Convergence Analysis of a Stereophonic Echo Canceling Algorithm Using Input Signals of All Channels

Masanori Kimoto† Toshihiro Furukawa†† Shinsaku Mori †

† Department of Electrical and Electronics Engineering, Nippon Institute of Technology
4-1, Gakuendai, Miyashiro-machi, Minamisaitama-gun, Saitama, 345-8501 Japan
E-mail: kimoto@nit.ac.jp, s_mori@nit.ac.jp
†† Department of Management Science, Science University of Tokyo
1-3, Kagurazaka, Shinjyuku-ku, Tokyo, 162-8601 Japan
E-mail: furukawa@ms.kagu.sut.ac.jp

Abstract

In the linear combination type stereophonic echo canceller, it is known not to converge the coefficient vector of the adaptive filter to a correct echo path. In this report, we analyze the convergence value of the filter coefficient vector of the stereo echo canceling algorithm usig input signals of all channels in relation to this problem. In this analysis, one of the two inputs to the unknown system and adaptive one are assumed to be a delayed and attenuated version of the other signal as a model of the input signal with a strong cross-correlation. As a result, it is shown for the coefficient vectors not to converge to echo paths, and nor to converge to the value which depends on the time delay and the attenuation of the input signal. We show that the computer simulation result are corresponding to our analytical results.

1. Introduction

Echo-canceling is an essential technique for constructing multimedia systems, such as a television conference system. Therefore many methods have so far been proposed. In this report we study the echo cancellation algorithm for a stereophonic system.

In 1984 Fujii and Shimada proposed a multichannel echo canceling algorithm of the linear combination type, and henceforth, several methods based on the similar system have been proposed. In particular the MC-LMS (Multichannel Least Mean Squares) method based on the LMS algorithm has an advantage of a small scale of architecture and less computation time for correction of coefficient vectors, and therefore it is still one of typical adaptive echo canceling algorithms.However since the correction of coefficient vectors by LMS-type algorithms utilizes only the corresponding channel signal but not whole input channel signals, the convergence speed is very slow. Here the convergence speed means the speed of minimizing output error (i.e. removal speed of residual echo).

To overcome this drawbacks Kimoto, et al. proposed the algorithm in which correction of coefficient vectors of each echo canceller are chosen from the entire input signal space [3].Thus this algorithm shows the rapid convergence speed and high ERLE(Echo Return Loss Enhancement) compared with the traditional including LMS-type alogorithms.

However, it is known not to converge the coefficient vector of the adaptive filter to a correct echo path in the linear combination type echo canceller including MC-LMS [1] [2] and Kimoto's method. It is a cause that the cross-correlation between input signals of all channels are strong. The filter coefficient error is not necessarily a corresponding even if the output error (i.e. residual echo) is zero, and this is an important problem of the multichannel echo canceller. The amount of the echo suppression is remarkably deteriorated only because a statistical character of the input signal changes even when the echo path does not change. For instance, the talker movement and the talker change, etc. on the near-end side. That is, the amount of the echo suppression depends on the transfer functions from the sound source to the microphone on the near-end side. Therefore, it is necessary to estimate each of the echo path accurately to improove this situation.

Furthermore, Hirano, et al. have indicated the convergence value of the filter coefficient vector of the MC-LMS analytical in relation to this problem in case of stereophonic [4]. In this analysis, one of the two inputs to the unknown system and adaptive one are assumed to be a delayed and attenuated version of the other signal as a model of the input signal with a strong cross-correlation. As a result, it is shown for the coefficient vectors not to converge to echo paths, and nor to converge to the value which depends on the time delay and the attenuation of the input signal. It is thought that this analysis has an important meaning when the mis-adjustment problem is improved in the MC-LMS.

This paper analyzes the convergence value of the coefficient vectors with stereophonic echo canceling algorithm using input signals of all channels (Kimoto's method). The input signal assumed the same relation as the analysis of Hirano et al.. And analyzed the convergence value of the coefficient vector of the algorithm.

Analitical results show that a part of the coefficient vector of the adaptive filter does not converge to the echo paths as well as MC-LMS. It is shown that an analytical result is corresponding to the computer simulation result.

2. Stereophonic Echo canceller

In this section we explain the system to be considered for the echo cancellation and also give some definitions.

The system of a linear combination type echo canceller is shown in Fig. 1, where we draw only paths for one channel. We consider the communication between Rooms A and B, and suppose that there are two microphones for two channels in Room A. In Room B the acoustic waves from two speakers propagate to two microphones there and then two signals on microphone are transmitted to Room A as shown in Fig.1. The echo cancellers for each channels are located between Rooms A and B as shown in Fig. 1(where we draw them only for channel L). In Room B, there are four echo paths between two speakers and two microphones.

