Buyer's EOQ model for deteriorating products under conditions of permissible delay in payments and quantity discounts for freight cost

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Abstract

This paper deals with the problem of determining the buyer's economic lot sizing policy for exponentially deteriorating products under trade credit. It is also assumed that the ordering cost consists of a fixed set-up cost and a freight cost, where the freight cost has a quantity discount offered due to the economies of scale. We formulate the mathematical model and the solution algorithm is developed based on the properties of an optimal solution.

I. Introduction

In deriving the economic order quantity(EOQ) formula, it is tacitly assumed that the retailer must pay for the items as soon as he receives them from a supplier. However, in practice, a supplier will allow a certain fixed period(credit period) for settling the amount the retailer owes to him for the items supplied. Trade credit would play an important role in the conduct of business for many reasons. For a supplier who offers trade credit, it is an effective means of price discrimination which circumvents antitrust measures and is also an efficient method to stimulate the demand of the product. For a buyer, it is an efficient method of bonding a supplier when the buyer is at the risk of receiving inferior quality goods or service and is also an effective means of reducing the cost of holding stocks. In this regard, a number of research papers appeared which deal with the EOQ problem under a fixed credit period. Chung[2] and Goyal[3] analyzed the effects of trade credit on the optimal inventory policy. Note that all the researches assumed that the ordering cost contains a fixed cost alone. But, in many practical situations, the order may be delivered in unit loads, i.e., trucks, containers, pallets, boxes, etc. and a quantity discount may occur in terms of the number of unit loads due to the economy of scale. In this regard, Shinn *et al.*[5] analyzed the joint price and lot size determination problem under conditions of permissible delay in payments and quantity discounts for freight cost. All the research works mentioned above implicitly assume that stock is depleted by customer's demand alone. This assumption is quite valid for products whose utility remains constant over time. However, there are numerous types of product whose utility does not remain constant over time. In this case, inventory is depleted not only by demand but also by deterioration. Chu *et al.*[1], and Hwang and Shinn[4] evaluated the effect of trade credit in determining the inventory policy of deteriorating products.

This paper deals with the buyer's economic lot size determination problem for an exponentially deteriorating product when the supplier permits delay in payments for an order of the product. It is also assumed that the ordering cost of the buyer contains not only a fixed cost but also a freight cost, which is a function of the lot-size.

II. Development of the mathematical model

In deriving the model, the following assumptions and notations are used:

(i) The demand for the items is constant with time.

(ii) Time period is infinite.

(iii) Shortages are not allowed.

(iv) Inventory is depleted not only by demand but also by deterioration with exponential distribution.

(v) The retailer pays the freight cost for the transportation of the quantity purchased where the freight cost has a quantity discount.

(vi) The supplier proposes a certain credit period and the purchasing cost of the items sold during the credit period is deposited in an interest bearing account with rate i. At the end of the credit period, the account is settled and the buyer starts paying for the interest charges on items in stock with rate r $(r \geq i)$.

- D : annual demand
- H : unit stock holding cost per item per year excluding interest charges
- tc : credit period set by the supplier
- r : interest charges per \$ investment in stocks per year
- i : interest rate which can be earned per \$ in a year
- C : unit purchase cost in \$
- S : fixed ordering cost of placing one order
- N_j : jth freight cost break quantity, $j = 1, 2, \cdots, n$, where $N_0 < N_1 < \cdots < N_n < N_{n+1}$ with $N_0 = 0$ and $N_{n+1} = \infty$.
- F_j : freight cost for Q, $N_{j-1} < Q \leq N_j$, where $F_{j-1} < F_j$ and $F_{j-1}/N_{j-1} > F_j/N_j$, $j = 1, 2, \cdots, n$.
- Q : size of one order
- T : time interval between successive orders

