◎ 초 청 강 연 ◎
(Invited Talk)

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Invited Talk	**Analytic Solutions of a Nonlinear Iterative Equation near Neutral Fixed Points and Poles**

Let f^n denote the n-th iterate of a map f, i.e., $f^1=f$, $f^2=f\circ f$, $f^n=f\circ f^{n-1}$ and $f^0(x):\equiv x$. The functional equation

$$F^m(x)=G\left(\sum_{k=0}^{m-1}a_k f^k(x)\right)+F(x),\quad m\geq 2,\quad x\in C, \tag{1}$$

where iteration of unknown function f is the main operation, is refered to as a nonlinear iterative equation. In general, $a_0, a_1,\cdots,a_{m-1}$ are complex constants and G, F are given complex-valued functions of a complex variable. Equation (1) includes a lot of important subjects as its special cases. In the theory of dynamical systems we are concerned with iterative roots of a homemorphism F, that is to find a function f such that

$$f^n(x)=F(x).$$

Invaiant curves of planar maps can be found by solving iterative equations, for instance,

$$f(x+f(x))=\rho(f(x)). \tag{2}$$

and

$$f(f(x))=2f(x)-x-\frac{1}{2}(g(f(x)+g(x)). \tag{3}$$

Both (2) and (3) are special cases of (1). In particular, the linear iterative equation

$$\lambda_1 f(x)+\lambda_2 f^2(x)+\cdots\lambda_m f^m(x)=F(x).$$

was discussed extensively. In this paper we generally study analytic solutions of equation (1) and give existence of analytic solutions of a nonlinear iterative equations in case all given functions are analytic and in case given functions have poles. As well as in many previous works, we reduce this problem to finding analyic solutions of a function equation without iteration of the unknown function f. For technical reasons, in previous works an indeterminate constant related to the eigenvalue of the linearized f at its fixed point O is required to fufil the Diophantine condition that O is an irrationally neutral fixed point of f. In this paper the case of rationally neutral fixed points is also discussed, where the Diphantine condition is not required.