A Realization Method of the Transfer Functions Containing Variable Parameter

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Abstract: In this paper, we propose a method for realizing transfer functions containing variable parameter, by the state-space method. By using this method, variable transfer functions (VTF) can be often realized with a minimal dimension. In case that a minimal realization can not be obtained, the realization dimension can be fairly reduced.

1. Introduction

Circuits containing variable elements are the general name of the circuits containing elements whose values can be varied independently to time and frequency, by varing the values continuously, the designated input-output characteristics can be obtained continuously. Such variable circuits take an important place in the measurement-control system and the communication system. Especially, in the design of the variable equalizers, many researches have been done so far [1] etc.

Though, various methods to synthesize the circuits containing variable elements mentioned above exist, the synthesis method in terms of variable transfer functions (VTF) which consist of the complex angular frequency s and the variable parameter λ is thought to be an interesting method from the point of view of the unification of the theory.

Some methods for realizing VTF have been reported [2],[3] and they are very interesting. But, these methods have the difficulty that the realization dimension is very high. Hence, the circuit synthesis is very complicated.

Therefore, in this paper, in order to overcome the difficulty of the above-mentioned methods, we propose a method for realizing VTF by the state-space method. In this method, we perform the statical feedback on the state-space realization form of separable-denominator VTF, so that we realize the general VTF. By using this method, VTF can be often realized with a minimal dimension.

In case that minimal realization can not be obtained, we propose a method that we perform the dynamical feedback on it. By using this method, the realization dimension can be fairly reduced.

2. Voltage Transfer Functions Containing Variable Parameter

Circuit functions of circuit containing m gearing variable elements, can be expressed by

$$F(s,\lambda)=\sum_{i=0}^{m} g_i(s)\lambda^i / \sum_{i=0}^{m} f_i(s)\lambda^i \tag{1}$$

$$=\sum_{j=0}^{n} \hat{g}_j(\lambda)s^j / \sum_{j=0}^{n} \hat{f}_j(\lambda)s^j \tag{2}$$

[4]. We call equation (1)((2)) variable transfer functions (VTF).

3. State-Space Realization Form of Separable-Denominator VTF

Separable-denominator VTF in the following, can be realized with a minimal dimension [5].

$$F_s(s,\lambda)=g(s,\lambda)/\{a(s)\cdot b(\lambda)\} \tag{3}$$

A minimal dimension is a degree with respect to two variables of its denominator.

Equation (3) can be realized by the following two types of state-space realization form, by the method in [5].

Controllable-Observable (C-O) state-space realization form:

$$\begin{cases}\begin{bmatrix}\dot{\mathbf{x}}\\ \mathbf{y}_i\end{bmatrix}=\begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{12}\\ \mathbf{0} & \mathbf{A}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \mathbf{u}_i\end{bmatrix}+\begin{bmatrix}\mathbf{b}_1\\ \mathbf{b}_2\end{bmatrix}u\\ y=\begin{bmatrix}0 & 1 & \mathbf{c}_2\end{bmatrix}\begin{bmatrix}\mathbf{x}^T & \mathbf{u}_i^T\end{bmatrix}^T+du\end{cases} \tag{4}$$

where

$$\mathbf{y}_i=\begin{bmatrix}y_1\cdots\cdots y_m\end{bmatrix}^T,\quad \mathbf{u}_i=\begin{bmatrix}u_1\cdots\cdots u_m\end{bmatrix}^T \tag{5}$$

$$y_i=\lambda u_i \quad (i=1,2,\cdots\cdots,m) \tag{6}$$

$$\mathbf{b}_2 = [\ 0 \quad 1]^{\mathrm{T}}. \tag{7}$$

Observable-Controllable (O-C) state-space realization form:

$$\begin{cases} \begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{y}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11}^{\mathrm{T}} & \mathbf{0} \\ \mathbf{A}_{12}^{\mathrm{T}} & \mathbf{A}_{22}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{u}_1 \end{bmatrix} + \begin{bmatrix} \mathbf{b}_2 \\ \mathbf{c}_2^{\mathrm{T}} \end{bmatrix} u \\ y = \begin{bmatrix} \mathbf{b}_1^{\mathrm{T}} & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x}^{\mathrm{T}} & \mathbf{u}_1^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} + du. \end{cases} \tag{8}$$

4. Minimal Realization of VTF

In (1), it is assumed that n>m.

Similarly, we can realize it in case that n<m.

4.1 Modification of VTF

We perform a bilinear transformation of the variable parameter in the following

$$\lambda = (c\lambda' + d)/(a\lambda' + b) \tag{9}$$

on equation (1). And, we decide values of a, b, c, d so that $\hat{g}_m(s)$ of

$$\hat{F}(s,\lambda') = \sum_{i=0}^{m} \hat{g}_i(s)\lambda'^i \Big/ \sum_{i=0}^{m} \hat{f}_i(s)\lambda'^i \tag{10}$$

is (n-1)-degree polynomial and $\hat{f}_m(s)$ of (10) is n-degree polynomial.