θ : positive number representing the stock deteriorating rate($0 \le \theta \le 1$)
$I(t)$: stock level at time t
In the case of exponential deterioration, as stated by Hwang and Shinn[4] the quantity ordered each cycle becomes

$$Q = D(e^{\theta T} - 1)/\theta. \qquad (1)$$

And the stock level at time t is

$$I(t) = D(e^{\theta(T-t)} - 1)/\theta, \ \ 0 \le t \le T. \qquad (2)$$

Also, for the formulation of the buyer's total cost with respect to T, we consider the freight cost discount schedule, $N_{j-1} < Q \le N_j$, $j = 1, 2, \cdots, n$. By equation (1), if we denote $L_j = \frac{1}{\theta} \ln\left(\frac{\theta}{D} N_j + 1\right)$, then the inequality $N_{j-1} < Q \le N_j$ can be rewritten as

$$L_{j-1} < T \le L_j \text{ for } j = 1, 2, \cdots, n. \qquad (3)$$

Therefore, depending on the relative size of tc to T, the buyer's total variable cost per year, $TC(T)$ has two different expressions as follows:
Case 1: $T \ge tc$

$$TC_{1,j}(T) = \frac{S+F_j}{T} + \frac{CD(e^{\theta T}-1)}{\theta T} + \frac{HD(e^{\theta T} - \theta T - 1)}{\theta^2 T} + \left(\frac{rCD(e^{\theta(T-tc)} - \theta(T-tc) - 1)}{\theta^2 T} - \frac{iCDtc^2}{2T}\right) \qquad (4)$$

$$, \ L_{j-1} < T \le L_j, \ j = 1, 2, \cdots, n.$$

Case 2: $T < tc$

$$TC_{2,j}(T) = \frac{S+F_j}{T} + \frac{CD(e^{\theta T}-1)}{\theta T} + \frac{HD(e^{\theta T} - \theta T - 1)}{\theta^2 T} + \left(\frac{iCDT}{2} - iCDtc\right) \qquad (5)$$

$$, \ L_{j-1} < T \le L_j, \ j = 1, 2, \cdots, n.$$

III. Determination of optimal policy

The problem is to find the buyer's economic lot size which minimizes $TC(T)$. Although the objective function can be differentiated, the resulting equation is mathematically intractable; that is, it is impossible to find the optimal solution in explicit form. Therefore, by using a truncated Taylor series expansion for the exponential term, the total cost function can be approximated as

$$TC_{1,j}(T) = \frac{S+F_j}{T} + CD + \frac{DT(H+\theta C)}{2} + \left(\frac{(r-i)CDtc^2}{2T} + \frac{rCDT}{2} - rCDtc\right), \qquad (6)$$

$$TC_{2,j}(T) = \frac{S+F_j}{T} + CD + \frac{DT(H+\theta C)}{2} + \left(\frac{iCDT}{2} - iCDtc\right). \qquad (7)$$

And so, there exists a unique value $T_{m,j}$, which maximizes $TC_{m,j}$ and they are:

$$T_{1,j} = \sqrt{(2(S+F_j) + (r-i)CDtc^2)/D(H+\theta C + rC)}, \qquad (8)$$

$$T_{2,j} = \sqrt{2(S+F_j)/D(H+\theta C + iC)}. \qquad (9)$$

Also, $T_{m,j}$ and $TC_{m,j}(T)$ have the following properties.

Property 1. For m given, $T_{m,j} < T_{m,j+1}$, $j = 1, 2, \cdots, n-1$.

Property 2. For any T, $TC_{m,j}(T) < TC_{m,j+1}(T)$, $m = 1, 2$ and $j = 1, 2, \cdots, n-1$.

Now, we present two observations, one for Case 1 and the other for Case 2. Based on these observations, we only need to consider a finite number of candidate values of T in finding an optimal value .

Observation 1. (for Case 1)

Suppose tc belongs to $(L_{a-1}, L_a]$ for some a. Let b be the largest index such that $T_{1,0} > L_b$. Also, let c be the larger value of a and b, and k ($k \geq c$) be the first index such that $T_{1,k} \leq L_k$, respectively.

(i) If the index k ($\leq n$) exists and $T_{1,k} < tc$, then T^* must be less than tc.

(ii) If the index k ($\leq n$) exists and $T_{1,k} \geq tc$, then we have to consider $T = L_c, L_{c+1}, \cdots, L_{k-1}, T_{1,k}$ only as candidates for T^*.

