Avoiding Indefiniteness in Criteria for Maximum Likelihood Bearing Estimation with Arbitrary Array Configuration

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Abstract: This paper presents a technique for avoiding indefiniteness in Maximum Likelihood (ML) criteria for Direction-of-Arrival (DOA) finding using a sensor array with arbitrary configuration. The ML criterion has singular points in the solution space where the criterion becomes indefinite. Solutions by iterative techniques for ML bearing estimation may oscillate because of numerical instability which occurs due to the indefiniteness, when bearings more than one approach to the identical value. The oscillation makes the condition for terminating iterations complex. This paper proposes a technique for avoiding the indefiniteness in ML criteria.

1. Introduction

The localization of multiple signal sources by a passive sensor array is of great importance in the areas of radar, sonar, seismology, radio-astronomy, etc. The basic problem in this context is to estimate bearings of narrow-band signal sources located in the far field of the array. To achieve high resolution under small data sample, low SNR conditions and/or coherent environments, several techniques has been introduced, such as Maximum Likelihood (ML) method and MUSIC method.

The ML technique, known to have excellent estimation accuracy, is superior to other methods concerned with the resolution. However the criterion has singular points in the solution space where the criterion becomes indefinite. Solutions by iterative techniques for ML bearing estimation may oscillate because of numerical instability which occurs due to the indefiniteness, when bearings more than one approach to the identical value. The oscillation makes the condition for terminating iterations complex.

This paper proposes a technique for avoiding the indefiniteness in ML criteria for an arbitrary array configuration.

2. ML Criterion

Under the assumption of additive Gaussian noises, an ML function for bearing estimation is described as a sum of norms of vectors which are projections of observed vectors onto the signal subspace [1]. The signal subspace is the subspace spanned by steering vectors associated with bearing to be estimated.

We consider a sensor array with arbitrary configuration. Supposed that arbitrary q different bearings have independent steering vectors. It is required to ensure the unique solution of bearing estimation. Let $\boldsymbol{a}(\theta, \phi)$ be a steering vector toward the direction (θ, ϕ), where θ and ϕ represent the azimuth and the elevation, respectively. If q steering vectors are lineally independent, the signal subspace has the q dimension. If two of them, say (θ_1, ϕ_1) and (θ_2, ϕ_2), have the same bearing and all others are different each other, it is apparent that the signal subspace has $q-1$ dimension.

On the other hand, taking the limit $\theta_1 \rightarrow \theta_2$ and $\phi_1 \rightarrow \phi_2$, the difference of the steering vectors $\boldsymbol{a}(\theta_2, \phi_2) - \boldsymbol{a}(\theta_1, \phi_1)$ belongs to the signal subspace. Hence a derivative $\boldsymbol{a}_\alpha(\theta_2, \phi_2)$ determined as follows belongs to the limit of the signal subspace.

$$\begin{aligned} \boldsymbol{a}_\alpha(\theta_2, \phi_2) &= \lim_{\Delta\gamma \to 0} \frac{\boldsymbol{a}(\theta_2, \phi_2) - \boldsymbol{a}(\theta_1, \phi_1)}{\Delta\gamma} \\ &= \cos\alpha \frac{\partial \boldsymbol{a}(\theta, \phi)}{\partial \theta} + \sin\alpha \frac{\partial \boldsymbol{a}(\theta, \phi)}{\partial \phi} \end{aligned} \quad (1)$$

where $\theta_2 - \theta_1 = \cos\alpha\Delta\gamma$ and $\phi_2 - \phi_1 = \cos\alpha\Delta\gamma$. Usually, the derivative $\boldsymbol{a}_\alpha(\theta_2, \phi_2)$ is linear independent of the steering vector $\boldsymbol{a}(\theta_2, \phi_2)$ and all other steering vectors. This indicates that the signal subspace has q dimension and the value of the ML function varies together with the signal subspace depending on α when the limit is taken. In other words, the ML criterion is indefinite at the point where more than one bearings are identical. Such points in the solution space are singular points of an ML function and numerically unstable for evaluating an ML function. It is hard to apply the gradient method when more than one bearings approach to the same bearing.

If all derivatives of steering vectors which appear when the limit is taken are ignored, the ML criterion can be determined uniquely. But it is always less than that of the case derivatives are taken into account. This indicates that $(\theta_1,\phi_1)=(\theta_2,\phi_2)$ is never a solution even if (θ_1,ϕ_1) climbing the ML function approaches to (θ_2,ϕ_2). In this case (θ_1,ϕ_1) wander around (θ_2,ϕ_2). Therefore ignoring all derivatives is not a solution for avoiding the indefiniteness of the ML function.

In the case of a uniform linear array, the singular point is avoidable and the formulation for avoiding the singular point has been proposed [2].

