Integer Programming-based Maximum Likelihood Method for OFDM Parameter Estimation

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Abstract: A problem of signal transmitted and received in OFDM systems is considered. In particular, an efficient solution to the problem of blind channel estimation based on Maximum Likelihood (ML) principle has been investigated. The paper proposes a new upper-bound cost, used in conjunction with a standard branch and bound integer programming technique for solving the ML problem. The tighter upper-bound cost exploits a finite-alphabet property of the transmitted signal. The proposed upper-bound cost was found to greatly speed up the ML algorithm, thus reducing computational complexity. Experimental results and discussion are included.

1. Introduction

Integer programming is a mathematical technique that can be applied in various problem such as those found in business, industrial and communications. This paper deals with its application to channel and symbol estimation in OFDM systems.

The method of Orthogonal Frequency Division Multiplexing (OFDM) modulation has gained much research interest recently, due to improved spectrum utilization efficiency as well as the elimination of Inter-Symbol Interference (ISI) through the use of guarding time. The scheme has been adopted in digital audio/video broadcasting standards, and is the candidate for use in future multimedia mobile communications services. However, to allow for coherent detection, an equalizer is needed at the receiving side. Current solutions to the problem include LMMSE (Linearly Minimum Mean Squares Error) or LSE (Least Squares Error) estimator [1-2], as well as some blind and semi-blind methods [3-5]. In this paper, we focus on the blind estimation method based on a Maximum Likelihood (ML) principle as described in [5]. The ML method is unique in a sense that it requires a received signal only from one OFDM symbol for blind channel and transmitted symbol estimation. This property should be desirable under fast fading envtronment. One major drawback of this method, however, is its high computational requirement. In this paper, we show that computation of this ML method can be greatly reduced by solving the problem using a standard branch and bound integer programming algorithm with the proposed improved upper-bound cost function. After presenting a brief description on the problem of signal received and transmitted in OFDM systems, as detailed in Section 2, the method to solve the problem is described in Sections 3. Section 4 presents experimental results and comparative discussion on the proposed method and the standard zero-one branch and bound method. Finally, concluding remarks are provided in Section 5.

2. Problem of Signal and Channel Parameter Estimation in OFDM Systems

In OFDM systems, data is transmitted over a multipath (fading) channel, where the transmitted signal is distorted. The received signal is affected by such channel factors as attenuation, signal reflection and refraction, as well as additive noise. Under this condition, the received signal at the i^{th} OFDM symbols, can be described by

$$x(t) = \sum_{m=0}^{M-1} h_{i,m} s(t - mT_s) + n(t) \qquad (1)$$

where M is the number of multipaths, and $s(t)$ is the signal transmitted over N subcarriers. The transmitted signal can be expressed as

$$s(t) = \sum_{n=0}^{N-1} b_{i,n} e^{jn\Delta\omega t} \qquad (2)$$

with $i(T_d + T_g) + T_g \le t \le (i+1)(T_d + T_g)$, for $i = 0,1,\ldots,$. Here $b_{i,n}$ is data transmitted on the n^{th} subcarrier over the i^{th} OFDM symbol duration. T_g, T_d, and T_s are guard time, OFDM symbol duration, and sampling time interval respectively. In addition, $n(t)$ is an additive noise and $h_{i,m}$ in Eq. (1) is a scaling factor corresponding to the m^{th} multipath at the i^{th} OFDM symbol duration. Note that, here the channel parameters are assumed to be the same over each OFDM symbol duration.

At the i^{th} OFDM symbol, given L samples of the received signal $x(t)$ obtained over time period T_d $(= T_s L)$, we can describe a vector of the received signal as

$$\begin{aligned} \mathbf{x}_i &= [x(iL) \;\; x(iL+1) \;\; \cdots \;\; x((i+1)L-1)]^T \\ &= \mathbf{A}_c \mathbf{B}_i \mathbf{A}_d \mathbf{h}_i + \mathbf{n}_i \end{aligned} \tag{3}$$

where

$$\mathbf{A}_c = [\mathbf{a}_{c,0} \;\; \mathbf{a}_{c,1} \;\; \cdots \;\; \mathbf{a}_{c,N-1}] \tag{4}$$

$$\mathbf{B}_i = \begin{bmatrix} b_{i,0} & & & \\ & b_{i,1} & & \\ & & \ddots & \\ & & & b_{i,N-1} \end{bmatrix} \tag{5}$$