We assume that each echo path is modelled by an FIR filter of order $N-1$ (N being known) and let $\boldsymbol{w}_N^{(ij)}\{i = L, R; j = L, R\}$ represent coefficient vectors of length N corresponding to four echo paths.

The linear combination type stereophonic echo canceller consists of two adaptive filters with coefficient vectors $\boldsymbol{h}_N^{(ij)}\{i = L, R; j = L, R\}$ of length N and identify the echo-path.

In arbitrary channel $j(= L$ or $R)$, Let $u^{(j)}(k)$ be an input signal for microphones in Room A at time k and we define the input matrix in Room A by an $N \times r$ matrix $X_{N,r}^{(j)}(k)$ as follows:

$$X_{N,r}^{(j)}(k) = \left[x_N^{(j)}(k), x_N^{(j)}(k-1), \cdots, x_N^{(j)}(k-r+1)\right] \quad (1)$$

where r is the length of a block, $x_N^{(j)}(k)$ is input vector as follows

$$x_N^{(j)}(k) = [u^{(j)}(k), u^{(j)}(k-1), \cdots, u^{(j)}(k-N+1)]^T. \quad (2)$$

The desired echo signal vector $d_r^{(j)}(k)$ at time k can be represented as:

$$d_r^{(j)}(k) = X_{N,r}^{(L)}(k)^T w_N^{(Lj)} + X_{N,r}^{(R)}(k)^T w_N^{(Rj)}. \quad (3)$$

On the other hand the output signal vector $y_r^{(j)}(k)$ of the estimators is given as :

$$y_r^{(j)}(k) = X_{N,r}^{(L)}(k)^T h_N^{(Lj)} + X_{N,r}^{(R)}(k)^T h_N^{(Rj)}. \quad (4)$$

From Eqs.(3) and (4) we have the output error vector $e_r^{(j)}(k)$ as follows:

$$e_r^{(j)}(k) = d_r^{(j)}(k) - y_r^{(j)}(k). \quad (5)$$

When we define as:

$$W^{(j)} \triangleq [{w_N^{(Lj)}}^T | {w_N^{(Rj)}}^T]^T, \quad \{j = L, R\} \quad (6)$$

$$H^{(j)}(k) \triangleq [h_N^{(Lj)}(k)^T | h_N^{(Rj)}(k)^T]^T, \quad \{j = L, R\} \quad (7)$$

$$X(k) \triangleq [X_{N,r}^{(L)}(k)^T | X_{N,r}^{(R)}(k)^T]^T, \quad (8)$$

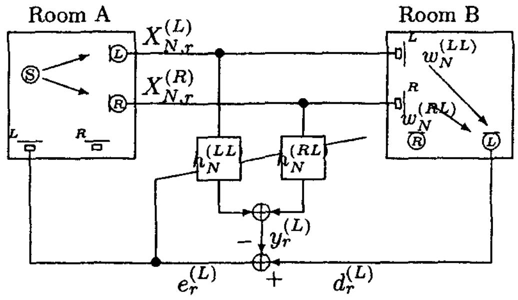


Figure 1: A 2-channel echo canceller based on linear combination

the coefficient error vector $\Gamma^{(j)}(k)$ is given by

$$\Gamma^{(j)}(k) = W^{(j)} - H^{(j)}(k). \quad (9)$$

$e_r^{(j)}(k)$ can be obtained as:

$$e_r^{(j)}(k) = X(k)^T \Gamma^{(j)}(k). \quad (10)$$

Then the procedure of the Kimoto's method is given as:

$$\begin{bmatrix} \Phi^{(Lj)}(k) \\ \Phi^{(Rj)}(k) \end{bmatrix} = A^+(k) e_r^{(j)}(k) \quad (11)$$

$$\Delta h^{(j)}(k) = \begin{bmatrix} \left[X_{N,r}^{(L)}(k) | X_{N,r}^{(R)}(k)\right] \Phi^{(Lj)}(k) \\ \left[X_{N,r}^{(L)}(k) | X_{N,r}^{(R)}(k)\right] \Phi^{(Rj)}(k) \end{bmatrix} \quad (12)$$

$$H^{(j)}(k+1) = H^{(j)}(k) + \Delta h^{(j)}(k) \quad (13)$$

where the $A(k)$ is given by:

$$A(k) = \left[X_{N,r}^{(L)}(k)^T X_{N,r}^{(L)}(k) | X_{N,r}^{(L)}(k)^T X_{N,r}^{(R)}(k) | X_{N,r}^{(R)}(k)^T X_{N,r}^{(L)}(k) | X_{N,r}^{(R)}(k)^T X_{N,r}^{(R)}(k)\right] \quad (14)$$

and superscript $^+$ means the Moore-Penrose type generalized inverse matrix.