3. Proposed Technique

In order to avoid the indefiniteness of the ML function, we take the approach to replace steering vectors with their derivatives if they are linearly dependent.

Define the following differential operator,

$$D(\alpha)=\cos\alpha\frac{\partial}{\partial\theta}+\sin\alpha\frac{\partial}{\partial\phi} \tag{2}$$

and notations

$$D(\alpha)\boldsymbol{a}(\theta,\phi)=\boldsymbol{a}_\alpha(\theta,\phi) \tag{3}$$

$$D(\alpha)D(\beta)\boldsymbol{a}(\theta,\phi)=\boldsymbol{a}_{\alpha\beta}(\theta,\phi) \tag{4}$$

The limit of the signal subspace spanned by several steering vectors approaching to one bearing (θ,ϕ) contains the following vectors. The left column represents parameters to be optimized for obtaining the maximum of the likelihood function.

2 steering vectors:

$$(\alpha_1)\qquad \boldsymbol{a}(\theta,\phi)\qquad \boldsymbol{a}_{\alpha_1}(\theta,\phi) \tag{5}$$

3 steering vectors:

$$(\alpha_1)\qquad \boldsymbol{a}(\theta,\phi)\qquad \boldsymbol{a}_{\alpha_1}(\theta,\phi)\qquad \boldsymbol{a}_{\alpha_1\alpha_1}(\theta,\phi) \tag{6}$$

$$()\qquad \boldsymbol{a}(\theta,\phi)\qquad \boldsymbol{a}_{\alpha_1}(\theta,\phi)\qquad \boldsymbol{a}_{\alpha_2}(\theta,\phi) \tag{7}$$

4 steering vectors:

$$(\alpha_1)\qquad \boldsymbol{a}(\theta,\phi)\quad \boldsymbol{a}_{\alpha_1}(\theta,\phi)\quad \boldsymbol{a}_{\alpha_1\alpha_1}(\theta,\phi)\quad \boldsymbol{a}_{\alpha_1\alpha_1\alpha_1}(\theta,\phi) \tag{8}$$

$$(\alpha_1)\qquad \boldsymbol{a}(\theta,\phi)\quad \boldsymbol{a}_{\alpha_1}(\theta,\phi)\quad \boldsymbol{a}_{\alpha_2}(\theta,\phi)\quad \boldsymbol{a}_{\alpha_1\alpha_1}(\theta,\phi) \tag{9}$$

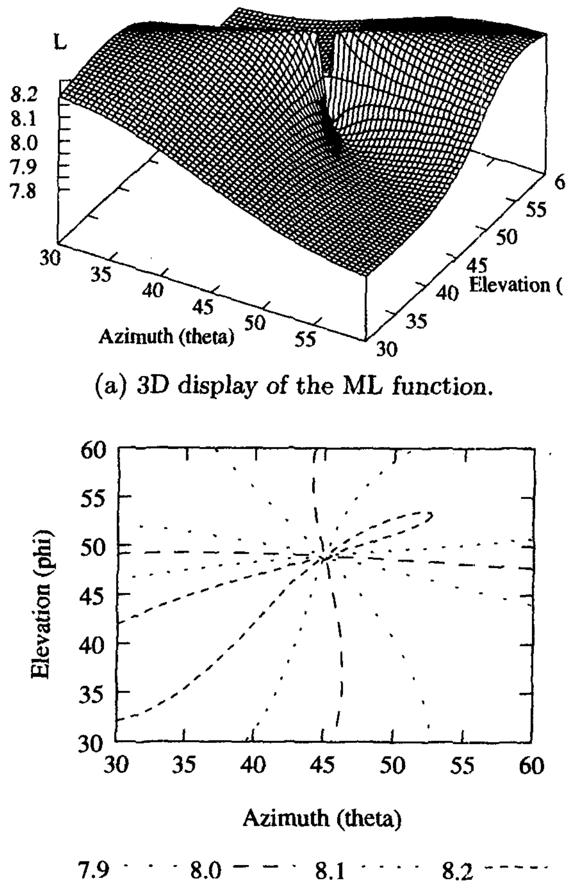
$$(\alpha_1,\alpha_2)\quad \boldsymbol{a}(\theta,\phi)\quad \boldsymbol{a}_{\alpha_1}(\theta,\phi)\quad \boldsymbol{a}_{\alpha_2}(\theta,\phi)\quad \boldsymbol{a}_{\alpha_1\alpha_2}(\theta,\phi) \tag{10}$$

Since the 1st order of derivative is belongs to a 2 dimensional subspace spanned by $\partial\boldsymbol{a}(\theta,\phi)/\partial\theta$ and $\partial\boldsymbol{a}(\theta,\phi)/\partial\phi$, no optimization is needed for Eq. (7).