$$\mathbf{A}_d = [\mathbf{a}_{d,0} \;\; \mathbf{a}_{d,1} \;\; \cdots \;\; \mathbf{a}_{d,M-1}] \tag{6}$$

$$\mathbf{h}_i = [h_{i,0} \;\; h_{i,1} \;\; \dots \;\; h_{i,M-1}]^T \tag{7}$$

$$\mathbf{n}_i = [n(iL) \;\; n(iL+1) \;\; \dots \;\; n((i+1)L-1)]^T \tag{8}$$

with

$$\mathbf{a}_{c,n} = \left[1 \;\; e^{jn\Delta\omega T_s} \;\; \dots \;\; e^{jn\Delta\omega T_s(L-1)}\right]^T, \quad n = 0, 1, \dots, N-1 \tag{9}$$

$$\mathbf{a}_{d,m} = \left[1 \;\; e^{-jm\Delta\omega T_s} \;\; \dots \;\; e^{-jm\Delta\omega T_s(N-1)}\right]^T, \quad m = 0, 1, \dots, M-1 \tag{10}$$

Let $\mathbf{b}_i \triangleq [b_{i,0} \;\; b_{i,1} \;\; \cdots \;\; b_{i,N-1}]^T$. For an OFDM signal using BPSK modulation scheme, from [5] a ML estimate of the transmitted signal is given by

$$\hat{\mathbf{b}}_i = \arg\max_{\mathbf{b}_i} \; \mathbf{b}_i^T \tilde{\mathbf{X}}_i^* \mathbf{A}_d \mathbf{A}_d^H \tilde{\mathbf{X}}_i \mathbf{b}_i^* \tag{11}$$

where

$$\tilde{\mathbf{X}}_i = \begin{bmatrix} X_{i,0} & & & \\ & X_{i,1} & & \\ & & \ddots & \\ & & & X_{i,N-1} \end{bmatrix} \tag{12}$$

From Eq. (12), $X_{i,l}$ is l^{th} element of $\mathbf{X}_i$ which is given by

$$\begin{aligned} \mathbf{X}_i &= \frac{1}{N}\mathbf{A}_c^H \mathbf{x}_i \\ &= \mathbf{B}_i \mathbf{A}_d \mathbf{h}_i + \mathbf{N}_i \end{aligned} \tag{13}$$

$$\mathbf{N}_i = \frac{1}{N}\mathbf{A}_c^H \mathbf{n}_i \tag{14}$$

Given the ML estimate of the transmitted signal, the ML estimate of the scaling factor can be obtained from

$$\hat{\mathbf{h}}_i = \frac{1}{N}\mathbf{A}_d^H \hat{\mathbf{B}}_i^* \mathbf{X}_i \tag{15}$$

where $\hat{\mathbf{B}}_i$ is constructed from Eq. (5) using the ML estimate of $\mathbf{b}_i$.

3. Integer Programming Method for ML OFDM Channel and Symbol Estimation

From Section 2, it can be seen that Eq. (11) is a pure zero-one integer optimization problem, because the feasible solution for each $b_{i,l}$ is either 1 or −1. Thus, a zero-one branch and bound method [6] can be applied in solving for a solution.

A zero-one branch and bound method is known to offer more efficient computation, when compared with a brute-force method. In a branch and bound strategy term, a partial solution is defined as a solution obtained where some variables are fixed, while the remaining ones are left undetermined. For each partial solution, there is at least one corresponding completion partial solution. Completion partial solutions are all feasible solutions, with values of the fixed component part of the variables given by the corresponding partial solution. Therefore the optimization cost corresponding to any partial solution is equal or better than the costs associated with its completion partial solutions.

Because the number of feasible values for each variable in zero-one branch and bound method is 2, branching is performed in a binary treelike fashion by creating 2 subsidiary partial solutions, both sharing the same previously fixed variables. First subsidiary partial solution is the current partial solution that fixes one more variable to one of the feasible values, while the second subsidiary partial solution is obtained by assigning the same variable to the other feasible value. A branch will be created when the optimization cost associated with the partial solution (at the node to be branched) is equal to or better than the best cost value among those of currently evaluated completion partial solutions. The branching process continues until no more branching is possible. Then, a completion solution with the best cost value is the final solution to the problem.

