Basic characteristics of super-multi-stabilized chaotic pulse-trains

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Abstract: Applying a higher frequency periodic control signal, a state of a chaotic pulse-train generator is quantized. The circuit has various co-existing super-stable periodic pulse-trains (ab. SSPTs) and generates one of them depending on the initial state. Also correlation characteristics of the SSPTs are analyzed precisely. We then consider application of the SSPTs to spread sequences of CDMA with pulse-train signals.

1. Introduction

The Chaotic Pulse-train Generator (ab. CPG) has been studied as a fundamental nonlinear system. The CPG can generate periodic and chaotic pulse-trains, and can exhibit various bifurcation phenomena. Applying a higher frequency periodic control signal, the state of the CPG is quantized and the pulse positions are restricted on a lattice. Then the CPG generates super-stable periodic pulse-trains (ab. SSPTs) governed by a quantized pulse position map (ab. Qmap) [3][4]. Various SSPTs co-exist and the CPG generates one of them depending on the initial state.

In this paper we analyze the characteristics of co-existing SSPTs and consider its application to CDMA communications. In order to consider correlation characteristics of the SSPTs, a multiplex correlation is defined. The correlation is analyzed precisely using the Qmap. We then suggest that the set of SSPTs can be useful as spread sequences of CDMA with pulse-train signals. Note that periods and correlation characteristics of SSPTs have not been analyzed sufficiently in the literatures. We have derived some basic results in [3].

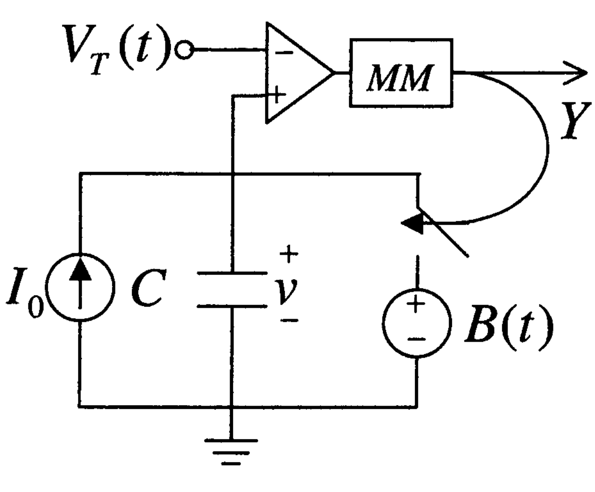

Figure 1. chaotic pulse-train generator.

The CPG can be also used to construct multiplex communications systems including ultrawide-bandwidth impulse-radio systems which have advantages of low power consumption, low probability of intercept and so on [5]. Then the analysis of our CPG may contribute to develop a flexible communication system based on the quantized pulse-train dynamics. A fusion of neural dynamics and communications systems has also begun to be considered [6].

2. Chaotic pulse-train generator and state quantization

Fig. 1 shows a chaotic pulse-train generator (ab. CPG) [2][3] . The circuit dynamics is described by

$$\begin{cases} C\dfrac{dv}{dt} = I_0, & \text{for } v < V_T(t), \\ v(t^+) = B(t^+), & \text{if } v(t) = V_T(t), \end{cases} \quad (1)$$

$$Y(t^+) = \begin{cases} -E, & \text{for } v < V_T(t), \\ E, & \text{if } v(t) = V_T(t), \end{cases}$$

