The Encoder Design of Punctured Turbo Trellis Coded Modulation applied to MPSK

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Abstract: This paper introduces an encoder design method of Turbo TCM (Trellis Coded Modulation) with symbol puncturing. TTCM consists of two simple trellis codes in parallel and modulator. To obtain an good encoder, we calculate the free distance by the assumption that the punctured symbol is transmitted from the subset that consist of signals with the same systematic bit at random. We develop a search program to find the component encoder which maximize the free distance. Especially, for 8-PSK with code rate 2/3, we search for the component codes. We find a new encoder which has better BER performance than that of Robertson's encoder. We verify the results through the simulation.

1. Introduction

Turbo codes can achieve remarkable error performance at a low signal-to-noise ratio (SNR) close to the Shannon capacity limit[1]. However, the powerful binary coding schemes are not suitable for bandwidth limited communication systems. In order to improve spectral efficiency, a general method is to combine the concepts of turbo code and trellis coded modulation (TCM). Recently, different approaches have been proposed to achieve large coding gains and high bandwidth efficiency for Gaussian channels [2],[4],[6]. Among them, Robertson's turbo trellis coding schemes are the least complex. The basic idea in this approach called Turbo Trellis Coded Modulation(or TTCM) is to concatenate two simple trellis codes in parallel and transmit the mapped symbol using the puncturing. In this case, interleaver operates on the group of bits.

The paper proposed by Robertson shows the structure of encoder with only memory 3 and 4. Also, as the number of memory is 4, we can find that Robertson's encoder is different with general TCM encoder which used by Ogiwara under the same system model[3]. Thus, we introduce the encoder design of TTCM which transmits the signal of component code to MPSK-symbol and

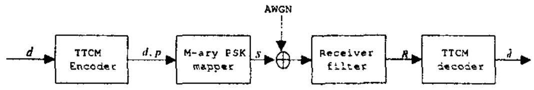

Figure 1. The transmitter and receiver of Turbo TCM

prove the encoder with the best performance.

The free distance is the main parameter on the design of the optimal encoder design. At punctured TTCM, the calculation of the free distance is different from that of TCM since the punctured symbol is not transmitted but at the receiver the symbol from the other component code is taken as the received symbol. We calculate the free distance by the assumption that the punctured symbol is transmitted from the subset that consist of signals with the same systematic bit at random. Section 2.explains the system model of Turbo TCM with the transmitter, and the receiver with the symbol MAP algorithm. Section 3describes the considerable factor to design the encoder of Turbo TCM and search algorithm to find the encoder that maximize the free distance. Section 4has the conclusion.

2. System Model

Figure 1 shows the encoding and decoding process of Turbo TCM. An information data sequence d is encoded by two parallel encoder of rate $n/(n+1)$. An information data stream d and punctured parity bit stream p is mapped to a symbol S by MPSK mapper. AWGN (Addictive White Gaussian Noise) channel is considered.

Generally, the transmitter of binary turbo code consists of two component encoders concatenated in parallel and interleaver, where component code is made of the recursive systematic convolutional code [1] . Turbo TCM is similar to binary turbo code, but it is the system that encoded into group of n bits and mapped into MPSK symbol. Also, interleaver operates on the group of bits. Figure 2 shows the structure of Turbo TCM transmit-

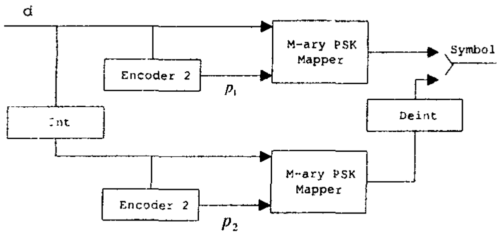

Figure 2. A structure of Turbo TCM transmitter

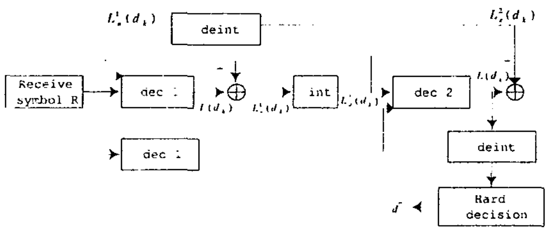

Figure 3. A structure of Turbo TCM decoder

ter [2]. First of all, n bits of input data stream d and parity bit p_1 obtained by encoder 1 are mapped into MPSK symbol. Then the interleaved input data stream and parity bit p_2 obtained by encoder 2 are mapped into MPSK symbol, where interleaver uses odd-even random interleaver. The symbol from encoder 2 is changed into the original position using deinterleaver. The reason is to prevent that each information symbol are transmitted more than twice and some are not transmitted [9] . The mapped symbol is alternately chosen from first mapper and second mapper. Figure 3 shows a structure of Turbo TCM decoder with information delivery process. The decoder 1 calculates an a posteriori value $L(d_k)$ using received symbol and initial priori value $L_a^1(d_k)$. The extrinsic value $L_e^1(d_k)$ is generated by subtracting a priori value from $L(d_k)$. This value is interleaved and used in a priori value $L_a^2(d_k)$ to the next decoder. The decoder 2 calculates an a posteriori value $L(d_k)$ using interleaved symbol and a priori value from the decoder 1. We do the hard decision with deinterleaving a posteriori probability $L(d_k)$ from the decoder 2, and the extrinsic value $L_e^2(d_k)$ is generated by subtracting a priori value from $L(d_k)$. $L_e^2(d_k)$ is used in a priori value to the decoder 1. Finally, decoder terminates the first iteration decoding. Using this iterative decoding, we can improve the reliability of the information reproduction.