4. Examples

4.1 Criterion

In Fig. 1, an example of the ML function is shown, when all derivatives are ignored. The scenario is shown



(a) 3D display of the ML function.

(b) Contour lines of the ML function on $(\theta_1,\ \phi_1)$ plane.

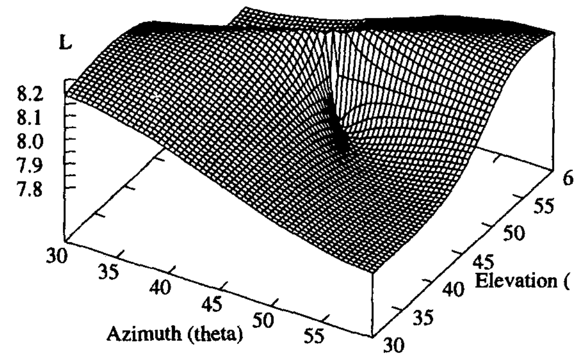
Figure 1. An example of an ML function ignoring all derivatives. 8 identical and omnidirectional sensors are located at corners of a cubic with edges of a half of wavelength on the horizontal plane. Two signals impinge on the sensor array from directions (30°, 40°) and (40°, 60°), respectively. Both SNR's are 0dB. The ML function is evaluated with one fixed bearing $(\theta_2,\ \phi_2)=(45°,\ 49°)$ and one variable bearing $(\theta_1,\ \phi_1)$. The ML function has a singular point at $(\theta_1,\ \phi_1)=(45°,49°)$.

in the figure caption. It can be found that the value of the ML function at $(\theta_1,\ \phi_1)=(\theta_2,\ \phi_2)=(45°,\ 49°)$ is smaller than other points around it.

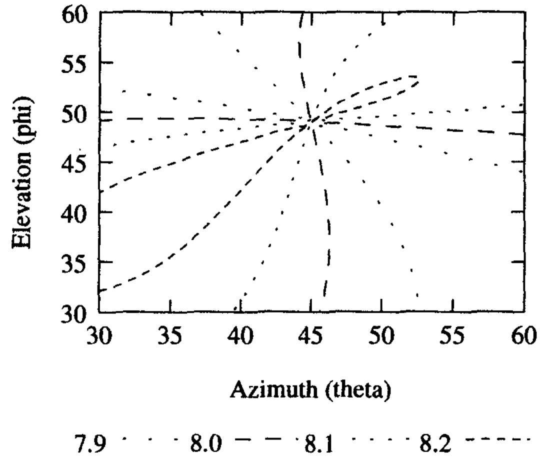
In Fig. 2, the graph of the maximum likelihood function is shown, when optimal derivatives are used. It can be found that the value of the ML function at $(\theta_1,\ \phi_1)=(\theta_2,\ \phi_2)=(45°,\ 49°)$ keeps continuity for the maximum value around it.

4.2 DOA finding

Using iterative techniques of the ML bearing estimation, the solution may show the behavior of oscillations due to the existence of singular points in the solution



(a) 3D display of the ML criterion.



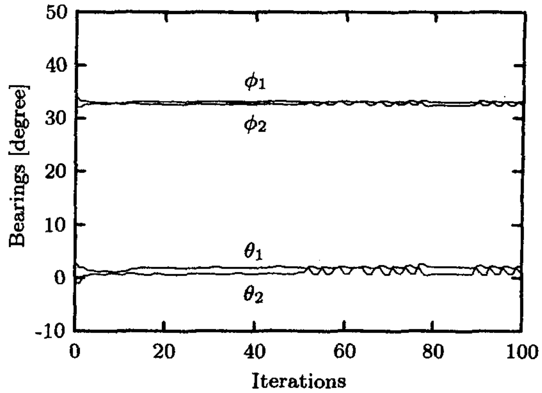
(b) Contour lines of the ML criterion on (θ_1, ϕ_1) plane.

Figure 2. An example of an ML function taking derivatives into consideration. The scenario is the same as Fig. 1.

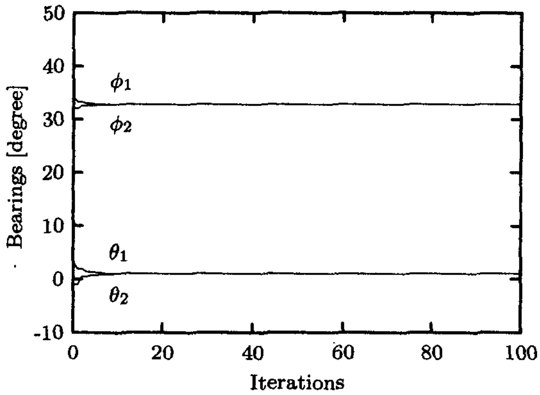
space. Figs 3 and 4 show examples of the ML bearing estimation. The scenario is shown in the caption of Fig. 3.

