Input Sequence Selection and Lookup Table for PAPR Reduction in OFDM Systems

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Abstract: Orthogonal Frequency Division Multiplexing or OFDM is a form of multi-carrier modulation technique. High spectral efficiency, robustness to channel fading, immunity to impulse interference, uniform average spectral density, capability of handling very strong echoes and less non linear distortion are among the favorite properties of OFDM. Even though there are many advantages of OFDM, it has two main drawbacks: high Peak to Average Power Ratio (PAPR) and frequency offset. In this paper, the issue of PAPR in OFDM is discussed. A new algorithm is proposed to reduce PAPR by selecting the input sequences properly using a lookup table.

1. Introduction

After more than thirty years of research and development carried out in different places, orthogonal frequency division multiplexing (OFDM) has been widely implemented in high speed digital communications. Due to the recent advancements in digital signal processing (DSP) and very large scale integrated circuits (VLSI) technologies, the initial obstacles of OFDM implementations do not exist any more. Meanwhile, the use of fast Fourier transform (FFT) algorithms eliminates arrays of sinusoidal generators and coherent demodulation required in parallel data systems and makes the implementation of the technology cost effective. In recent years OFDM has gained a lot of interest in diverse digital communication applications. This has been due to its favorable properties like high spectral efficiency and robustness to channel fading. Today, OFDM is mainly used in digital audio broadcasting system (DAB) that was initiated by CCETT in France and developed into a new major broadcasting standard by a collaborative project [1], COFDM in digital video broadcasting system (DVB) [2] enables an end-to-end digital transmission system, which is spectrally efficient and rugged against channel distortions. This can be used for services such as HDTV, offering increased capacity for program broadcasting.

In conventional serial data transmission system, an information symbols are transmitted sequentially. And each symbol occupies entire available spectrum bandwidth. But for OFDM system the information is converting to n parallel sub channel and sends with low rate using frequency division multiplexing. Sub carrier frequency spacing is selected carefully such that each sub carrier is located on the other sub carriers zero crossing points. Then there are overlapping among sub carriers but will not interfere with each other, if they are sampled at the sub carrier frequencies. This means that all sub carriers are orthogonal. Among many advantages of OFDM, there are some disadvantages also. The frequency offset of the sub carriers and the high PAPR are major drawbacks of OFDM [3]. In this paper PAPR problem will be addressed. A new algorithm is proposed to reduce PAPR by selecting the input sequence using a lookup table. Rest of the paper is organized as follows. In section 2, an introduction to OFDM system is given. Section 3 presents a detail analysis of PAPR and section 4 discusses the related work of PAPR reduction. Section 5 includes the proposed scheme and section 6 describes the conclusions.

2. Introduction to OFDM

An OFDM symbol consists of N sub carriers by the frequency spacing of Δf. Thus, the total bandwidth B will be dividing into N equal spacing sub carriers. And all sub carriers are orthogonal to each other within a time interval of length $T=1/\Delta f$. Each sub carrier can be modulated independently with the complex modulation symbol $X_{m,n}$, where m is a time index and n is a sub carriers index. Then within time symbol T the following signal of the m-th OFDM block is described in equation (1).

$$x_m(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{m,n} g_n(t - mT) \tag{1}$$

where $g_n(t)$ is defined through equation (2).

$$g_n(t) = \begin{cases} \exp(j2\pi n\Delta f t), & 0 \le t \le T \\ 0, & else \end{cases} \tag{2}$$

Where $g_n(t)$ is a rectangular pulse applied to each sub carrier [4]. The total continuous time signal $x(t)$ consisting of all OFDM blocks is given by equation (3)

$$x(t) = \frac{1}{\sqrt{N}} \sum_{m=0}^{\infty} \sum_{n=0}^{N-1} X_{m,n} g_n(t - mT) \tag{3}$$

Now, consider a single OFDM symbol ($m = 0$) without loss of generality. This can be shown because there is no overlap between different OFDM symbols. Since $m = 0$, $X_{m,n}$ can replace with X_n .Then OFDM signal can be described as follows.