$$V_T(t) = \alpha(t - \frac{T}{2M}) + V_0, \text{ for } 0 \le t < \frac{T}{M},$$

$$V_T(t + \frac{T}{M}) = V_T(t),$$

$$B(t) = -\beta(t - \frac{T}{2}) - B_0, \text{ for } 0 \le t < T,$$

$$B(t+T) = B(t).$$

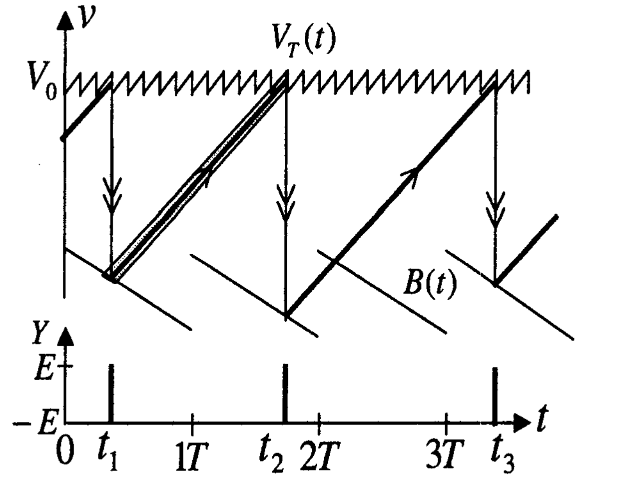

Figure 2. Basic dynamics of the CPG. Quantization frequency $M = 7$.

$B(t)$ is a sawtooth periodic signal with period T and is called the base. $V_T(t)$ is a periodic sawtooth signal with period $\frac{T}{M}$ and is called the threshold. M is a positive integer and is called the quantization frequency. The threshold $V_T(t)$ synchronizes with the base $B(t)$. As shown in Fig. 2, the state v increases by integrating the current I_0. If the state v hits the threshold, the mono-stable multi-vaibrator (ab. M.M.) outputs a pulse $Y = E$ and closes the switch instantaneously. At this moment, the state v is reset to the base $B(t)$. Repeating this manner, the CPG generates a pulse-train $Y(t)$. Let t_n be the n-th pulse position. If the threshold is constant ($V_T(t) = V_0$), the pulse position is governed by the following chaotic pulse position map as shown in Fig. 3.:

$$t_{n+1} = f(t_n) = t_n - \frac{I_0}{C}B(t_n), \qquad (2)$$

$$f : \boldsymbol{R}^+ \to \boldsymbol{R}^+,$$

Where, $\boldsymbol{R}^+$ is a positive real number. Since $f(t+T) = f(t) + T$, the pulse-train dynamics can be characterized by the following return map F:

$$\theta_{n+1} = F(\theta_n) \equiv \theta_n - \frac{I_0}{C}B(\theta_n) \pmod{\mathrm{T}}, \qquad (3)$$

$$F : [0,T) \to [0,T).$$

Note that adjusting the shape of the base $B(t)$, various pulse position maps can be obtained. As the sawtooth signal $V_T(t)$ is applied, the state v can hit the threshold $V_T(t)$ only at the vertical parts, because all the states v in the shaded region is reset to the base at the same moment t_2 as shown in Fig. 2. Hence, the pulse position t_n is quantized and is restricted on a lattice $\boldsymbol{L} \equiv \{0, \frac{1}{M}T, \frac{2}{M}T, \cdots\}$. Therefore the pulse position t_n is super-stable. The pulse position t_n is described by the following quantized pulse-position map (ab. Qmap) as shown in Fig. 4:

$$t_{n+1} = g(t_n) \equiv \frac{T}{M}Int(\frac{M}{T}f(t_n) + \frac{T}{2}), \qquad (4)$$

$$g : \boldsymbol{L} \to \boldsymbol{L} \equiv \{0, \frac{1}{M}T, \frac{2}{M}T, \cdots\}.$$

Since $g(t+T) = g(t)+T$, the SSPT can be characterized by the following quantized return map G:

$$\theta_{n+1} = G(\theta_n) \equiv g(\theta_n) \pmod{\mathrm{T}}, \qquad (5)$$

$$G : \boldsymbol{L}_0 \to \boldsymbol{L}_0 \equiv \{0, \frac{1}{M}T, \frac{2}{M}T, \cdots, \frac{M-1}{M}\}.$$

In this paper, we consider role of the parameter M, and other parameters are fixed: $V_0 = \frac{3}{2}\frac{I_0}{C}T$, $\alpha = \frac{I_0}{C}$, $B_0 = 0$, and $\beta = \frac{I_0}{C}$. We give the some definitions on the Qmap g.

