T} + \frac{CD(e^{\theta T}-1)}{\theta T} + \frac{HD(e^{\theta T}-\theta T-1)}{\theta^2 T} + \left(\frac{iCDT}{2} - iCDtc_j\right) \tag{5}$$

$$, \quad v_{j-1} \le CDT < v_j, \quad j=1,2,\cdots,m.$$

Also, by using a truncated Taylor series expansion for the exponential function, the total cost function can be approximated as

$$TC_{1,j}(T) = \frac{S}{T} + CD + \frac{DT(H+\theta C)}{2} + \left(\frac{(r-i)CDtc_j^2}{2T} + \frac{rCDT}{2} - rCDtc_j\right), \tag{6}$$

$$TC_{2,j}(T) = \frac{S}{T} + CD + \frac{DT(H+\theta C)}{2} + \left(\frac{iCDT}{2} - iCDtc_j\right). \tag{7}$$

And so, there exists a unique value $T_{n,j}$, which minimizes $TC_{n,j}(T)$ and they are:

$$T_{1,j} = \sqrt{(2S+(r-i)CDtc_j^2)/D(H+\theta C+rC)}, \tag{8}$$

$$T_{2,j} = \sqrt{2S/D(H+\theta C+iC)}. \tag{9}$$

Also, $T_{n,j}$ and $TC_{n,j}(T)$ have the following properties.

Property 1. $T_{1,j} < T_{1,j+1}$ holds for $j=1,2,\cdots,m-1$.

Property 2. $T_{2,j} = T_{2,j+1}$ holds for $j=1,2,\cdots,m-1$.

Property 3. For any T, $TC_{n,j}(T) > TC_{n,j+1}(T)$, $n=1,2$ and $j=1,2,\cdots,m-1$.

Now, from the above properties, we can make the following observations about the characteristics of the total cost function for T, $T \in TI_j = \{T | v_{j-1}/CD \le T < v_j/CD\}$, $j=1,2,\cdots,m$. These observations simplifies our search process such that only a finite number of candidate values of T needs to be considered to find T^* for the approximate model. Let k be the smallest index such that $T_{2,j} < tc_j$.

Observation 1.

For $T \in TI_j$, $j \ge k$, we can consider the following three cases for $T_{2,j}$; $T_{2,j} < v_{j-1}/CD$, $v_{j-1}/CD \le T_{2,j} < v_j/CD$, and $v_j/CD \le T_{2,j}$.

(i) If $T_{2,j} < \frac{v_{j-1}}{CD}$, then $T = \frac{v_{j-1}}{CD}$ yields the minimum total cost for $T \in TI_j$.

(ii) If $\frac{v_{j-1}}{CD} \le T_{2,j} < \frac{v_j}{CD}$, then $T = T_{2,j}$ yields the minimum total cost for $T \in TI_j$.

(iii) If $\frac{v_j}{CD} \le T_{2,j}$, then we do not need to consider T for $T \in TI_j$ to find T^*.

Observation 2.

For $T \in TI_j$, $j < k$, we can consider the following four cases for $T_{1,j}$; $T_{1,j} < \frac{v_{j-1}}{CD}$, $\frac{v_{j-1}}{CD} \le T_{1,j} < \frac{v_j}{CD}$, $tc_j < \frac{v_j}{CD} \le T_{1,j}$ and $\frac{v_j}{CD} \le tc_j \le T_{1,j}$.

(i) If $T_{1,j} < \frac{v_{j-1}}{CD}$, then $T = \frac{v_{j-1}}{CD}$ yields the minimum total cost for $T \in TI_j$.

(ii) If $\frac{v_{j-1}}{CD} \le T_{1,j} < \frac{v_j}{CD}$, then $T = T_{1,j}$ yields the minimum total cost for $T \in TI_j$.

(iii) If $tc_j < \frac{v_j}{CD} \le T_{1,j}$, then $T = \frac{v_j^-}{CD}$,where $v_j^- = v_j - \varepsilon$ and ε is a very small positive number, yields the minimum total cost for $T \in TI_j$.

(iv) If $\frac{v_j}{CD} \le tc_j \le T_{1,j}$, then we do not need to consider T for $T \in TI_j$ to find T^*.

Observation 3. (Search Stopping Rule)

(i) If $T = T_{1,j}$ yields the minimum total cost for $T \in TI_j$, then $T^* \ge T_{1,j}$.

(ii) If $T = \frac{v_j^-}{CD}$ yields the minimum total cost for $T \in TI_j$, then $T^* \ge \frac{v_j^-}{CD}$.

Solution algorithm

Step 1: Compute $T_{2,j} = \sqrt{\frac{2S}{D(H+\theta C+iC)}}$ and let k be the smallest index such that $T_{2,j} < tc_j$.

If $T_{2,j} \ge tc_j$ for all $1 \le j \le m$, then set $k = m+1$ and go to Step 3.

Otherwise, find the index l satisfying $v_{l-1} \le T_{2,j}CD < v_l$ and go to Step 2.

Step 2: 2.1 If $k > l$, then go to Step 2.3. Otherwise, go to Step 2.2.

2.2 Compute the total annual variable cost for $T = T_{2,j}, \frac{v_l}{CD}, \frac{v_{l+1}}{CD}, \cdots, \frac{v_{m-2}}{CD}$ and $\frac{v_{m-1}}{CD}$, and go to Step 3.

2.3 Compute the total annual variable cost for $T = \frac{v_{j-1}}{CD}$, $j = k, k+1, \cdots, m$ and go to Step 3.

Step 3: 3.1 Set $j = k - 1$.

3.2 If $tc_j \ge \frac{v_j}{CD}$, then go to Step 3.4. Otherwise, go to Step 3.3.

3.3 In case

i) $T_{1,j} < \frac{v_{j-1}}{CD}$, compute the total cost for $T = \frac{v_{j-1}}{CD}$ and go to Step 3.4.

ii) $T_{1,j} < \frac{v_j}{CD}$, compute the total cost for $T = T_{1,j}$ and go to Step 4.

iii) $T_{1,j} \geq \frac{v_j}{CD}$, compute the total cost for $T = \frac{v_j^-}{CD}$, $v_j^- = v_j - \varepsilon$ where ε is a very small positive number and go to Step 4.

3.4 Reset $j = j - 1$ and go to Step 3.2.

Step 4: Select the one that yields the minimum total annual variable cost as T^* for the approximate model and stop.

IV. Conclusions

In this paper, we evaluated the buyer's economic lot sizing model for an exponentially deteriorating product when the supplier permits delay in payments for an order of a product. Recognizing that a major reason for the supplier to offer trade credit is to stimulate the demand of the product, it is assumed that the length of delay is a function of the buyer's total amount of purchase. The availability of order-size-dependent delay in payments can be justified by the principle of economies of scale, and tends to make the buyer's order size larger by inducing him to qualify for a longer credit period in his payments.

References

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