Training Adaptive Equalization With Blind Algorithms

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Abstract: A good performance on communication systems is obtained by decreasing the length of training sequence in the initial stage of adaptive equalization. This paper presents a new approach to accomplish this, with the use of a training adaptive equalizer. The approach is based on combining the training and tracking modes, in which the training equalizer is updated by the LMS algorithm with the training sequence and then updated by a blind algorithm. By computer simulations, it is shown that a class of the proposed equalizers provides better performance than the conventional training equalizer.

1. Introduction

One of the major problems in digital communication systems is that of existence of intersymbol interference(ISI). To overcome ISI, various types of adaptive equalizers have been proposed so far [1][2]. In the initial stage of the equalizers, a certain length of training sequence is usually transmitted to adjust the equalizer coefficient vector. And then, a data sequence is transmitted with the adaptation of the equalizer coefficient vector. Hence, a shorter length of training sequence increases the number of data to be transmitted. However, as the training sequence length is decreased, the task of equalization is not succeeded and a higher error rate in symbol detection is caused. In addition, the training sequence length required for the equalizer is changed dependently on ISI. This is typical with the use of the LMS algorithm [3] for the equalizer coefficient adaptation. The LMS is the most popular adaptive algorithm for the equalization scheme involving the training sequence, so-called training equalization. However, the convergence speed of the LMS depends on the spread of eigenvalues in the input autocorrelation matrix and a large spread of those invokes a slow convergence [4], in which a long training sequence is required. This deteriorates the efficiency of the communication systems. Thus, if the training sequence length is fixed as being comparatively short for the communication systems regardless of the channel characteristics, a highly efficient communication may be realized.

More recently, there has been much interests in blind equalization, where no training sequence is used [5][6]. Blind equalization is attractive in the sense of increasing the efficiency of communications, but it has a problem which is not shared with the training equalization. It is necessary for the blind equalizer to set the initial values of the coefficient vector without training. If these values are not suitable for the channel, the convergence of the equalizer becomes quickly worse.

In this paper, we describe a new approach for the training equalization, which employs a blind adaptive algorithm. At first, a training sequence (generally whose length may be insufficient for the conventional training equalizer) is transmitted to update the coefficient vector of the equalizer, and then the updated coefficient values are modified and used as the initial setting of the blind algorithm. And then a data sequence is transmitted with the adaptation of the equalizer coefficient vector based on the blind algorithm. Since the training sequence length used is comparatively short for this scheme, it is expected to increase the efficiency of the communication system.

This paper is organized as follows. In Section 2, we describe some conventional algorithms of adaptive equalization. In Section 3, we provide the new approach for the training equalization. We demonstrate computer simulations in Section 4. Finally, conclusions are drawn in Section 5.

2. Conventional Algorithms

2.1 Training Adaptive Equalization

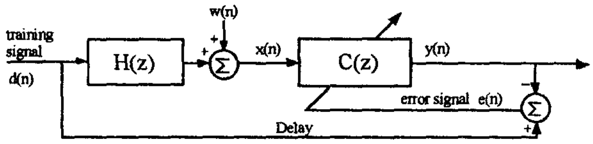

Figure 1. Training adaptive equalizer in the training mode

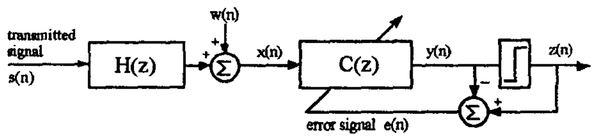

Figure 2. Training adaptive equalizer in the tracking mode

A general structure of the training adaptive equalizer is shown in Figures 1 and 2. The input vector $X(n)$ is defined as

$$X(n) = [x(n)\ \ x(n-1)\ \ ...\ \ x(n-M+1)]^T \quad (1)$$

where T denotes transpose and $x(n)$ is the channel output given by

$$x(n) = \sum_{i=-\infty}^{\infty} h_i d(n-i) + w(n). \quad (2)$$

The $d(n)$ is assumed to be the transmitted sequence, which is used as the training sequence at the receiver side. The $w(n)$ is additive noise and h_i denotes the channel impulse response. The equalizer coefficient vector is defined as

$$C(n) = [c_1(n)\ c_2(n)\ \dots\ c_M(n)]^T. \quad (3)$$

Then, the equalizer output is obtained as

$$y(n) = X(n)^T C(n). \quad (4)$$

The training equalizer starts its adaptation with the known training sequence during the initial stage. Generally, the coefficient vector of the equalizer is initialized to a zero vector, that is $C(0) = 0$. And then, various types of adaptive algorithms can be used to adjust the equalizer coefficient vector by minimizing the mean square error (MSE) between the equalizer output, $y(n)$, and the training sequence with a delay, $d(n-D)$. If the LMS algorithm is used for the adaptive algorithm, then the adaptation of the coefficient vector is described as

$$e(n) = d(n-D) - y(n) \quad (5)$$
$$C(n+1) = C(n) + \mu e(n)X(n) \quad (6)$$

where μ is a positive constant called "step-size parameter" that controls the rate of convergence of the LMS algorithm.