Definition A pulse position t^* is said to be super-stable periodic pulse position (ab. SSPP) with period pT if p is the minimum integer such that $g^q(t^*) - t^* = pT$ for some positive integer q, where g^q denotes the q-fold composition of g. A pulse-train Y^* is said to be super-stable periodic pulse-train (ab. SSPT) with period pT if Y^* is represented by the sequence of SSPPs $(t^*, g(t^*), \cdots, g^{q-1}(t^*))$ and $g^q(t^*) - t^* = pT$. A pulse position t^e is said to be eventually periodic point (ab. EPP) if t^* is not periodic but $g^k(t^e)$ is periodic for some integer k.

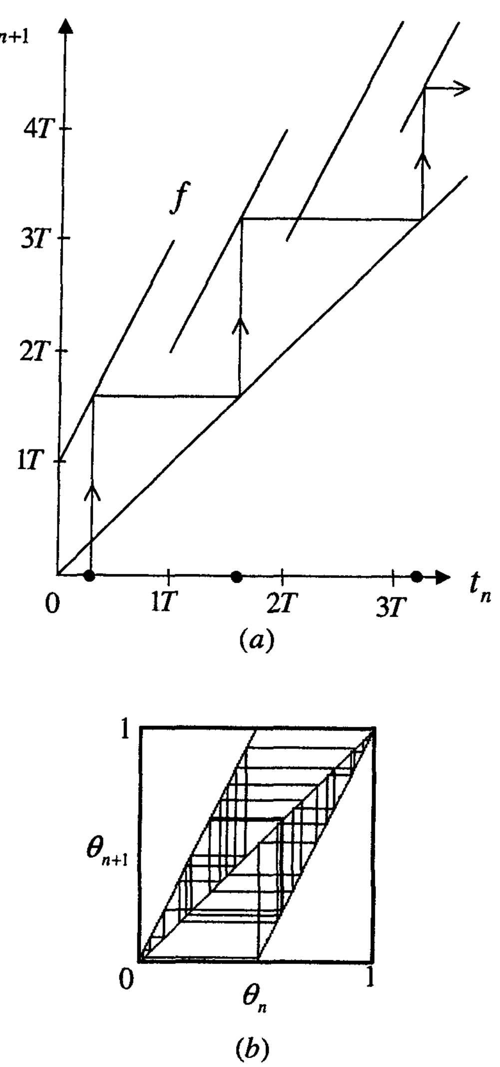

Figure 3. (a)Chaotic pulse position map f. (b)Chaotic return map F.

The Qmap has various co-existing SSPTs and generates one of them depending on the initial state as shown in Fig. 4. In this paper we focus on SSPTs with the maximum period. Let N be the number of the SSPTs with the maximal period, where all the shift invariant SSPTs are counted as one SSPT. We present an algorithm that calculate the number N for some quantization frequency M.

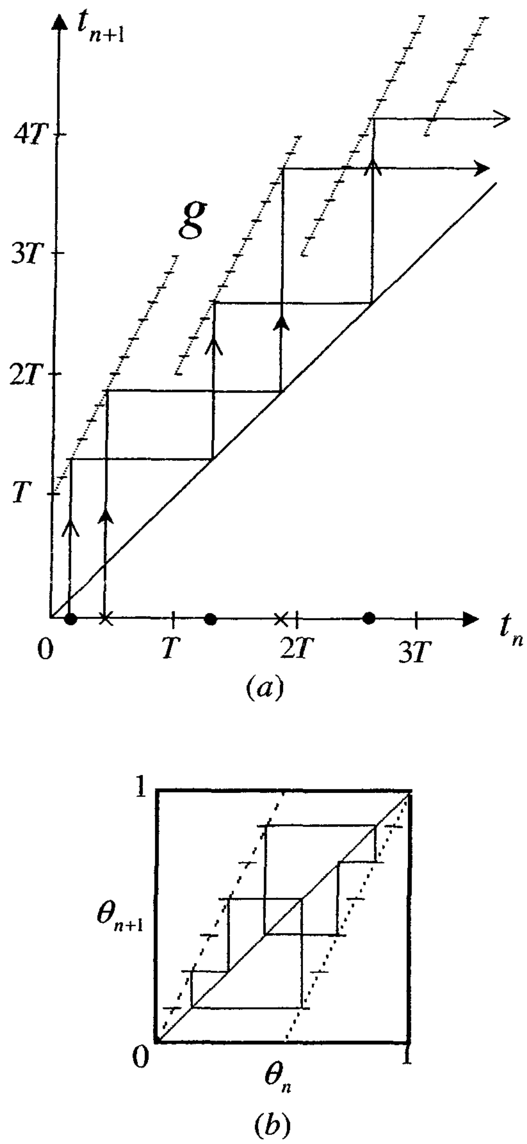

Figure 4. (a) Quantized pulse position map g for $M = 7$. The dotted line is the chaotic pulse position map f. (b) quantized return map G for $M = 7$.