3. Convergence Analysis of the Kimoto's method

In this section, we analyze the convergence value of the coeficient vector with the Kimoto's method. In the following analysis, the relation between two input signals is given by

$$x_N^{(R)}(k) = \alpha x_N^{(L)}(k-\tau), \quad (15)$$

where τ and α are time delay between $x_N^{(R)}(k)$ and $x_N^{(L)}(k)$ and attenuation factor, respectively. And $u^{(L)}(k)$ is assumed the white Gaussian signal with zero mean. A white Gaussian signal is characterized by

$$E\left[u^{(L)}(i) u^{(L)}(j)\right] = \begin{cases} \sigma_x^2 & (i = j) \\ 0 & (i \neq j) \end{cases}, \quad (16)$$

where $E[\cdot]$ denotes ensemble average. Moreover, $x_N^{(L)}(k)$ is independent of $\Gamma^{(L)}(k)$ and $A^+(k)$, block length τ equals 1 and only L channel path in Room B is considered are assumed in the following analysis.

Substituting the (10) into (11), we have

$$\begin{bmatrix} \Phi^{(LL)}(k) \\ \Phi^{(RL)}(k) \end{bmatrix} = A^+(k) \begin{bmatrix} x_N^{(L)}(k)^T | \alpha x_N^{(L)}(k-\tau)^T \end{bmatrix} \Gamma^{(L)}(k). \tag{17}$$

When we define as:

$$A^+(k) = [a_1, a_2, a_3, a_4]^T \tag{18}$$

Substituting (18) into (17) gives

$$\begin{bmatrix} \Phi^{(LL)}(k) \\ \Phi^{(RL)}(k) \end{bmatrix} = \begin{bmatrix} a_1 \left[x_N^{(L)}(k)^T | \alpha x_N^{(L)}(k-\tau)^T\right] \\ a_2 \left[x_N^{(L)}(k)^T | \alpha x_N^{(L)}(k-\tau)^T\right] \\ a_3 \left[x_N^{(L)}(k)^T | \alpha x_N^{(L)}(k-\tau)^T\right] \\ a_4 \left[x_N^{(L)}(k)^T | \alpha x_N^{(L)}(k-\tau)^T\right] \end{bmatrix} \Gamma^{(L)}(k). \tag{19}$$

Upper N rows of $\Delta h^{(L)}(k)$ (i.e. $\Delta h_N^{(LL)}(k)$) derived from (19) and (12) as

$$\Delta h_N^{(LL)}(k) = \left[x_N^{(L)}(k) | \alpha x_N^{(L)}(k-\tau)\right] \begin{bmatrix} a_1 \left[x_N^{(L)}(k)^T | \alpha x_N^{(L)}(k-\tau)^T\right] \\ a_2 \left[x_N^{(L)}(k)^T | \alpha x_N^{(L)}(k-\tau)^T\right] \end{bmatrix} \Gamma^{(L)}(k) \tag{20}$$

$$= \left[a_1 x_N^{(L)}(k) x_N^{(L)}(k)^T + a_2 \alpha x_N^{(L)}(k-\tau) x_N^{(L)}(k)^T, \; a_1 \alpha x_N^{(L)}(k) x_N^{(L)}(k-\tau)^T + a_2 \alpha^2 x_N^{(L)}(k-\tau) x_N^{(L)}(k-\tau)^T\right] \Gamma^{(L)}(k). \tag{21}$$

For example, by taking an ensemble average of (21) with the case of $N = 3$ and $\tau = 1$, $E\left[\Delta h_N^{(LL)}(k)\right]$ becomes

$$E\left[\Delta h_N^{(LL)}(k)\right] = \begin{bmatrix} a_1\sigma_x^2 & a_2\alpha\sigma_x^2 & 0 & a_2\alpha^2\sigma_x^2 & 0 & 0 \\ 0 & a_1\sigma_x^2 & a_2\alpha\sigma_x^2 & a_1\alpha\sigma_x^2 & a_2\alpha^2\sigma_x^2 & 0 \\ 0 & 0 & a_1\sigma_x^2 & 0 & a_1\alpha\sigma_x^2 & a_2\alpha^2\sigma_x^2 \end{bmatrix} E\left[\Gamma^{(L)}(k)\right], \tag{22}$$

