On the Selection of Burst Preamble Length for the Symbol Timing Estimate in the AWGN Channel

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Abstract: For detection of digitally modulated signals, the receiver must be provided with accurate carrier phase and symbol timing estimates. So far, lots of algorithms have been suggested for those purposes. In general, a interpolation filter with TED(Timing Error Detection) like Gardner algorithm is popularly used for symbol timing estimate of digital communication receiver. Apart from the perfomance point of view, a multiplicative operation of any interpolation filter limits the symbol rate of the system. Hence, we suggest a new symbol timing estimate algorithm for high speed burst-mode fixed wireless communication system and analyze its performance in the AWGN channel.

1. Introduction

In general, channel characteristics have great influence on not only the structure of communication system but also the performance of it. The AWGN channel, which has no fading and no non-linearity, is the basic channel model that is familiar with most communication system designers and is the start point of various analysis such as Rayleigh fading, Rician fading and so on.

Unfortunately, there are no AWGN channels in our real world. Then, what are we doing with performance analysis of any system in the AWGN channel?

We think that fixed LOS(line-of-sight) wireless channel of which carrier frequency is above 10GHz has almost theoretical AWGN channel properties because there are no fading and no multipath. There only exists a path loss that is the function of distance. It provides us with a good reason for considering a fixed LOS wireless channel over 10GHz as merely an AWGN channel. Hence, from now on, we focus our attention on the fixed LOS wireless system whose carrier frequency is above 10GHz.

Buy the way, a burst transmission system has its unique preamble pattern for many purposes such as packet detection, timing and frequency estimates, etc. We suggest a new preamble pattern that is only suitable for above channel environment.

If we use a square wave pulse train as preamble of burst transmitted packet, the output of any pulse shaping filter in the transmitter will be a form of a single tone since the first frequency component of that pulse train only remains after passing low pass filter. Thanks to the nature of any single tone, the preamble is not affected by the channel distortion. In other words, the receiver can always receive the preamble as a form of single tone with mere amplitude change and without any distortion caused by the channel and RF circuit of both transmitter and receiver. It means that there are no ISI components in the filtered square wave pulse train preamble sequence. However, we can directly use a sinusoid in the portion of preamble instead. On the basis of these properties, the main idea of this new algorithm is summarized as follows.

If we detect the maximal point of a single tone, we can directly find the exact symbol timing estimate. The maximal point-searching algorithm will be simply materialized with a group of sample delay blocks, symbol interval integrators and maximal selector, not multiplicative device.

However, in most cases, we do symbol timing estimate under the existence of frequency and phase errors. Effects of these errors on the symbol timing estimate will be investigated in this paper.

This paper is organized as follows. In Section 2, we present the system model. Section 3 is for the mathematical analysis of that system. The discussion is done in Section 4. Finally, Section 5 contains our conclusions and remarks.

2. System Models

The proposed structure of symbol timing estimate block in the receiver is shown in Figure 1.

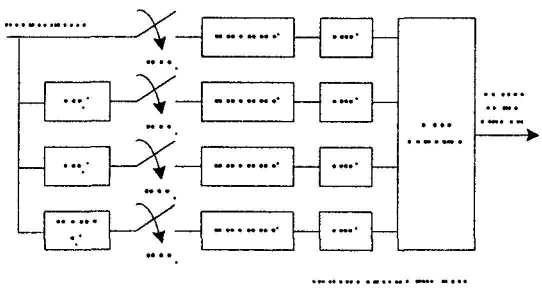

Figure 1. The structure of symbol timing estimate

We assumed in Figure 1 that there exist N samples per symbol, the sampling frequency is f_s and symbol period is T_s. The integration time of symbol integrator is dependent both on the required energy to find the maximal point with sufficiently high detection probability and on the frequency error of the system.

We can write the burst signal transmitted as (1)

$$s(t)=\left\{\sum_{n=0}^{M-1}P_n+\sum_{n=M}^{N}I_n\right\}\cdot h(t-nT_s)\cdot\cos(2\pi f_c t)+\sum_{n=M}^{N}Q_n\cdot h(t-nT_s)\cdot\sin(2\pi f_c t) \tag{1}$$

Where $\{P_n\}$ represents the preamble sequence, $\{I_n\}$ and $\{Q_n\}$ are discrete information-bearing sequences of symbols, *h(t)* is the impulse response of the pulse shaping filter, f_c is the carrier frequency.

Note that the preamble sequence is BPSK signal while the information-bearing sequence is QPSK. Let us assume that $\{P_n\}$ is square wave pulse train. For the convenience, we just focus on the portion of the preamble of the transmitted signal because the receiver only handles that part for the symbol timing estimate.