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n\Delta f t} \tag{4}$$

If the bandwidth of the OFDM signal is $B = N \times \Delta f$ and the $x(t)$ signal is sampled by the sampling time of $\Delta t = \frac{1}{B} = \frac{1}{N\Delta f}$, then the OFDM signal is in discrete time form and can be written as shown in equation (5).

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi kn/N}, \quad k = 0,1,\ldots,N-1 \qquad (5)$$

Where n denotes the index in frequency domain and X_n is the complex symbol in frequency domain. Furthermore, Equation (5) can be expressed using the IFFT [5].

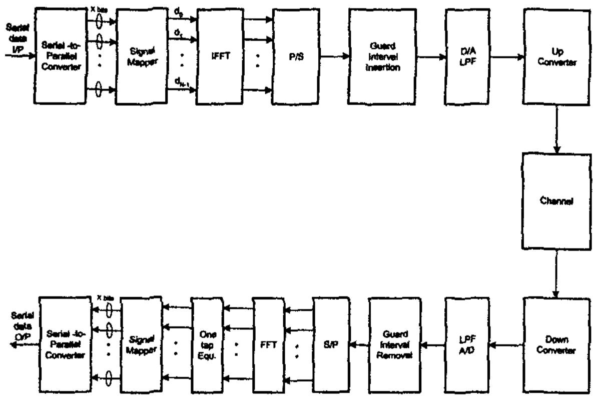

Figure 1: Basic FFT base OFDM

Figure 1 shows a typical system block diagram for an OFDM system. The serial input data bit stream are converged to N parallel sub channels and mapped with selected modulation scheme by a signal mapper, result in N sub channels contain an information in complex number form. These complex values are sent to the N channel IFFT. Then signal is converted back to a serial sequence again by using a P/S device. The function of the guard interval is to reduce the effect of ISI caused by multipath propagation. Finally, the signal is converted to an analog signal and up converted to suite for the transmission. At the receiver, a reverse procedure is used to demodulate the OFDM signal with the FFT block. In OFDM system, only a simple equalizer, a single tap equalizer, is needed at the receiver to take out the ISI.

3. PAPR

As explained earlier, one of the major drawbacks of OFDM is the very high peak to average power ratio (PAPR). PAPR of OFDM is increasing exponentially with the number of sub carriers. If power amplifiers are not operated with large power back-offs, it is impossible to keep the out-of-band power below the specified limits. This situation leads to very inefficient amplification and expensive transmitters so it is highly desirable to reduce PAPR. Therefore PAPR is a main problem in OFDM.

In this section, detailed mathematical analysis for PAPR is presented. RMS magnitude of the OFDM signal is defined as the root of the time average of the envelope power ($\sqrt{\overline{P}}$), and $\overline{P}$ is defined by equation (6).

$$\overline{P} = \frac{1}{T}\int_{t=0}^{T} |x(t)|^2 \, dt = \frac{1}{N}\sum_{n=0}^{N-1} |X_n|^2 \qquad (6)$$

where $x(t)$ is OFDM signal defined in equation (3). The value $\overline{P}$ in this case corresponds to a single OFDM symbol, and depend on the sequence of information carrying coefficients $\{X_n\}$. The average power of OFDM symbols can be write as $P_{av} = E\{\overline{P}\}$. Thus PAPR of OFDM signal can be defined as

$$\xi = \frac{\max_{t \in [0,T)} |x(t)|^2}{P_{av}} = \frac{\max_{t \in [0,T)} |x(t)|^2}{E\left\{ |x(t)|^2 \right\}} \qquad (7)$$

