Even-phase ZCD codes for MAI Cancelled DS-CDMA Systems

Jae-sang Cha
Department of Information and Communication, SeoKyeong University,
16-1, Chongnung-Dong Songbuk-Ku, Seoul 136-704, Korea
Tel.: +82-2-940-7468, Fax: +82-2-940-7468
e-mail: chajs@skuniv.ac.kr

Abstract: Multiple access interference (MAI) and multipath interference (MPI) degrade the system performance in the DS-CDMA (direct-sequence code-division multiple-access) systems. In this paper, a generalized construction method for $2^n (n = 1, 2, 3)$ phase preferred pairs (PP) with zero-correlation duration (ZCD) of $(0.5N + 1)$ chips is proposed. $2^n (n = 1, 2, 3)$ phase ZCD code sets with ZCD and enlarged family sizes are generated by carrying out a chip-shift operation of the preferred pairs. The properties of the proposed codes are effective for the cancellation of MAI and MPI in DS-CDMA Systems.

1. Introduction

For any two complex spreading codes of period $N$, $C^{(x)}_N = (c_0^{(x)}, \ldots, c_{N-1}^{(x)})$ and $C^{(y)}_N = (c_0^{(y)}, \ldots, c_{N-1}^{(y)})$, the periodic correlation function with a shift $\tau$ is defined as

$$\theta_{x,y}(\tau) = \sum_{k=0}^{N-1} c_k^{(x)} c_{k+\tau \mod N}^{(y)}$$

where $\tau$ denotes a complex conjugate. Eqn (1) becomes the autocorrelation function (ACF) when $x = y$ and the crosscorrelation function (CCF) when $x \neq y$.

Since the maximum magnitude of periodic ACF side-lobes ($\theta_{x,x}$) and the maximum magnitude of periodic CCF ($\theta_x$) are bounded by theoretical limits [1], spreading codes with both zero $\theta_{x,x}$ and zero $\theta_x$ cannot be constructed. However, it is possible to construct spreading codes with both zero $\theta_{x,x}$ and zero $\theta_x$ at the local duration around $\tau = 0$. This local duration is defined as the zero-correlation duration (ZCD). This ZCD property can cancel MAI at the uplink of DS-CDMA systems, and it can be found in the binary codes [2-6] and a class of 4-phase codes [6]. In this paper, we propose new 8-phase ZCD codes and present a generalized construction method for $2^n (n = 1, 2, 3)$ phase ZCD codes.

At first, we present a generalized construction method for $2^n (n = 1, 2, 3)$ phase ZCD PP that have the ZCD of $(0.5N + 1)$ chips and periods.

2. Construction method of $2^n (n = 1, 2, 3)$ phase ZCD codes

The $2^n (n = 1, 2, 3)$ phase ZCD codes are constructed by using the chip-shift operation of the $2^n (n = 1, 2, 3)$ phase ZCD PP. $2^n (n = 1, 2, 3)$ phase ZCD PP is started from the initial $2^n (n = 1, 2, 3)$ phase ZCD PP generated by the initial basic matrix and its periods are extended by the period-extension matrix $E$ as defined below.

1) Definition of the initial basic matrix

The initial basic matrices to construct preferred pairs are defined for the 2-phase, 4-phase and 8-phase cases respectively. The initial basic matrices to make 2-phase PP, 4-phase PP and 8-phase PP are defined as Eqn (2), Eqn (3), Eqn (4), respectively.

$$G_{2P} = \begin{bmatrix}
1 & 1 & 1 & -1 \\
1 & -1 & 1 & 1 \\
1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1
\end{bmatrix}$$

$$G_{8P} = \begin{bmatrix}
1 & 1 & -j & j \\
1 & -j & 1 & -j \\
j & j & -1 & 1 \\
-j & -j & 1 & 1
\end{bmatrix}$$

$$G_{8P} = \begin{bmatrix}
1 & 1 & -j & j \\
1 & -j & 1 & -j \\
j & j & -1 & 1 \\
-j & -j & 1 & 1
\end{bmatrix}$$

$$G_{8P} = \begin{bmatrix}
1 & 1 & -j & j \\
1 & -j & 1 & -j \\
j & j & -1 & 1 \\
-j & -j & 1 & 1
\end{bmatrix}$$

$$G_{8P} = \begin{bmatrix}
1 & 1 & -j & j \\
1 & -j & 1 & -j \\
j & j & -1 & 1 \\
-j & -j & 1 & 1
\end{bmatrix}$$