In general, a square wave pulse train with symbol duration T_s can be described by Fourier series expansion.[1] If we used a root raised-cosine filter as pulse shaping filter, the output of $\{P_n\}$, filtered by *h(t)*, will be a single tone whose frequency is $f_p=\frac{1}{2T_s}$. This property enables us to simplify the mathematical expression of the preamble part as $\cos(2\pi f_p t)$.

Then, the received preamble sequence can be represented as

$$r_p(t)=\cos(2\pi f_p t)\cdot\cos(2\pi f_c t+\theta)+n(t) \tag{2}$$

Where *n(t)* represents the additive white Gaussian noise with zero mean and spectral density $\frac{N_0}{2}$. If we use the receiver bandpass filter of bandwidth B($\approx\frac{1}{T_s}$), *n(t)* can be represented by

$$n(t)=n_c(t)\cos(2\pi ft)-n_s(t)\sin(2\pi ft) \tag{3}$$

Note that $n_c(t)$ and $n_s(t)$ are lowpass Gaussian processes with zero mean and variance N_0B. [5]

3. Mathematical Analysis

Figure 2 shows the proposed quadrature receiver structure that performs the noncoherent detection of the preamble sequence.

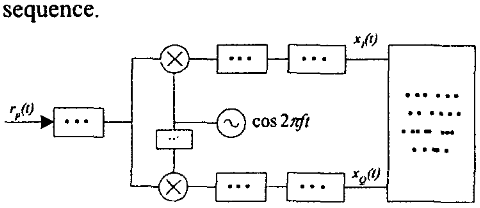

Figure 2. The proposed quadrature receiver

Let us assume that there is an error $\Delta f=f_c-f$ in carrier frequency between transmitter and receiver, then we can write $x_I(t)$ as

$$x_I(t)=\frac{1}{2}\cos(2\pi f_p t)\cdot\cos(2\pi\Delta ft+\theta)+\frac{1}{2}n_c(t) \tag{4}$$

Similarly,

$$x_Q(t)=\frac{1}{2}\cos(2\pi f_p t)\cdot\sin(2\pi\Delta ft+\theta)+\frac{1}{2}n_s(t) \tag{5}$$

If the over-sampling ratio is L, *l*-th input to symbol integrator, after sample and hold in Figure 1, $y_l(nT_s)$ *is*

$$y_{Il}(nT_s)=\frac{1}{2}\cos\left\{\left(n-\frac{l}{L}\right)\pi\right\}\cdot\cos\left\{2\pi\Delta fT_s\left(n-\frac{l}{L}\right)+\theta\right\}+\frac{1}{2}n_c\left\{\left(n-\frac{l}{L}\right)T_s\right\} \tag{6}$$

$$y_{Ql}(nT_s)=\frac{1}{2}\cos\left\{\left(n-\frac{l}{L}\right)\pi\right\}\cdot\sin\left\{2\pi\Delta fT_s\left(n-\frac{l}{L}\right)+\theta\right\}+\frac{1}{2}n_s\left\{\left(n-\frac{l}{L}\right)T_s\right\} \tag{7}$$

Note that $2\pi f_p=\frac{\pi}{T_s}$ and $f_s=\frac{L}{T_s}$.

But for $\Delta fT_s \ll 1$, the effect of phase shift can be ignored. In addition, statistical characteristics of additive white Gaussian noise will be the same irrespective of the time shift. Namely, it is stationary.

Hence, we can rewrite (6) and (7) as

$$y_{Il}(nT_s)=\frac{1}{2}\left[\cos\left\{\left(n-\frac{l}{L}\right)\pi\right\}\cdot\cos(2\pi\Delta fnT_s)+n_c(nT_s)\right] \tag{8}$$

$$y_{Ql}(nT_s)=\frac{1}{2}\left[\cos\left\{\left(n-\frac{l}{L}\right)\pi\right\}\cdot\sin(2\pi\Delta fnT_s)+n_s(nT_s)\right] \tag{9}$$

After summation over P symbols in the integrators, we have

$$Y_{Il}=\sum_{n=0}^{P-1}y_{Il}(nT_s) \tag{10}$$

$$Y_{Ql}=\sum_{n=0}^{P-1}y_{Ql}(nT_s) \tag{11}$$

To eliminate the phase ambiguity of $\pm\pi$ radians, we choose only even terms among P symbols in (10) and (11), then

$$Z_{Il}=Y_{Il}^{even}=\sum_{k=0}^{N_p-1}y_{Il}(2kT_s) \tag{12}$$

$$Z_{Ql}=Y_{Ql}^{even}=\sum_{k=0}^{N_p-1}y_{Ql}(2kT_s) \tag{13}$$

To find the optimal symbol timing estimate irrespective of frequency is to select maximal values of equation (14)