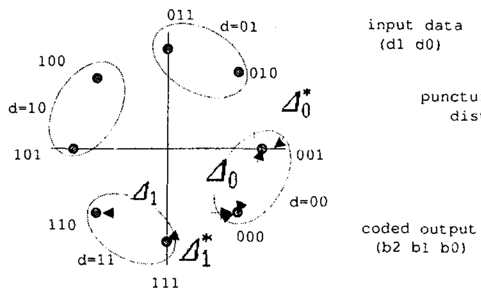

Figure 4. Signal constellation of 8-PSK symbol assuming a puncturing

3. The encoder design of TTCM

3.1 Design criteria

3.11 Symbol mapping and distance recalculation

Just as TCM, Turbo TCM uses Ungerboeck mapping with signal set partition. Figure 4 shows an example for 8-ary PSK mapping. In Eq.[15b] of [5], it is stated that minimal distance is bounded by

$$d_{free}^2 \geq \Delta_{free}^2 = min \sum_{i=k}^{k+L} \Delta_{q(e_i)}^2 \equiv min \delta^2[e(D)] \quad (1)$$

minimizing over all nonzero code sequences $e(D)$. The variable $q(e_i)$ is the number of trailing zeros in e_i . The value $\Delta_0^2, \Delta_1^2, \Delta_2^2, \cdots$ are the squared minimal Euclidean distances between signals of each subset, and must be replaced by $\Delta_0^{*2}, \Delta_1^{*2}, \Delta_2^{*2}, \cdots$ just as figure 4, when the transmitted symbol was punctured. This distance be calculated by assuming that the random parity bit takes its worst case value and minimizes the distance between elements of the subsets. At the figure 4, four dotted ellipse express the subset of the same information symbol, when the transmitted symbol was punctured , without consideration of parity bit. Thus, we can define a value of priori probability to the distance metric between symbol on the signal constellation (under consideration that generation probability of parity bit(0 or 1) is 1/2) and received symbol.

3.12 Connection setting of the encoder with m memory

Figure 5 shows a generalized model of trellis code encoder. The primitive polynomial selects the backward polynomial h_0. The forward connections are set as follows in order to maximize the separation between signal

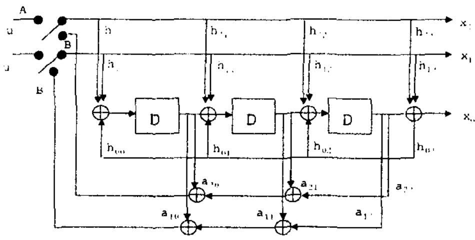

Figure 5. The encoder of Turbo TCM(rate $b/(b+1)$)