Dividing the denominator and numerator of equation (10) by the coefficient of s^n of $\hat{f}_m(s)$, we obtain

$$\hat{F}(s,\lambda') = \sum_{i=0}^{m} \tilde{g}_i(s)\lambda'^i \Big/ \sum_{i=0}^{m} \tilde{f}_i(s)\lambda'^i \tag{11}$$

$$= \sum_{j=0}^{n} \bar{g}_j(\lambda')s^j \Big/ \sum_{j=0}^{n} \bar{f}_j(\lambda')s^j. \tag{12}$$

4.2 Realization of separable-denominator VTF containing unknown quantities

Consider the case of

$$\hat{F}_s(s,\lambda',\mathbf{a}_i) = \sum_{i=0}^{m} \tilde{g}_i(s)\lambda'^i \Big/ \{(\sum_{i=0}^{n-1} a_i s^i + s^n)\cdot \bar{f}_n(\lambda')\} \tag{13}$$

where a_i (i=0,1,——,n-1) are unknown quantities.

We obtain the C-O state-space realization form of equation (13).

It can be expressed as

$$\begin{cases} \begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{y}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11}(\mathbf{a}_i) & \mathbf{A}_{12}(\mathbf{a}_i) \\ \mathbf{0} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{u}_1 \end{bmatrix} + \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} u \\ y = \begin{bmatrix} 0 & 1 & \mathbf{c}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}^{\mathrm{T}} & \mathbf{u}_1^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \end{cases} \tag{14}$$

4.3 Statical feedback containing unknown quantities

We perform the following statical feedback on (14).

$$u = \mathbf{f}(\mathbf{a}_i)\mathbf{x} + v \tag{15}$$

$\mathbf{f}(\mathbf{a}_i)$ in (15) can be calculated by the method in [6].

Then, the following system can be obtained.

$$\begin{cases} \begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{y}_1 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{A}}_{11}(\mathbf{a}_i) & \mathbf{A}_{12}(\mathbf{a}_i) \\ \hat{\mathbf{A}}_{21}(\mathbf{a}_i) & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{u}_1 \end{bmatrix} + \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} v \\ y = \begin{bmatrix} 0 & 1 & \mathbf{c}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}^{\mathrm{T}} & \mathbf{u}_1^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \end{cases} \tag{16}$$

Now,

$$\det(s\mathbf{I}_n - \hat{\mathbf{A}}_{11}(\mathbf{a}_i)) = \tilde{f}_m(s) \tag{17}$$

$$\det(\lambda'\mathbf{I}_m - \mathbf{A}_{22}) = \bar{f}_n(\lambda') \tag{18}$$

are satisfied.

4.4 Calculation of unknown quantities

We calculate the following characteristic polynomial of (16),

$$\det\begin{bmatrix} s\mathbf{I}_n - \hat{\mathbf{A}}_{11}(\mathbf{a}_i) & -\mathbf{A}_{12}(\mathbf{a}_i) \\ -\hat{\mathbf{A}}_{21}(\mathbf{a}_i) & \lambda\mathbf{I}_m - \mathbf{A}_{22} \end{bmatrix}$$

$$= \det\{s\mathbf{I}_n - \mathbf{A}_{11}(\mathbf{a}_i) - \mathbf{b}_1\mathbf{f}(\mathbf{a}_i)\}\cdot\det(\lambda'\mathbf{I}_m - \mathbf{A}_{22})$$

$$+\hat{h}(s,\lambda',\mathbf{a}_i). \qquad (19)$$

From (17) and (18), (19) can be expressed as

$$\tilde{f}_m(s)\bar{f}_n(\lambda') + \hat{h}(s,\lambda',\mathbf{a}_i). \qquad (20)$$

Then, VTF of the system in (16) is

$$F(s,\lambda',\mathbf{a}_i) = \sum_{i=0}^{m}\tilde{g}_i(s)\lambda'^i / \left\{\tilde{f}_m(s)\bar{f}_n(\lambda') + \hat{h}(s,\lambda',\mathbf{a}_i)\right\}. \qquad (21)$$

We express denominator polynomial of equation (11) as

$$\tilde{f}_m(s)\bar{f}_n(\lambda') + h(s,\lambda'). \qquad (22)$$

Comparing (22) with (20), we calculate the values of a_i (i =0,1,——,n-1) so that

$$h(s,\lambda') = \hat{h}(s,\lambda',\mathbf{a}_i). \qquad (23)$$

In case that such a_i (i =0,1,——n-1) don't exist, it is impossible to realize equation (1) with a minimal dimension, by this method. Then, we apply the realization method which we will propose in **5**.