$$= \begin{bmatrix} a_1\sigma_x^2\gamma_{(1)}^{(LL)} + a_2\alpha\sigma_x^2(\gamma_{(2)}^{(LL)} + \alpha\gamma_{(1)}^{(RL)}) \\ a_1\sigma_x^2(\gamma_{(2)}^{(LL)} + \alpha\gamma_{(1)}^{(RL)}) + a_2\alpha\sigma_x^2(\gamma_{(3)}^{(LL)} + \alpha\gamma_{(2)}^{(RL)}) \\ a_1\sigma_x^2(\gamma_{(3)}^{(LL)} + \alpha\gamma_{(2)}^{(RL)}) + a_2\alpha^2\sigma_x^2\gamma_{(3)}^{(RL)} \end{bmatrix}, \tag{23}$$

where $E\left[\Gamma^{(L)}(k)\right]$ defined by

$$E\left[\Gamma^{(L)}(k)\right] \triangleq \left[\gamma_{(1)}^{(LL)}, \gamma_{(2)}^{(LL)}, \cdots, \gamma_{(N)}^{(LL)}, \gamma_{(1)}^{(RL)}, \gamma_{(2)}^{(RL)}, \cdots, \gamma_{(N)}^{(RL)}\right]^T. \tag{24}$$

By the same calculation, $E\left[\Delta h_N^{(RL)}(k)\right]$ becomes

$$E\left[\Delta h_N^{(RL)}(k)\right] = \begin{bmatrix} a_3\sigma_x^2\gamma_{(1)}^{(LL)} + a_4\alpha\sigma_x^2(\gamma_{(2)}^{(LL)} + \alpha\gamma_{(1)}^{(RL)}) \\ a_3\sigma_x^2(\gamma_{(2)}^{(LL)} + \alpha\gamma_{(1)}^{(RL)}) + a_4\alpha\sigma_x^2(\gamma_{(3)}^{(LL)} + \alpha\gamma_{(2)}^{(RL)}) \\ a_3\sigma_x^2(\gamma_{(3)}^{(LL)} + \alpha\gamma_{(2)}^{(RL)}) + a_4\alpha^2\sigma_x^2\gamma_{(3)}^{(RL)} \end{bmatrix} \tag{25}$$

We assume that $E\left[\Delta h_N^{(LL)}(\infty)\right]$ and $E\left[\Delta h_N^{(RL)}(\infty)\right]$ equals 0_N. The result for $\gamma_{(j)}^{(iL)}\{i = L, R; j = 1, 2, \cdots, N\}$ are derived from (23) and (25) as

$$\gamma_{(1)}^{(LL)} = 0 \tag{26}$$

$$\gamma_{(2)}^{(LL)} + \alpha\gamma_{(1)}^{(RL)} = 0 \tag{27}$$

$$\gamma_{(3)}^{(LL)} + \alpha\gamma_{(2)}^{(RL)} = 0 \tag{28}$$

$$\gamma_{(3)}^{(RL)} = 0. \tag{29}$$

(26) and (29) show that $h_{(1)}^{(LL)}$ and $h_{(3)}^{(RL)}$ converge to the optimum value.The other elements are not converge to the optimum values. When solving $E\left[\Delta h_N^{(LL)}(\infty)\right] = 0_N$ in case of $N = 3$ and $\tau = 2$ by the same process as the example, $\gamma_{(j)}^{(iL)}$ becomes

$$\gamma_{(1)}^{(LL)} = 0 \tag{30}$$

$$\gamma_{(3)}^{(LL)} + \alpha\gamma_{(1)}^{(RL)} = 0 \tag{31}$$

$$\gamma_{(2)}^{(LL)} = 0 \tag{32}$$

$$\gamma_{(2)}^{(RL)} = 0 \tag{33}$$

$$\gamma_{(3)}^{(RL)} = 0 \tag{34}$$

(30), (32), (33) and (34) show that $h_{(1)}^{(LL)}$, $h_{(2)}^{(LL)}$,$h_{(2)}^{(RL)}$ and $h_{(3)}^{(RL)}$ converge to the optimum value.