$$\max_l Z_l=Z_{Il}^2+Z_{Ql}^2 \tag{14}$$

Note that Z_{Il} and Z_{Ql} are statistically independent Gaussian random variables with means m_{Il} and m_{Ql} and common variance σ_l^2 and that Z_l has a noncentral chi-square distribution with noncentrality parameter

$$s_l^2=m_{Il}^2+m_{Ql}^2 \tag{15}$$

Now, we define a new random variable $R_l = \sqrt{Z_l}$. The pdf of R_l has Rician distribution as (16)

$$P_{Rl}(r) = \frac{r}{\sigma_l^2}\exp\left(-\frac{r^2+s_l^2}{2\sigma_l^2}\right)\cdot I_0\left(\frac{rs_l}{\sigma_l^2}\right) \quad ,r \ge 0 \qquad (16)$$

Where $I_0(\cdot)$ is zero-order modified Bessel function.
However,

$$m_{Il} = E[Z_{Il}] = \frac{1}{2}\cos\left(\frac{l}{L}\pi\right)\sum_{k=0}^{N_p-1}\cos(4\pi\Delta f kT_s) \qquad (17)$$

Similarly,

$$m_{Ql} = E[Z_{Ql}] = \frac{1}{2}\cos\left(\frac{l}{L}\pi\right)\sum_{k=0}^{N_p-1}\sin(4\pi\Delta f kT_s) \qquad (18)$$

Since sampled and delayed version of $n(t)$ has the same spectral density N_0 and each $n(2kT_s)$ is statistically independent, then

$$\sigma_l^2 = \mathrm{var}[Z_{Il}] = \mathrm{var}[Z_{Ql}] = \frac{N_pN_0}{4} \qquad (19)$$

$$s_l^2 = \frac{1}{4}\cos^2\left(\frac{l}{L}\pi\right)\cdot\left[\frac{\sin(N_p 2\pi\Delta f T_s)}{\sin(2\pi\Delta f T_s)}\right]^2 \qquad (20)$$

Hence, the signal degradation with frequency error is

$$D(\Delta f) = \frac{1}{N_p{}^2}\left[\frac{\sin(N_p 2\pi\Delta f T_s)}{\sin(2\pi\Delta f T_s)}\right]^2 \qquad (21)$$

But for N_p>>1 and $\pi\Delta f T_s << 1$, the degradation due to frequency error is [3]

$$D(\Delta f) \approx \left[\frac{\sin(N_p 2\pi\Delta f T_s)}{N_p 2\pi\Delta f T_s}\right]^2 \qquad (22)$$

Note that, in equation (17) and (18), l=0 is the index of the maximal point, that is, exact symbol timing estimate. Then, the detection probability is as (23)

$$p_D = \prod_{i=1}^{L-1}(P_0 > P_i) \qquad (23)$$

Where P_i is the probability density function of i-th sample point.
For large SNR, we may make the approximation [4]

$$I_0\left(\frac{rs_l}{\sigma_l^2}\right) \approx \exp\left(\frac{rs_l}{\sigma_l^2}\right)\Big/\left(\frac{2\pi rs_l}{\sigma_l^2}\right)^{\frac{1}{2}} \qquad (24)$$

so that $$\ln I_0\left(\frac{rs_l}{\sigma_l^2}\right) \approx \frac{rs_l}{\sigma_l^2} \qquad (25)$$

Then we can approximate (23) as

$$p_D = \prod_{i=1}^{L-1}\int_{\frac{s_i+s_0}{2}}^{\infty}P_0(r)dr \approx \int_{\frac{s_1+s_0}{2}}^{\infty}P_0(r)dr \qquad (26)$$

4. Discussion

For the analysis of the preamble detector, we assume that the symbol rate is 10Mbps and L=4.

Figure 3 shows the signal degradation with respect to the frequency errors with varying the integration time.

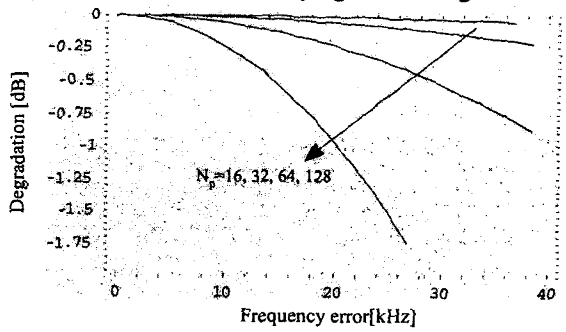

Figure 3. The signal degradation with varying the integration time

We observe that the signal degradation is proportional to both the frequency error and the integration time N_p.
This tells us that large integration time is not always profitable to the performance of the estimate process. If there is no or few frequency error, we can get more energy which guarantees the high detection probability of exact symbol timing. But the frequency error limits the integration time. However, the amount of the signal degradation of each output of symbol integrator will be the same. It means that, a maximal point is always indicates the exact symbol timing in spite of frequency errors. There may be some changes in detection probability when a frequency error exists.
The relationship between the detection probability and the integration time is shown in Figure 4 when the frequency error is 1kHz.