$$G_{8P} = \begin{bmatrix}
1 & 1 & -j & j \\
1 & -j & 1 & -j \\
j & j & -1 & 1 \\
-j & -j & 1 & 1
\end{bmatrix}$$
In Eqn (3), \( G_{4, p} \) has 4-phase elements of \( \{i, j, -i, -j\} \) or \( \left\{ \begin{array}{c} 1+j \ 1-j \ -1+j \ -1-j \\ \frac{1}{ \sqrt{2}} \ 
 \ -\frac{1}{ \sqrt{2}} \end{array} \right\} \) with \( j = \sqrt{-1} \). In Eqn (4), \( G_{8, p} \) has 8-phase elements of \( \{i, A, j, B, -1, C, -j, D\} \), where  
\[ A = \frac{1}{ \sqrt{2}} (1+j) \quad B = \frac{-1}{ \sqrt{2}} (1+j) \quad C = \frac{1}{ \sqrt{2}} (1-j) \quad D = \frac{-1}{ \sqrt{2}} (1-j) \].

2) Definition of the period-extension matrix

For the convenience, we denote \( 2^a(n=1,2,3) \) as \( 2^{a,1,2,3} \).

Using any given initial PP of \( \{2^{a,1,2,3}(a), 2^{a,1,2,3}(b)\} \) have the period \( m \) and ZCD of \( (0.5 \times m + 1) \) chips, the period-extension matrix \( E \) is written as

\[
E = \begin{bmatrix}
X & Y & X & -Y \\
X & Y & -X & Y \\
X & -Y & X & Y \\
- X & Y & X & Y \\
\end{bmatrix}
\]

where \( X = (e_0^{(a)}, \ldots, e_{(m-1)}^{(a)}) \), \( Y = (e_0^{(a)}, \ldots, e_{(m-1)}^{(a)}) \) and \( m = 4 \times 2^k \). Here \( i \) is nonnegative integer if \( n = 1 \) or 2, or positive integer if \( n = 3 \). Any row of \( \pm E \) is \( 2^{a,1,2,3}(a) \) and \( 2^{a,1,2,3}(b) \) with the period of \( 2^m \).

Furthermore, any row of \( \pm j E \) can also be \( 2^{a,1,2,3}(a) \) in \( 2^{a,1,2,3}(b) \) is generated from \( 2^{a,1,2,3}(a) \), where  
\[
e_p^{(a)} = (1)^{p} e_p^{(a)} \quad (p = 0,1,2,3, m-1) 
\]

\( 2^{a,1,2,3}(a), 2^{a,1,2,3}(b) \) is a \( 2^{a,1,2,3} \) phase PP with ZCD of \( (0.5 \times m + 1) \) chips.

3) Step(ii): The construction of \( 2^{a,1,2,3} \) phase ZCD PP

The pairs of \( 2^{a,1}(a), 2^{a,1}(b) \) and \( 2^{a,2,1}(a), 2^{a,2,1}(b) \) that have ZCD of \( (0.5 \times 4 + 1) \) chips are defined as the initial 2-phase ZCD PP and the initial 4-phase ZCD PP, respectively. where \( 2^{a,1}(a) \) is \( (x_0^{(a)}, s_1^{(a)}, s_2^{(a)}, s_3^{(a)}) \) and  
\[
2^{a,2,1}(a) = (q_0^{(a)}, q_1^{(a)}, q_2^{(a)}, q_3^{(a)}) \]

is any row of \( \pm G_{4, p} \) and \( \pm G_{8, p} \).  

4) \( \theta \) \( ( \theta = (s_1^{(a)}, s_2^{(a)}, s_3^{(a)}) \) and \( 2^{a,2,1}(b) = (q_0^{(b)}, q_1^{(b)}, q_2^{(b)}, q_3^{(b)}) \) are generated from \( 2^{a,1}(a) \) and \( 2^{a,2}(a) \), respectively, where  
\[
s_p^{(a)} = (1)^{p} s_p^{(a)} \quad q_p^{(a)} = (1)^{p} q_p^{(a)} \quad (p = 0,1,2,3) 
\]

Moreover, a pair of \( 2^{a,1}(a), 2^{a,1}(b) \) with ZCD of \( (0.5 \times 8 + 1) \) chips is defined as the initial 8-phase ZCD PP, where \( 2^{a,1}(a) \) is any row of \( \pm G_{8, p} \) or \( \pm G_{8, p} \) and  
\[
2^{a,1}(b) = (d_0^{(a)}, d_1^{(a)}, d_2^{(a)}, d_3^{(a)}) \]

is also generated from \( 2^{a,1}(a) \), where  
\[
d_p^{(a)} = (1)^{p} d_p^{(a)} \quad (p = 0,1,2,3) 
\]

In utilizing initial \( 2^{a,1,2,3} \) phase ZCD PP and Eqn (5), \( 2^{a,1,2,3} \) phase ZCD PP of longer period are constructed recursively. Thus, for the period  
\[ N = 4 \times 2^k \quad (i = 0,1,2,3, \ldots \quad o = i,1,2,3, \ldots) \]

with ZCD of \( (0.5 N + 1) \) chips can be constructed.