Now, it is assumed that such a_i (i =0,1,——,n-1) ((23) is satisfied) exist.

Substituting those a_i into (16), (16) is a minimal realization of equation (11).

In case that n<m in equation (1), we consider a separable-denominator VTF in the following.

$$\hat{F}_s(s,\lambda',\mathbf{a}_i) = \sum_{i=0}^{m}\tilde{g}_i(s)\lambda'^i / \left\{\tilde{f}_m(s)\cdot(\sum_{i=0}^{m-1}a_i\lambda'^i + \lambda'^m)\right\} \qquad (24)$$

We obtain O-C state-space realization form of equation (24), and after that, we calculate the values of

a_i (i =0,1,——,m-1) through the similar steps.

5. Realization Method by Dynamical Feedback

5.1 Modification of VTF

We perform a bilinear transformation of the variable parameter in the following

$$\lambda = (g\lambda'' + h)/(e\lambda'' + f) \qquad (25)$$

on equation (1). And, we decide the values of e, f, g, h so that the following transformed VTF

$$\tilde{F}(s,\lambda'') = \sum_{i=0}^{m}q_i(s)\lambda''^i / \sum_{i=0}^{m}p_i(s)\lambda''^i \qquad (26)$$

satisfies the following two conditions.

①. $q_m(s)$ is (n-1)-degree polynomial..

②. $p_m(s)$ is a Hurwitz polynomial..

Dividing the denominator and numerator of equation (26) by the coefficient of s^n in $p_m(s)$, we obtain

$$\tilde{F}(s,\lambda'') = \sum_{i=0}^{m}\hat{q}_i(s)\lambda''^i / \sum_{i=0}^{m}\hat{p}_i(s)\lambda''^i. \qquad (27)$$

Performing the steps in **4.2**, **4.3** on equation (27), we obtain the system corresponding to (16).

5.2 Calculation of dynamical feedback

The system (corresponding to (16)) obtained in **5.1**, can be expressed as

$$\begin{cases} \dot{\mathbf{x}} = \hat{\mathbf{A}}_{11}(\mathbf{a}_i)\mathbf{x} + \begin{bmatrix}\mathbf{A}_{12}(\mathbf{a}_i) & \mathbf{b}_1\end{bmatrix}\begin{bmatrix}\mathbf{u}_i \\ v\end{bmatrix} \\ \begin{bmatrix}\mathbf{y}_i \\ y\end{bmatrix} = \begin{bmatrix}\hat{\mathbf{A}}_{21}(\mathbf{a}_i) \\ \mathbf{0} \quad 1\end{bmatrix}\mathbf{x} + \begin{bmatrix}\mathbf{A}_{22} & \mathbf{b}_2 \\ \mathbf{c}_2 & 0\end{bmatrix}\begin{bmatrix}\mathbf{u}_i \\ v\end{bmatrix} \end{cases} \qquad (28)$$

From (28), we calculate backward

$$\begin{bmatrix} \mathbf{y}_i \\ y \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{A}}(s,\mathbf{a}_i) & \hat{\mathbf{b}}(s,\mathbf{a}_i) \\ \hat{\mathbf{c}}(s,\mathbf{a}_i) & \hat{d}(s) \end{bmatrix} \begin{bmatrix} \mathbf{u}_i \\ v \end{bmatrix}. \tag{29}$$

From (29), we calculate

$$\hat{p}_m(s)\cdot\det\left\{\lambda^n \mathbf{I}_m - \hat{\mathbf{A}}(s,\mathbf{a}_i)\right\}$$
$$=\sum_{i=0}^{m-1}\tilde{h}_i(s,\mathbf{a}_i)\lambda^{ni} + \hat{p}_m(s)\lambda^{nm}. \tag{30}$$

Then, VTF of the system (28) is

$$F(s,\lambda^n,\mathbf{a}_i) = \sum_{i=0}^{m}\hat{q}_i(s)\lambda^{ni}$$
$$/\left\{\sum_{i=0}^{m-1}\tilde{h}_i(s,\mathbf{a}_i)\lambda^{ni} + \hat{p}_m(s)\lambda^{nm}\right\}. \tag{31}$$

By performing the following dynamical feedback on (28), we can let the denominator polynomial of VTF coincide with the denominator polynomial of (27),

$$v = \mathbf{f}(s,\mathbf{a}_i)\mathbf{u}_i + w. \tag{32}$$

$\mathbf{f}(s,\mathbf{a}_i)$ in (32) can be calculated by the method in [6].

6. Conclusion

In this paper, we proposed a method for realizing VTF, by the state-space method. By using this method, VTF can be realized with a very low dimension.

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