After such a training process, the equalizer coefficient vector becomes close to the desired one, because much of the ISI caused by the channel is removed in the training process. In the next stage, a real data transmission is accomplished through a memoryless decision device (slicer). The receiver switches into the tracking mode where the equalizer output $y(n)$ is sent to the slicer and the slicer output $z(n)$ is used in place of the training sequence to update the equalizer coefficient vector. In this case, since a decision of the equalizer output is used in the training mode, the tracking mode results in a decision-directed (D-D) mode. The LMS adaptation in the D-D mode is described by

$$z(n) = \mathrm{sgn}(y(n)) \quad (7)$$
$$e(n) = z(n) - y(n) \quad (8)$$
$$C(n+1) = C(n) + \mu e(n)X(n). \quad (9)$$

The LMS algorithm works well in the D-D mode when the MSE level of the equalizer output is considerably reduced in the training mode. Thus, in general, a long training sequence is prepared for the training adaptive equalizer.

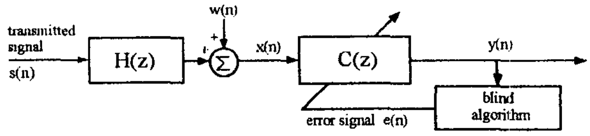

Figure 3. Blind adaptive equalizer

2.2 Blind Adaptive Equalization

A general structure of the blind adaptive equalizer is shown in Figure 3. For the blind equalizer, there is no training mode. Thus, immediately a real data transmission begins. If the Sato algorithm [7] is used for the adaptation of the blind equalizer, the coefficient vector is updated as follows:

$$e(n) = y(n) - \gamma\ \mathrm{sgn}(y(n)), \quad (10)$$
$$C(n+1) = C(n) - \mu e(n)X(n) \quad (11)$$

where γ is a constant determined by

$$\gamma = \frac{E|s(n)|^2}{E|s(n)|}. \quad (12)$$

The $s(n)$ is the transmitted sequence and E denotes expectation.

It is essential for the blind algorithms that $C(0)$ should not be set to a zero vector. If such a setting is used, the equalizer does not work effectively. To avoid this situation, the initial coefficient vector of the blind equalizer should be set to a vector suitable for the channel. The initial coefficient vector is typically set to that in which only one element is 1 and the other elements are 0s. However, we have no accurate knowledge about the channel a priori in many cases.

3. Training Adaptive Equalization With Blind Algorithms

As described in Section 2, both the training and blind equalizers have each problem. The training equalizer needs a training sequence whose length is proportional to the degree of ISI. On the other hand, the blind equalizer needs a knowledge of the channel characteristics by which the initial coefficient vector of the blind algorithm is set up. To solve these problems, we propose a new strategy for the channel equalization, where a blind algorithm is applied to the training adaptive equalizer in the tracking mode.

Now suppose that for the training equalizer a constant length of the training sequence is required in the training mode. We denote this length by N_{tr}. Generally, N_{tr} is set to a large number not to fail the task of equalization. However, for the proposed approach, N_{tr} can be less than a common used in the conventional training equalizers.

After the training of the equalizer based on the training sequence, we modify the coefficient vector as depicted in Figure 4. One element which gives the maximum value among the coefficient values is selected and defined as MAX. Then, at the N_{tr} iterations of the adaptive algorithm,

$$c_{MAX}(N_{tr}) = \alpha \tag{13}$$

$$c_i(N_{tr}) = 0 \quad i \neq MAX \tag{14}$$

are operated. After this, by using this coefficient vector as the initial vector setting, a blind algorithm is implemented in the tracking mode.

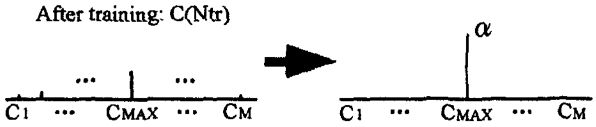

Figure 4. *Coefficients modification*

The idea of this approach comes from the fact that the training equalizer should make the same main tap as the blind equalizer in an ideal case. Thus, if the coefficient vector of the equalizer trained in the training mode is utilized for the setting of the initial coefficient vector of a blind equaliser, the blind equalizer could behave robustly without the training sequence. This directly results in shortening the training sequence length required.