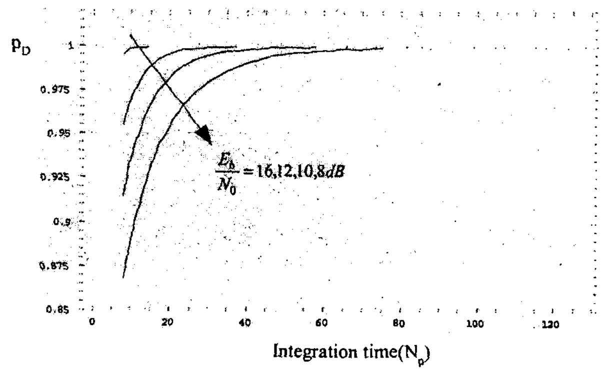

Figure 4. The detection probability of symbol timing estimate with varying bit energy to noise ratio

With a fixed frequency error, as we increase the integration time, we can get more exact symbol timing estimate. In other words, the longer the integtration time is, the higher the detection probability of selecting the maximal sample point will be guaranteed. But the preamble sequence itself is a redundancy that lowers the packet efficiency. Hence, we can choose the proper integration time. From figure 4, we can see that N_p=32 is sufficient to choose the optimal symbol timing with almost no occurrence of missing in case $\frac{E_b}{N_0}$ is greater than 12dB. If we want decrease the preamble length upto about N_p =20, we should assign the required $\frac{E_b}{N_0}$ above 16dB in the link budget design.

Figure 5 shows the effect of frequency errors on the detection probability when $\frac{E_b}{N_0}$ is 16dB.

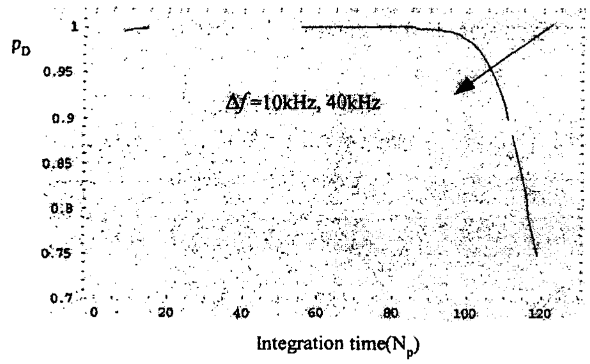

Figure 5. The detection probability with varying frequency errors

Under Np=80, the frequency error has no effect on the detection probability. Note that high frequency error with large integration time causes the detection probability low. This coincides with the result shown in Figure 1.

Figure 6 shows the effect of $\frac{E_b}{N_0}$ on the detection probability when the frequency error is 4kHz.

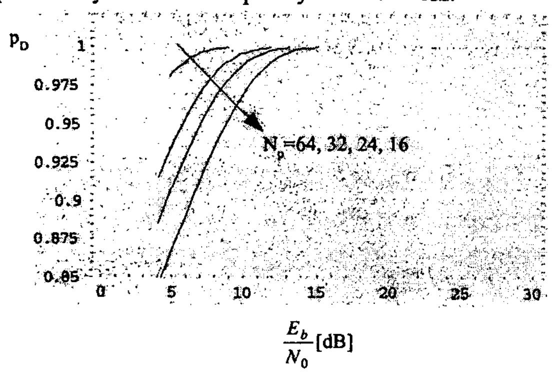

Figure 6. The detection probability with varying integration time

Like the result in Figure 4, N_p=16 is sufficient in case $\frac{E_b}{N_0}$ is above 16dB. A short preamble is required in most systems for the sake of the data efficiency.

5. Conclusions

In this paper, we presented a new structure for symbol timing estimate and analyzed its performance under various conditions. These results in section 4 will be good references for choosing the preamble length under specific environment if we use the suggested algorithms.

There have been a lot of good algorithms for symbol timing estimate. But, sometimes, it is difficult to apply these algorithms directly to hardware materialization with various kinds of reasons like hardware complexity, operation speed, cost and so on. The suggested algorithm in this paper can be applied to some systems. It is all up to both a hardware designer's choice and a system specification whether to use the algorithm suggested or not.

References

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