Equations (8)-(11) clearly explain that PAPR is less than or equal to N. If the input data power is normalized, then $E\left\{ |x(t)|^2 \right\} = 1$. Then we get,

$$\xi = \max_{t \in [0,T)} |x(t)|^2 \qquad (8)$$

$$\xi = \max_{t \in [0,T)} \left| \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n \Delta f t} \right|^2 \qquad (9)$$

$$\le \frac{1}{N} \left| \sum_{n=0}^{N-1} X_n e^{j2\pi n \Delta f t} \right|^2 \qquad (10)$$

$$\le N \qquad (11)$$

It clearly shows that the maximum PAPR is equal to the number of sub carriers N. For M-PSK modulation, there are only M^2 sequences having maximum PAPR N as described in [6]. These mean, the number of sequence that give very high PAPR is not very high. If the number of sub channels are increased, the ratio of the sequence that give very high PAPR and all distinct sequences is decreasing rapidly. The overall number of distinct sequence for N sub carriers OFDM system with M-PSK is M^N sequences. Thus the ratio can be obtained by equation (12)-(13).

$$R = \frac{M^2}{M^N} \qquad (12)$$

$$R = M^{2-N} \qquad (13)$$

For example, a system with 256 sub carriers and 4-PSK modulation has the maximum PAPR equal to $10 \times \log_{10} 256 = 24$ dB. The number of 24 dB PAPR sequences is equal to $4^2 = 16$ sequences, and the number of all distinct sequences is 4^{256} that is very large compared to 16. The ratio $R = 4^{2\text{-}256} = 4^{-254}$. It is clear that the probability of $\xi = N$ is negligible when the number of sub carriers is large, as is the case in practice. But, ξ can still be very large and must be reduced in order to have an efficient OFDM transmission.

4. Related Work

Several algorithms [7-13] have been proposed to handle this PAPR problem. However, non of these algorithms have produced significant reduction of PAPR in OFDM systems.

Partial Transmit Sequence (PTS) was proposed in [7]. In PTS the information bearing subcarrier block is subdivided into disjoint carrier subblocks and introduced rotation factors for each subblock and modified subcarrier amplitude vector. Thereby, PAPR was reduced with different rotation factors for different subblocks. This needs number of iterations to find the optimum combination of factors for subblocks. Adaptive PTS [8] was proposed to reduce the number of iteration by setting up a desired threshold and trial for different weighing factor until the PAPR drop under the threshold. With this approach we can reduce the number of iteration and the complexity of the system by only 0.1% loss in reduction of PAPR. In [8] 256 subcarriers are considered with QPSK. Results showed that PAPR can be reduced by 4.1 dB and 4.0 dB without adaptive PAPR and with adaptive PAPR respectively. However these two approaches need to send side information to the receiver. This mean the bandwidth efficiency was reduced.

Method	Advantage	Disadvantage
PTS	Low PAPR	Side information needed, Long calculation time (high complexity)
Adaptive PTS	Low complexity	Side information needed
PTS with embedded side information	High bandwidth efficiency	Introduce word error rate in detection
QPSK with RM codes	Low PAPR	Can not use with high order constellation
Rotation of redundancy bit position in sub-block code word	PAPR only 3.54 dB (for word length 4)	Side information needed
Oversized IFFT	PAPR is limited to the clipping ratio without out of band noise	In band distortion, shrinking of constellation and noise like component
Companding transform	Low PAPR and less delay	Higher BER

Table 1: Comparison of existing schemes for PAPR reduction

PTS with embedded side information [9] is another approach that can be combined with both conventional and adaptive PTS. This approach is embedding a combined knowledge within the transmitted data, so no extra bits are sent. But these introduce the word error at a detection of the sequence information. Some approach used the properties of block coding to solve this problem.

A simple Encodable/Decodable OFDM QPSK [10] used Reed-Muller code with QPSK. This can reduce the PAPR to be less than 6 dB but it can not be used with high order constellation. OFDM PAPR reduction by a Rotation of redundancy bit position in sub block code word scheme was proposed in [11]. In this method the redundant bit position of sub-block coding are rotated and the lowest PAPR codeword is chosen by a feed back scheme. However, the side information for bit position is required.