4. Computer Simulations

In this section, we will illustrate the performance of the proposed equalizer and compare it with the conventional one. In the following simulations, we used a raised-cosine channel [4], whose impulse response is described by

$$h_i = \begin{cases} \frac{1}{2}[1+\cos(\frac{2\pi}{W}(i-2))], & i = 1,2,3 \\ 0, & otherwise \end{cases} \tag{15}$$

where W is a parameter to control the magnitude of ISI induced by the channel. Specific parameters for the simulations are as follows;

- channel parameter $W = 2.9, 3.2, 3.5$
- additive noise $w(n)$: white Gaussian noise by which the signal-to-noise ratio (SNR) of the channel output is set to 40dB
- transmitted signal $s(n)$: random sequence with the values of ± 1
- filter order $M = 11$
- delay $D = 7$
- step-size $\mu = 0.025$ (only for LMS + Godard, $\mu = 0.015$)

In Table 1, the training sequence length required for the training equalizer with the LMS adaptation has been investigated. On each channel determined by the parameter setting of W, we found the minimum length of the training sequence, N_{tr}, for which the training equalizer in the D-D mode provides the lowest MSE level in the steady state. It is observed that as the channel parameter W is increased, a longer length of the training sequence is required. This suggests that if we have no knowledge about the channel a priori, an sufficiently long length of the training sequence should be prepared. Such setting for the equalizer, obviously, is not required on the channel with a small W.

Table 1. Relation between the channel parameter and the minimum length of the training sequence for the LMS training equalizer

W	N_{tr}	MSE
2.9	34	-4.05
3.2	47	-4.47
3.5	94	-6.14

On the other hand, the MSE included in Table 1 shows that obtained at the N_{tr}-th iterations of the LMS algorithm for each of the channel parameters. From this, we see that by reducing the MSE level to about -5dB in the training mode, the training equalizer works effectively in the D-D mode.

Next, we implemented the proposed training equalizer in three types of connection of adaptive algorithms. One of the three proposed equalizers is that in which the LMS algorithm is commonly used in the training and tracking modes[1], but the coefficient vector is modified at the end of the training mode. This is denoted by LMS + LMS. In the other two proposed equalizers, blind algorithms are used. One of them is that in which the LMS algorithm is used in the training mode and the Sato algorithm is used in the tracking mode (LMS + Sato). The other is that in which the LMS algorithm is used in the training mode and the Godard algorithm [8] is used in the tracking mode (LMS + Godard). In Table 2, a performance comparison of these proposed equalizers has been made with a common length of the training sequence, N_{tr}. The MSE included shows that obtained with the coefficients modification technique of (13)(14) at the N_{tr} iterations of the LMS algorithm for each of the channel parameters. It is observed that the MSE level achieved for each channel is nearly the same for the three equalizers. However, by comparing Table 2 with Table 1, we notice that at the end of the training mode, the proposed equalizers provide a significant improvement in the MSE level. To obtain this, the training sequence length required for the proposed equalizers is only 20, which is significantly smaller than that required for the conventional training equalizer, as found in Table 1. Thus, we deduce that the training sequence length required could be decreased by relying on the proposed equalizers.

An example of the convergence curves the proposed equalizers provide is shown in Figure 5, where the conventional training equalizer is compared with the proposed equalizers. This figure shows that by means of the coefficients modification technique employed in the proposed equalizer, a significant reduction in the MSE

[1] The LMS algorithm used in the D-D mode can be recognized as a blind algorithm.

Table 2. Relation between the channel parameter and the minimum length of the training sequence for the proposed training equalizer

Algorithm	W	N_{tr}	MSE
LMS+Sato	2.9		-10.22
	3.2		-7.26
	3.5		-6.65
LMS+Godard	2.9		-10.22
	3.2	20	-7.26
	3.5		-5.26
LMS+LMS	2.9		-10.21
	3.2		-7.26
	3.5		-6.65

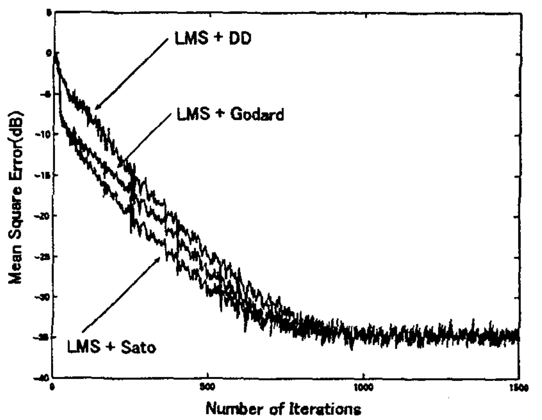

Figure 5. Convergence curves in the case of $W = 3.2$. The LMS +DD denotes the conventional training equalizer in which the LMS algorithm is used in the training mode and the LMS algorithm is again used without the coefficients modification in the tracking mode.

levels is obtained and results in a faster convergence. Also, this figure suggests that the Sato algorithm is preferred as the blind algorithm connected from the training mode.

5. Conclusions

In this paper, for the purpose of increasing the efficiency of a communication system, a new approach for the training equalization has been developed where a blind algorithm is applied in the tracking mode of the training adaptive equalizer. Through computer simulations, we found that the task of the training adaptive equalizer is satisfied with reduction to an MSE level of ~5 dB in the training mode. Also, it was confirmed that the proposed equalizers shorten the length of the training sequence.

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