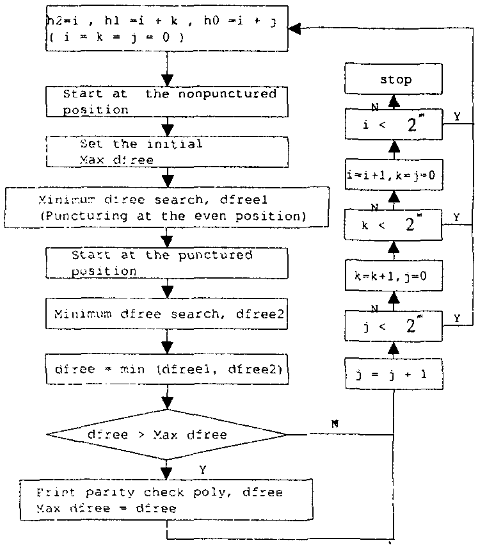

Figure 6. Search algorithm to find the free distance of Turbo TCM

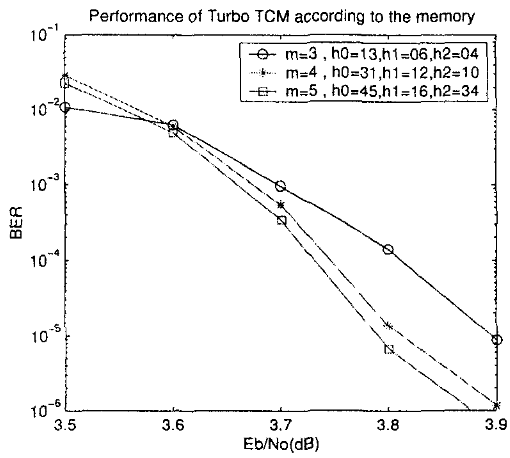

Figure 7. Performacne analysis of the Turbo TCM according to memory

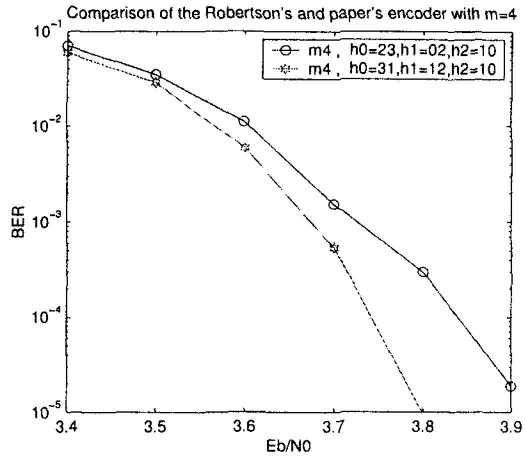

Figure 8. Performance comparison of the Robertson's and optimal encoder (m=4)

points when diverging from a state and when remerging to a state and have no parallel transition [7].

$$h_{i,0} = h_{i,m} = 0 \quad (i = 1, \cdots, b) \tag{2}$$

3.13 Tail termination

Tail termination is performed by setting the switch shown in figure 5 in position B. The coefficient $a_{i,0}, \cdots, a_{i,m-1}$ $(i = 1, 2, \cdots, b)$ can be obtained by repeated use of Eq.(3) [6],[7]. The trellis can be terminated in state zero with at least and at most clock cycles.

$$S^k(D) = [\sum_{i=1}^{b} u_i^k h_i(D) + DS^{k-1}(D)] mod h_0(D) \tag{3}$$

3.2 The free distance of Turbo TCM

The free distance calculation of Turbo TCM with symbol puncturing is different to that of TCM. The minimum distances of paths diverging from a odd position and paths from the even position are different. Under considering this point, we developed a search program to find the encoder that maximizes the free distance of the component code with puncturing and designed the optimal encoder. The free distance search consist of the algorithm suggested in [8]. We must pay attention to the distance at the punctured position. The free distance is defined as the minimum distance of all the possible coded sequence pairs.

Table 1. The optimal encoder and free distances of Robertson's Turbo TCM

Code	b	$H^0(D)$	$H^1(D)$	$H^2(D)$	d^2_{free}
8-PSK, 8state	2	11	02	04	1.7573
8-PSK, 16state	2	23	02	10	1.7573

Table 2. The optimal encoder and free distances of Turbo TCM through program

Code	b	$H^0(D)$	$H^1(D)$	$H^2(D)$	d^2_{free}	$N_d free$
8-PSK, 8state	2	13	06	04	1.7573	0.31250
8-PSK, 16state	2	31	12	10	2.3431	0.47265
8-PSK, 32state	2	45	16	34	2.9289	0.52368

The exhausted search has been done for all the possible connections in figure 5. For the encoders with the same distance we choose the one with the minimum $N(d_{free})$ which denotes the average multiplicity of error events. Table.(1) shows the encoder of Robertson's Turbo TCM mapped by 8-ary PSK [2]. Table.(2) shows a structure of encoder obtained by the search program. The Robertson's paper gives the encoders with only 8 and 16 states. We note that for the encoder with 16 states in Robertson's, the squared free distance has 1.7573 which is equal to the distance of 8 state. However, the squared free distance of our new encoder with 16 states has 2.3431 which is larger than that of 8 states. We also find the optimum encoder for 32 states, which is not given in [2]. We have performed the simulation to verify the performance of our new encoders. We choose the odd-even random interleaver. Figure 7 shows the simulation results of Turbo TCM with the encoders proposed in table.(2). The number of iteration is set to 9. Performance becomes better as the memory becomes larger. In the system with four memory, bit error rate of 10^{-5} is realized at $E_b/N_0 = 3.8dB$. Figure 8 shows the comparison of the Robertson's encoder and our new encoder for the size of memory, $m = 4$. The BER performance of our encoder is better than that of the Robertson's encoder by 0.1 dB at the BER of 10^{-5}.

4. Conclusion

We have described the design method of the optimum component code of TTCM with symbol puncturing. In the receiver we use the symbol mapped by the output from the other component encoder for punctured symbol instance. This symbol contains the information on the systematic bits. Therefore the free distance calculation of punctured component code is different to that

Table 3. A structure of Tail termination of Turbo TCM(octal notation)

Code	♯ of input bit	$a_1(D)$	$a_2(D)$
8-PSK, 8state	2	4	2
8-PSK, 16state	2	13	02
8-PSK, 32state	2	30	06

of normal TCM. In this paper we present a method to design the good component code and give design results for 8-PSK cases which has better BER perfrmance than the previous work.

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