Oversized IFFT [12] is proposed as another scheme to solve this problem. In oversized IFFT, clipping and filtering are done by forward and inverse FFT. This can avoid out of band power but with some in band distortion, overall shrinking of constellation and noise like component are introduced. Companding transform [13], compress a large signal while enhancing a small signal that can achieve a desire PAPR but bit error rate (BER) is increased. Table 1 shows a brief comparison between these major schemes.

5. Proposed Scheme

We have studied the effect of BPSK and QPSK modulation schemes on PAPR, for systems with different number of channels (2^k number of channels in the set), and found that QPSK is a better option compare to BPSK. Table 2 summarized the information about 0 dB PAPR sequences found in the study with QPSK as the modulation format. The study is limited to maximum of 16 channels and further work is in progress to generalize the idea and methodology for any number of channels of the form 2^k.

k	Number of channels	Number of all distinct input sequences	Number. Of sequences which 0 dB PAPR output
1	2	16	8
2	4	256	32
3	8	65,536	128
4	16	4,294,967,296	512

Table 2: The number of QPSK input sequences that gives 0 dB PAPR

Using the information given in Table 2, we can find an expression for the number of sequence in each case as follows.

Let k is an integer then

Number of channel = 2^k

Number of all posible input sequence = 4^{2^k}

Number of sequence with 0 dB PAPR = 2×4^k

Now, consider the 8 channels system in Table 2. We can generate the 0 dB PAPR OFDM system with 8 sub carriers and information rate of 7/16, by grouping the input bit stream into a group of 6 or 8 bits. The output of QPSK signal mapper is grouped with either 3 complex number or 4 complex numbers (each complex number corresponds to a symbol) and each group is compared with a table containing 0 dB PAPR sequences corresponding to the system. The matching sequence is sent to the 8 channel IFFT.

Figure 2 shows the block diagram of the proposed algorithm.

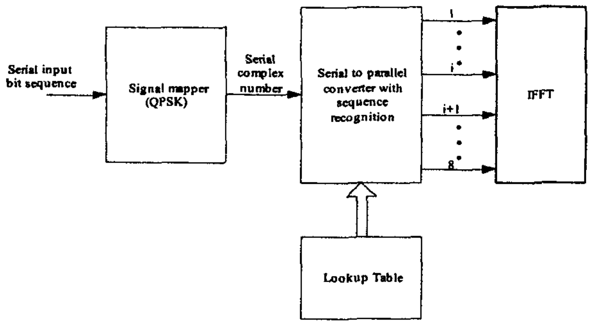

Figure 2: Block diagram for the input sequence selecting

In figure 2, channel 1 to i are the information carrying channels and rest contains residual values to make the sequence 0 dB PAPR. As proved above, i can be 3 or 4 depending on the serial complex number sequence. Thus, the information rate is $\log_4(128)/8$, which is equal to 7/16.

Since most of the practical systems can afford to have a PAPR of more than 0 dB, we can increase the information rate until the PAPR reaches the maximum tolerable level of the system. Then the number of input sequences in the lookup table can be increased as shown in table 3.

Maximum PAPR(dB)	Number of useful Input sequences	i (average value)	Information rate
0	128	3.5	7/16
0.97	1152	5.0	5/8
1.76	2177	5.5	11/16
2.32	2433	5.6	28/40
2.68	2945	5.7	57/80
2.92	5504	6.2	62/80
3.01	12096	6.7	67/80

Table 3 Maximum PAPR and information rate

An added advantage of this method is it can detect possible errors as the system transmits only a specific set of sequences and hence any sequence out of this set can clearly be concluded as an error. An educated guess can be considered to be a coarse error correction.

6. Conclusion

In this paper we proposed a scheme to reduce PAPR in an OFDM system. We use the algorithm comparing input sequence with the sequence that gives low PAPR in the lookup table. Results show that PAPR can be reduced to 0 dB with information rate of 7/16 or 1 dB with rate of 5/8. For a system with 8 channels OFDM with QPSK modulation scheme. Future work is required to extend this algorithm to a system with large number of sub channels.

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