(iii) If $T_{1,j} > L_j$ for all $c \leq j \leq n$, then we have to consider $T = L_c, L_{c+1}, \cdots, L_n$ as candidates for T^*.

Observation 2. (for Case 2)

Suppose tc belongs to $(L_{a-1}, L_a]$ for some a. Let b be the largest index such that $T_{2,0} > L_b$ and k ($k > b$) be the first index such that $T_{2,k} \leq L_k$, respectively.

(i) If $L_b \geq tc$, L_{a-1} becomes the only candidate for T^*.

(ii) If the index k ($\leq a$) exists and $T_{2,k} < tc$, then we have to consider $T = L_b, L_{b+1}, \cdots, L_{k-1}, T_{2,k}$ as candidates for T^*.

(iii) If $T_{2,j} > L_j$ for all $b \leq j < a$ and $T_{2,a} \geq tc$, then we have to consider $T = L_b, L_{b+1}, \cdots, L_{a-1}$ as candidates for T^*.

Solution algorithm

Step 1. This step identifies all the candidate values T_o of T satisfying $T_o \geq tc$.

1.1. Compute $T_{1,0} = \sqrt{(2S + (r-i)CDtc^2)/D(H + \theta C + rC)}$ and find index b such that $T_{1,0} \in (L_b, L_{b+1}]$.

1.2. Find index a such that $tc \in (L_{a-1}, L_a]$ and let c = Max[a, b].

1.3. Compute $T_{1,j}$ and find the first index k ($k \geq c$) such that $T_{1,k} \leq L_k$.

1.4. If the index k ($\leq n$) exists, then go to Step 1.5.
Otherwise, compute the total cost for $T_o = L_c, L_{c+1}, \cdots, L_n$ and go to Step 2.

1.5. If $T_{1,k} < tc$, then go to Step 2.
Otherwise, compute the total cost for $T_o = L_c, L_{c+1}, \cdots, L_{k-1}, T_{1,k}$ and go to

Step 2.

Step 2. This step identifies all the candidate values T_o of T satisfying $T_o < tc$.

2.1. Compute $T_{2,0} = \sqrt{2S/D(H+\theta C+iC)}$ and find index b such that $T_{2,0} \in (L_b, L_{b+1}]$.

2.2. If $L_b \geq tc$, then compute the total cost for $T_o = L_{a-1}$ and go to Step 3. Otherwise, compute $T_{2,j}$ and find the first index k $(k > b)$ such that $T_{2,k} \leq L_k$ and go to Step 2.3.

2.3. If the index k $(\leq a)$ exists and $T_{2,k} < tc$, then compute the total cost for $T_o = L_b, L_{b+1}, \cdots, L_{k-1}, T_{2,k}$ and go to Step 3. Otherwise, compute the total cost for $T_o = L_b, L_{b+1}, \cdots, L_{a-1}$ and go to Step 3.

Step 3. Select the buyer's economic lot size among T_o found in steps 1 and 2 which gives the minimum total cost.

IV. Conclusions

In this paper, we have analyzed the economic lot sizing policy of a buyer in an environment in which the freight cost has a quantity discount and the supplier provides a certain fixed credit period for settling the amount the retailer owes to him. The ordering cost sometimes depends upon the ordering quantity, owing to discounts allowed by a shipping company for large order. In this regard, we think that the model presented in this paper may be more realistic for some real world problems.

References

1. Chu, P., Chung, K. J., and Lan, S. P., 1998, Economic order quantity of deteriorating items under permissible delay in payments, Computers & Operations Research 25, 817-824.
2. Chung, K. J., 1998, A theorem on the determination of economic order quantity under conditions of permissible delay in payments, Computers & Operations Research 25, 49-52.
3. Goyal, S.K., 1985, Economic order quantity under conditions of permissible delay in payments, Journal of Operational Research Society 36, 335-338.
4. Hwang, H. and Shinn, S. W., 1997, Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments, Computers and Operations Research 24, 539-547.
5. Shinn, S. W., Hwang, H., and Park, S. S., 1996, Joint price and lot size determination under conditions of permissible delay in payments and quantity discounts for freight cost, European Journal of Operational Research 91, 528-542.