For the problem of signal received and transmitted in OFDM systems, we denote C_k as a cost associated with a partial solution at node k, which is given by

$$C_k = \max_{\mathbf{B}_i} \; \mathbf{B}_i^T \tilde{\mathbf{X}}_i^* \mathbf{A}_d \mathbf{A}_d^H \tilde{\mathbf{X}}_i \mathbf{B}_i^* \tag{16}$$

where $\mathbf{B}_i$ is constructed from Eq. (5).

From Eq. (16), it can be said that a partial solution cost C_k is a cost associated with the leaf node which provides the best solution among those underneath the node k. In practice, this value can only be obtained after evaluating all possible solutions under node k. However, to reduce the need to evaluate all leaf nodes, the upper-bound on C_k obtained from the following equation may be used instead.

$$\tilde{C}_k = \sum_{n=0}^{L_k-1}\sum_{m=0}^{L_k-1} \hat{b}_{i,m}^k \hat{b}_{i,n}^k A_{m,n} + 2\sum_{n=L_k}^{N-1}\sum_{m=0}^{L_k-1}\left|A_{m,n}\right| + \sum_{n=L_k}^{N-1}\sum_{m=L_k}^{N-1}\left|A_{m,n}\right| \tag{17}$$

where $\hat{b}_{i,l}^k$ is a value of $b_{i,l}$ assigned at a node k, $l = 0,1,\cdots, L_k - 1$. Here L_k is the number of fixed variables at a node k. For example, from Figure 1, at node $k = 6$, and $L_k = 3$, $\hat{b}_{i,0}^6 = 1$, $\hat{b}_{i,1}^6 = 1$ and $\hat{b}_{i,2}^6 = -1$.

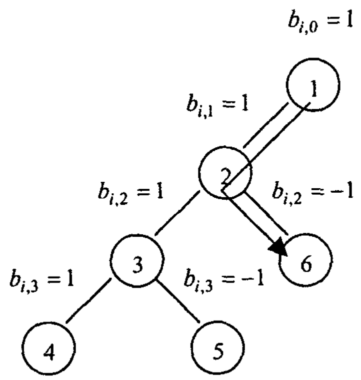

Figure 1. Branch-and-bound tree with $k = 6, L_k = 3$

Here, the partial solution will be branched with preorder traversing (depth first search). Initially, we may assign the current best cost value as $-\infty$, or equals to cost value corresponding to a randomly chosen feasible solution. The partial solution is branched out from a node of which its upper-bound cost is greater than a current best cost value (since we are dealing with a maximum optimization problem). Once no more branching is possible, the current best solution is the ML estimate of the transmitted data.

Next, we propose another upper-bound cost function, which is tighter than the one just mentioned. By using the proposed upper-bound cost function, the probability of cutting the infeasible partial solution is increased, thus reducing computational complexity.

From Eq. (16), a tighter upper-bound on C_k is given by

$$\tilde{\tilde{C}}_k = \sum_{n=0}^{L_k-1}\sum_{m=0}^{L_k-1} \hat{b}_{i,m}^k \hat{b}_{i,n}^k A_{m,n} + 2\sum_{n=L_k}^{N-1}\left|\sum_{m=0}^{L_k-1} \hat{b}_{i,m}^k A_{m,n}\right| + \sum_{n=L_k}^{N-1}\sum_{m=L_k}^{N-1}\left|A_{m,n}\right| \tag{18}$$

Note that, the above upper-bound exploits a finite-alphabet property of the signal transmitted using BPSK modulation. However, its extension to other PSK modulation methods is straightforward. It will be shown in the next section that this upper-bound cost can improve computational efficiency of the Integer Programming ML method.

4. Experimental Results

In this section, some simulation results are provided to show performance gained by the proposed method. In our computer simulation experiment, the system operated at the 1 GHz band and the transmission bandwidth is 640 KHz. Data was modulated with each subcarrier using the BPSK modulation scheme. The system operated under a Rayleigh-fading channel with no Doppler effect. The signal is transmitted using different numbers of subcarries N = 8, 12, ..., 32. Figure 2 shows the averaged number of partial solutions needed to be evaluated using each of the three methods. The first method performed is the total enumeration (brute-force) method. The second method employed a branch and bound integer programming algorithm with the upper-bound cost as described by Eq. (17). The third method is the same as the second method, but using Eq. (18) for an upper-bound cost calculation instead. From the figure, it is seen that the second method requires significantly less computation as compared with a standard total enumeration method. Moreover, using Eq. (18) as an upper-bound cost in the third method further reduces the method's computational requirement.