Next, we think about the case of arbitrary N and τ. $E\left[\Delta h_N^{(LL)}(k)\right]$ becomes

$$E\left[\Delta h_N^{(LL)}(k)\right] = [B|C]\, E\left[\Gamma^{(L)}(k)\right] \tag{35}$$

where B and C are defined by

$$B = \begin{bmatrix} a_1\sigma_x^2 & 0 & \overbrace{\cdots}^{\tau-1} & 0 & a_2\alpha\sigma_x^2 & & \mathbf{0} \\ & a_1\sigma_x^2 & 0 & & 0 & \ddots & \\ & & a_1\sigma_x^2 & 0 & & \ddots & a_2\alpha\sigma_x^2 \\ & & & \ddots & \ddots & & 0 \\ & & & & \ddots & \ddots & \vdots \\ & \mathbf{0} & & & & \ddots & 0 \\ & & & & & & a_1\sigma_x^2 \end{bmatrix} \tag{36}$$

$$C = \begin{bmatrix} a_2\alpha^2\sigma_x^2 & & & & & & \\ 0 & a_2\alpha^2\sigma_x^2 & & & \mathbf{0} & & \\ \vdots & 0 & a_2\alpha^2\sigma_x^2 & & & & \\ 0 & & 0 & \ddots & & & \\ a_1\alpha\sigma_x^2 & 0 & & \ddots & \ddots & & \\ & a_1\alpha\sigma_x^2 & \ddots & & \ddots & \ddots & \\ \mathbf{0} & & a_1\alpha\sigma_x^2 & 0 & \underbrace{\cdots}_{\tau-1} & 0 & a_2\alpha^2\sigma_x^2 \end{bmatrix} \quad (37)$$

Finally, $\gamma_{(j)}^{(iL)}$ becomes

$$\left.\begin{matrix} \gamma_{(1)}^{(LL)} \\ \vdots \\ \gamma_{(\tau)}^{(LL)} \end{matrix}\right\} = 0 \quad (38)$$

$$\left.\begin{matrix} \gamma_{(\tau+1)}^{(LL)} + \alpha\gamma_{(1)}^{(RL)} \\ \gamma_{(\tau+2)}^{(LL)} + \alpha\gamma_{(2)}^{(RL)} \\ \vdots \\ \gamma_{(N)}^{(LL)} + \alpha\gamma_{(N-\tau)}^{(RL)} \end{matrix}\right\} = 0 \quad (39)$$

$$\left.\begin{matrix} \gamma_{(N-\tau+1)}^{(RL)} \\ \vdots \\ \gamma_{(N)}^{(RL)} \end{matrix}\right\} = 0 \quad (40)$$

Therefore, first τ element of $h_N^{(LL)}$ and the last τ element of $h_N^{(RL)}$ converge to the echo paths.

4. Computer Simulations

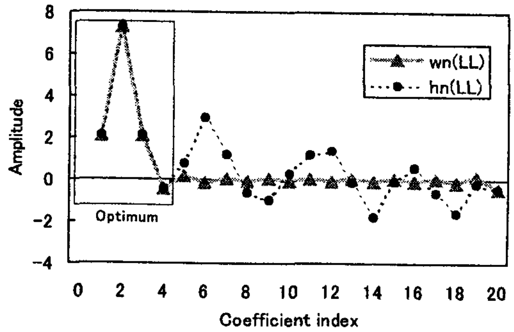
Simulations have been carried out in order to examine the validity of the analysis.

4.1 Condition

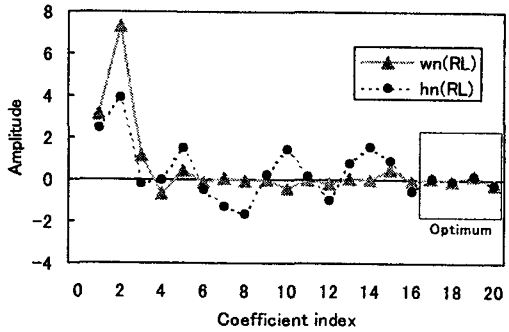
- Input signal $u(k)$ is white Gaussian signal with the variance 1/12.
- We assume that there is the relation between these two input signals as follows:
  $x_N^{(R)}(k) = 0.9x_N^{(L)}(k-4)$. $\{$i.e. $\tau = 4\}$
- The systems and the estimators are both FIR filters of order $19 (N = 20)$.

4.2 Results

It is understood that the first τ element of $h_N^{(LL)}$ and the last τ element of $h_N^{(RL)}$ converge to the echo paths $w_N^{(iL)}\{i = L, R\}$. As for channel R, this result is the same.



(a)$h_N^{(LL)}(\infty)$



(b)$h_N^{(RL)}(\infty)$

Figure 2: Coefficient vector after convergence

5. Conclusion

This paper shows the convergence value of the coefficient vector with the stereophonic echo canceling algorithm using input signals of all channels. In the analysis, it is shown that a part of coefficient vector decided by size of τ converge to the echo paths. Moreover, the analytical result is corresponding to the computer simulation result.

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