In all simulations, Alternating Projecting (AP) algorithm [1] is applied. In the update phase of the AP algorithm, first two dimensional search on a mesh of the area $0 \le \theta < 360°$, $-90° \le \phi \le 90°$ is carried out, and then a fine search on a two dimensional mesh of a restricted area is carried out.

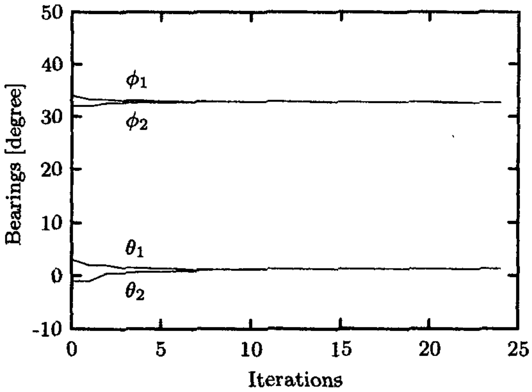
Oscillation of solution is seen in AP algorithm as shown in Fig. 3 (a). The oscillation can be suppressed as shown in Fig. 3 (b) by keeping the bearing obtained in the previous update phase when the bearing giving a greater value of the ML function could not be found in the current update phase. Although in this case the ML criterion is monotonically increasing as shown in Fig. 4, it approaches to less value than the ML criterion of Fig. 3 (a). This may be happen because one bearing is blocked to approach to another bearing due to the



(a) Normal AP algorithm (AP).



(b) AP algorithm restricted so that the ML criterion never decreases in update phase (AP monotonic).



(c) AP algorithm applying the proposed technique and also restricted so that the ML criterion never decreases in update phase (APM monotonic).

Figure 3. Examples of bearing estimations. The array configuration of sensors is the same as in Fig. 1. Independent signals are impinging from two directions (32°,0°) and (35°,5°) with SNR's 5 dB for both. Using 200 snapshots, bearings are estimated by the alternating projecting (AP) algorithm. Explanations for difference in (a), (b) and (c) are described above.

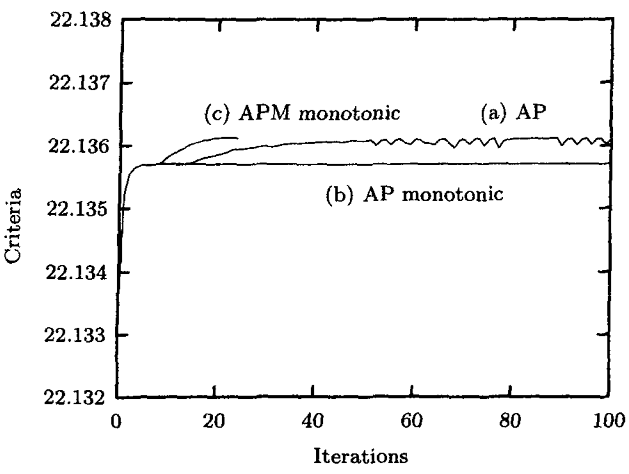


Figure 4. Examples of bearing estimations. The scenario is the same as Fig. 3. The labels in the figure correspond to the simulations in Fig. 3.

singularity of duplicated bearings.

The use of the technique of avoiding singular points, not only the oscillation is suppressed as well as Fig. 3 (b), but also less iterations are required than Fig. 3 (b). Furthermore the ML criterion approaches to the greater value than both Fig. 3 (a) and (b), since the proposed technique does not prevent one bearing from approaching to another bearing.

5. Conclusions

This paper has presented a technique for avoiding indefiniteness in Maximum Likelihood (ML) criteria for Direction-of-Arrival (DOA) finding using a sensor array with arbitrary configuration. Simulation results has been shown to demonstrate the validity of the proposed technique. The application of the proposed technique to the bearing estimation has been shown. The numerical instability in neighborhood of singular points is still remained, since this paper has shows the technique avoiding only singular points. The numerical instability may cause undesired behavior of convergence in iterative techniques of DOA finding. Further investigation is required.

References

[1] I. Ziskind and M. Wax, "Maximum likelihood localization of multiple sources by alternating projection," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. 36, No. 10, pp. 1553-1560, Oct. 1988.

[2] M. Suzuki, H. Sanada, N. Nagai: "An Improvement of Alternating Projection Algorithm for Bearing Estimation Using a Uniform Linear Array," *in Proc. IEEE ISPACS 2000*, Vol. 2, pp.870-875, Honolulu (Nov. 2000).