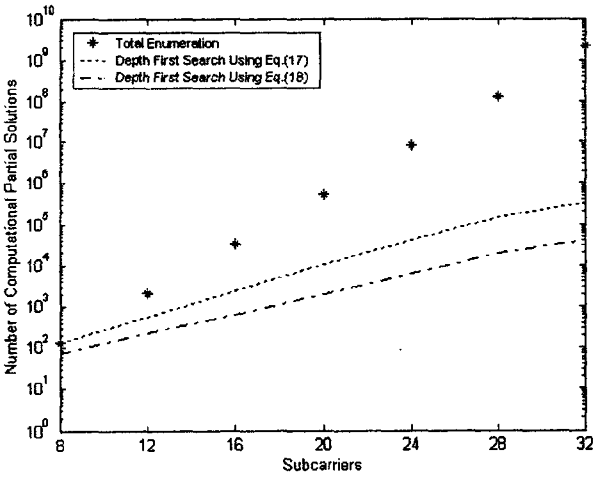

Figure 2. Computational complexity by each of the three methods

The performance of the proposed method also depends on how nodes (and the corresponding partial solutions) are branched. By using the best bound-cost value (as called best first search) strategy, performance can be improved. With this strategy, at each branching step, the node with the highest bound-cost is chosen for branching. Figure 3 shows the resulting averaged number of partial solutions (i.e., the number of branching) which must be computed when using the proposed upper-bound cost function with the depth first search strategy, compared with that of the same method used with the best first search strategy. From the figure, the best first search strategy offers reduced computational complexity.

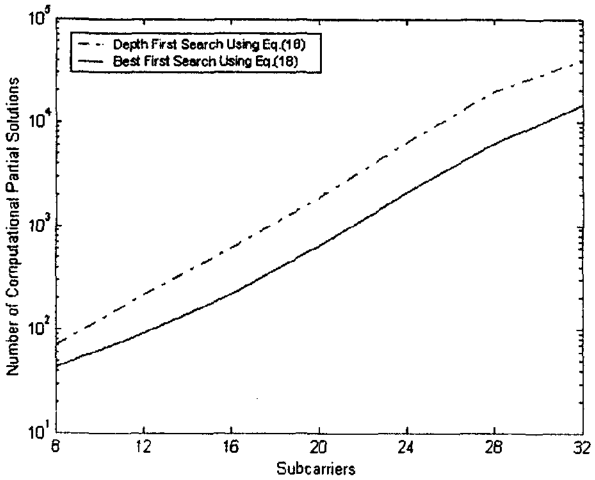

Figure 3. Computational requirements for different branching techniques using the same improved upper-bound cost function

Next, we consider the situation with various SNRs for illustrating the impact of SNRs on the method's computational complexity. We simulated the same scenario as previously described, with SNRs = 0, 5, 10, ..., 30. Figure 4 shows the results from the simulation using 16 subcarriers. From the figure, it is seen that the computational requirement at high SNRs is lower than that at low SNRs.

5. Conclusion

In this paper, we have shown that a standard branch and bound integer programming algorithm with the proposed improved upper-bound cost function can greatly reduce the complexity of computation in obtaining ML OFDM estimate. Simulation results demonstrate the performance of the improved upper-bound cost function compared with the simple upper-bound cost function. It has been shown that the improved upper-bound cost function can also be speed up by using the best first search branching strategy. In addition, the simulation result shows that computation complexity of the method increases while decreasing SNRs.

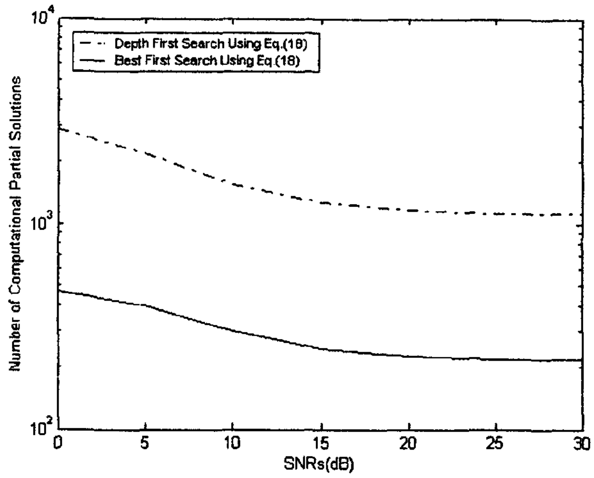

Figure 4. Computational requirements at various SNRs

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