Algorithm (1)

Step 1:
Set the counter value $n = 0$, $i = 1$ and choose the initial pulse-position $l = 0 \in \boldsymbol{L_0}$. Then go to Step 2.

Step 2:
Iterate G from l. If $G^j(l) \neq G^e(l)$, $(1 < e < j)$ is "don't care", we declare $(l, \cdots, G^j(l))$ as "don't care". Then go to Step 3. If G satisfies $G^d(l) = G^h(l)$ $(0 < d < h)$, we get a sequence of quantized return map $(l, G(l), \cdots, G^d(l), \cdots, G^{h-1}(l))$. We then declare all the pulse positions in the sequence as "don't care". A sequence of Qmap $(g^d(l), \cdots, g^{h-1}(l))$ is a SSPT. Its map period q_i can be calculated by $q_i = h - d$, and period p_i can be calculated by $p_i = g^h(l) - l$. Renew the counter vaule $i = i + 1$. Go to the Step 3

Step 3:
Renew the counter value $n = n+1$ and choose the initial pulse-position $l = \frac{n}{M}T \in \boldsymbol{L_0}$. Go to the Step 4.

Step 4:
If l is "don't care", then go to the Step 3. If we find l that is not "don't care", then go to Step 2. If $n = M$, then the algorithm is terminated. Let $K = i$ is the number of co-existing SSPTs.

Algorithm (2)

Step 1:
Using the algorithm (1), calculate the period p_i $(i = 1, \cdots, K)$ of co-existing SSPTs.

Step 2:
Let $p_{max} = max_i\{p_i\}$ be a maximum period of co-existing SSPTs. N is the number of the SSPTs such that the period $p_i = p_{max}$. Terminate the algorithm.

In Fig. 5, there exist 9 SSPTs ($N = 9$) with the maximum period $21T$. Using the algorithms, we can calculate N for various parameter values as shown in Fig 5. We can see that the graph of N has peaks at $M = 2^k+1$, $k = 1, 2, \cdots$. The arrow in Fig. 6 shows the quantization frequency $M = 2^7 = 129$. In this case, all the pulse positions t_n are SSPPs and there are no EPPs, because $M = 2^k + 1$, $k = 1, 2, \cdots$. is odd number [3]. Fig. 5 shows the SSPTs for $M = 2^7 + 1 = 129$.

3. Correlation characteristics and CDMA

In this section, we consider correlation characteristics of the SSPTs and consider their application to CDMA communications systems. Let us chose L SSPTs from all the co-existing N SSPTs, and let $\boldsymbol{Y}$ be the set of these SSPTs, where $L \leq N$. Also let $P(t)$ be a multiplex pulse-train defined by:

$$P(t) = \bigcup_{Y_i(t) \in \boldsymbol{Y}} Y_i(t). \tag{6}$$

There exist p^{L-1} kinds of multiplex pulse-trains $P(t)$ depending of the initial pulse position of each SSPT $Y_i(t)$. We define a multiplex-correlation $R_m(t_s)$ which is a correlation of $P(t)$ and $Y_m(t)$ as the following. Let $t_s \in \{\frac{0}{M}, \frac{1}{M}T, \cdots, \frac{pM-1}{M}T\}$ be the phase shift. Let $K_m(t_s)$ be the number of pulse-positions $t \in \{0, \frac{1}{M}T, \cdots, \frac{pM-1}{M}T\}$ such that $Y_m(t + t_s) = E$ and $P(t) = E$. Let $L_m(t_s)$ be the number of pulse-positions $t \in \{0, \frac{1}{M}T, \cdots, \frac{pM-1}{M}T\}$ such that $Y_m(t + t_s) = E$ and $P(t) = -E$. Let q_m be the number of pulses of Y_m for one period. Then we define the multiplex-correlation $R_m(t_s)$:

$$R_m(t_s) = \frac{K_m(t_s) - L_m(t_s)}{q_m}. \tag{7}$$

Here, let us consider the multiplex pulse-train $P(t)$ of 9 SSPTs ($L = N = 9$) in Fig. 5. We can calculate the multiplex-correlation $R_m(t_s)$ precisely by the computer as shown in Fig. 7. The largest peak $R_1(t_s) = 1$ appears at the phase shift $t_s = 0$ and $21T$. Let Q_m be

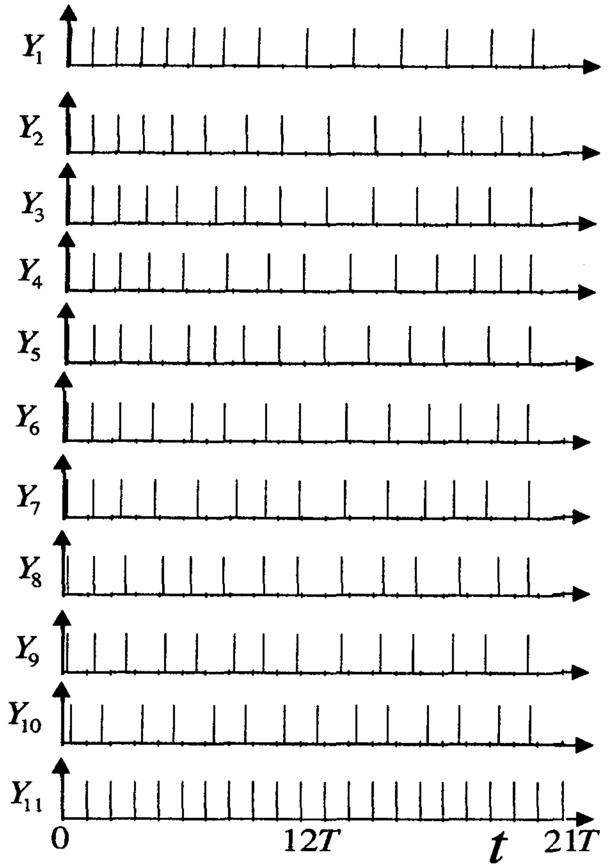

Figure 5. Co-existing SSPTs for $M = 129$. 9 SSPTs with period $21T$ and 1 SSPT with period T co-exist

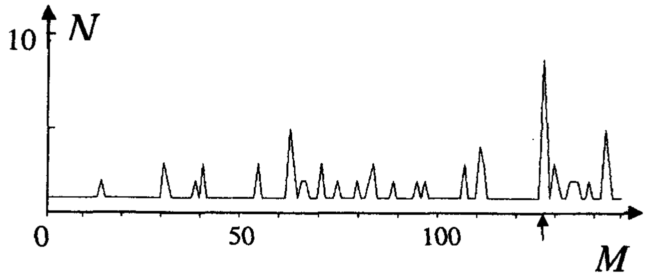

Figure 6. The number N of the SSPTs with the maximal period.

Figure 7. The multiplex-correlation for $M = 129$.

the second largest peak of $R_m(t_s)$. In Fig. 7, the second peak $Q_1 = -0.500$ is much smaller than the largest peak 1. Also each multiplex-correlation R_m, $m = 1, 2, \cdots, L$, has the largest peak $R_m(0) = 1$ and the second peak satisfies $-0.428 \leq Q_m \leq -0.285$. This implies small cross-correlation against another SSPT Y_n, $n \neq m$. Hence the set $\boldsymbol{Y}$ of SSPTs is useful for spread sequences of CDMA with pulse-train signals [5]. The multiplex-correlation depends on which SSPTs are chosen to make the multiplex pulse-trains $P(t)$.

4. Conclusions

We have considered the CPG and its sate quantization. Based on the analysis of the multiplex-correlation, we have shown that the SSPTs can be useful for CDMA communications systems. Future problems include development of an automatic synthesis method of the set of SSPTs for CDMA and synthesis of a practical communication circuit.

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