The 4-phase ZCD PP \( \{2^{a,2,1}(a), 2^{a,2,1}(b)\} \) present the ZCD of \( (0.5 N + 1) \) chips as shown in Fig. 1.

Step(ii): Construction of sets of \( 2^{a,1,2,3} \) phase ZCD codes

Let \( 2^{a,1,2,3} \) \( P(N,M,Z_L) \) represent a set of \( 2^{a,1,2,3} \) phase ZCD codes having a code period of \( N \) and family size \( M \), where any pair of \( 2^{a,1,2,3} \) \( P(N,M,Z_L) \) has the common ZCD-length of \( Z_L \). By using the chip-shift operation of \( \{2^{a,1,2,3}(a), 2^{a,1,2,3}(b)\} = 2^{a,1,2,3} \) \( P(N,2,0.5N+1) \), a set of \( 2^{a,1,2,3} \) \( P(N,M \geq 2, Z_L \leq 0.5N+1) \) can be constructed. Let \( T \) be the chip-shift operator, which shifts a code cyclically to the left by \( i \) chips, \( 2^{a,1,2,3} \) \( P(N,M \geq 2, Z_L \leq 0.5N+1) \) can be generated from \( \{2^{a,1,2,3}(a), 2^{a,1,2,3}(b)\} \) as  
\[ 2^{a,1,2,3} \] \( P(N,M \geq 2, Z_L \leq 0.5N+1) \] = \[ \{2^{a,1,2,3}(a), 2^{a,1,2,3}(b), T^{(k)} \{2^{a,1,2,3}(a), 2^{a,1,2,3}(b)\}, \ldots, T^{(k-1)} \{2^{a,1,2,3}(a), 2^{a,1,2,3}(b)\}, T^{(k-1)} \{2^{a,1,2,3}(a), 2^{a,1,2,3}(b)\}\} \]

where \( \Delta \) is a chip-shift increment and \( \Delta \) is a non-negative integer. The various examples of 2-phase ZCD code sets are shown in Table 1. Fig. 2 shows the family size against ZCD of \( 2^{a,1} \) phase (binary) ZCD codes with a period of 128. When ZCD > three chips, it is clear that proposed codes have larger family sizes than that of binary code pairs with ZCD [2] or QSO(G+G) codes generated from orthogonal Gold codes [3]. Here, large family sizes produce a larger CDMA user number and large ZCDs produce longer cell radius of MAI-cancelled DS-CDMA systems, respectively.

3. Conclusion

We have proposed novel \( 2^{a,1,2,3} \) phase codes with sufficient ZCD and family sizes. The proposed \( 2^{a,1,2,3} \) phase codes can be usefully employed in the DS-CDMA systems with Interference Cancellation.

References


Fig. 1 ACF and CCF of a 4phase ZCD PP of period 64

Fig. 2 Family size against ZCD of 2-phase ZCD codes of period 128
Table 1. Various examples of 2-phase ZCD code sets.

\[ 2^m \mathbf{P}(N = 16, M = 2, Z_L = 9) : \]

\[ S_{16}^a = (+ + - + + + - - - + + + + - - - ) \]
\[ S_{16}^b = (+ - - - + + + - + - + + + + - - ) \]

\[ 2^m \mathbf{P}(N = 64, M = 2, Z_L = 33) : \]

\[ S_{64}^{(a)} = (u \ v \ u \ v \ u \ v \ u \ v \ u \ v \ u \ v \ u \ v \ u \ v \ u \ v) \]
\[ S_{64}^{(b)} = (c \ d \ c \ d \ c \ d \ c \ d \ c \ d \ c \ d \ c \ d \ c \ d \ c \ d) \]

where \( u = (- --), v = (-+--), c = (-+-+), \) and \( d = (++--) \).

\[ 2^m \mathbf{P}(N = 32, M = 6, Z_L = 6) : \]

\[ S_{32}^1 = (- - + + + + - - - + + + + - - - + + + + - - - + + + + - - - + + + + -) \]
\[ S_{32}^2 = (- + + + - - - + + + + - - - + + + + - - - + + + + - - - + + + + -) \]
\[ S_{32}^3 = (- + + + - - - + + + + - - - + + + + - - - + + + + - - - + + + + -) \]
\[ S_{32}^4 = (+ - + + - - - - + + + + - - - + + + + - - - + + + + - - - + + + + -) \]
\[ S_{32}^5 = (+ - + + - - - - + + + + - - - + + + + - - - + + + + - - - + + + + -) \]
\[ S_{32}^6 = (+ + + + - - - - + + + + - - - + + + + - - - + + + + - - - + + + + -) \]