◎ 함 수 방 정 식 ◎
(Funtional Equation)

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FE-1 **Unitary Interpolation Problems in Tridiagonal Algebras**

Given operators X and Y acting on a Hilbert space $\mathcal{H}$, an interpolating operator is a bounded operator A such that $AX = Y$. An interpolating operator for n-operators satisfies the equation $AX_i = Y_i$, for $i = 1,2,\cdots,n$. Given vectors x and y in a Hilbert space, an interpolating operator is a bounded operator T such that $Tx = y$. An interpolating operator for n vectors satisfies the equation $Tx_i = y_i$, for $i = 1,2,\cdots,n$.

Theorem A. Let $X=(x_{ij})$ and $Y=(y_{ij})$ be operators acting on such that $x_{i\sigma(i)} \neq 0$ for all i. Then the following statements are equivalent.

(1) There exists a unitary operator A in Alg$\mathcal{L}$ such that $AX = Y$ and every E in $\mathcal{L}$ reduces A.

(2) $\sup \left\{ \dfrac{\left\| \sum_{i=0}^{n} E_i Y f_i \right\|}{\left\| \sum_{i=0}^{n} E_i Y f_i \right\|} : n \in N,\ E_i \in \mathcal{L} \text{ and } f_i \in H \right\} < \infty$

and $\dfrac{|y_{i\sigma(i)}|}{|x_{i\sigma(i)}|} = 1$ for all $i = 1,2,\cdots,n$.

Theorem B. Let $x=\{x_i\}$ and $y=\{y_i\}$ be two vectors in a separable complex Hilbert space $\mathcal{H}$ such that $x_i \neq 0$ for all $i = 1,2,\cdots,n$.

Then the following statements are equivalent.

(1) $\sup \left\{ \dfrac{\left\| \sum_{k=1}^{l} \alpha_k E_k y \right\|}{\left\| \sum_{k=1}^{l} \alpha_k E_k y \right\|} : l \in N,\ \alpha_k \in C \text{ and } E_k \in \mathcal{L} \right\} < \infty$

and $|y_n||x_n|^{-1} = 1$ for all $i = 1,2,\cdots,n$.

(2) There exists an operator A in Alg$\mathcal{L}$ such that $Ax = y$, A is a unitary operator and every E in $\mathcal{L}$ reduces A, where Alg$\mathcal{L}$ is a tridiagonal algebra.