A Detailed Routing Algorithm for Switch Boxes

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Abstract: A k-side switch box with W terminals on each side is said to be *hyper universal*, denoted by (k, W)-HUSB, if it is routable for any global routing with density at most W. In [5], we have proposed a switch box design strategy and designed a near optimum $(4, W)$-HUSB $F(W)$ with $6.\bar{6}W$ switches. In this paper, we design, analyze and implement an efficient detailed routing algorithm for the S-box $F(W)$. This router can accommodate all routing requirement topologies.

1. Introduction

Field Programmable Gate Array (FPGA) is one of the fast developed product type of VLSIs in last decade, and has been used in a great amount of digital equipment today.

A typical $2D$-FPGA architecture is shown in Figure 1. The functional blocks (or logic cells) are marked by L, which are separated by vertical and horizontal channels. There are W (called channel density) prefabricated parallel wire segments running between each pair of adjacent L-cells in both vertical and horizontal channels. There are C-boxes in the channel between adjacent L-cells. A Switch Box (S-box), located at each intersection of a vertical channel and a horizontal channel, contains programmable switches to connect wire segments running from its surrounding C-boxes.

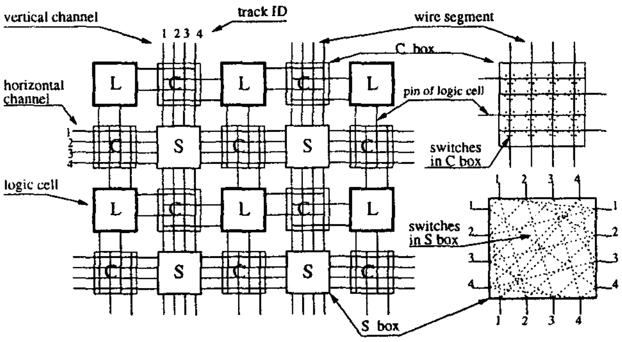

Figure 1. The architecture of a 2D-FPGA.

Conventionally, the routing process is divided into two subsequent steps, global routing and detailed routing. Global routing specifies the connection topologies for all nets, while the detailed routing decides the exact assignment of wire segments and switches used to materialize the complete routing. A global routing is called *routable* if there is a detailed routing for it.

There are two important issues in the design of FPGAs: the area and time efficiency and the routability. Switch boxes, which are the main programmable component and have great effects on the two issues of FPGA chips, have been the crucial part in the FPGA architecture designs [1], [2], [4].

In [5], we have proposed a switch box design strategy and designed a near optimum $(4, W)$-HUSB $F(W)$, and proved theoretically that $F(W)$ reflects a good balance of the area efficiency and routability issues. Therefore, it would be desirable if we provide an efficient detailed routing algorithm for the S-box $F(W)$. This is the purpose of this paper.

2. The S-box $F(W)$

We first briefly describe the terminology and notations we used in the designing of HUSBs in [5], [6].

A k-side switch box with W terminals on each side is denoted by (k, W)-SB. We label the k sides by $1, \ldots, k$ respectively. Then a net connecting the sides $i_1, \ldots, i_r$ can be represented as a set $\{i_1, \ldots, i_r\}$. A *k-way (balanced) global routing of density d*, or briefly (k, d)-GR ((k, d)-BGR), is a collection $\{N_i | i = 1, \ldots, v\}$ of subsets of $\{1, 2, \ldots, k\}$ such that each i $(1 \leq i \leq k)$ appears at most d (exactly d) sets in $\{N_i | i = 1, \ldots, v\}$; N_i is also referred to as a *net* of the global routing. Let R be a (k, d)-GR and R' be a sub-collection of R. If R' is a (k, d')-GR with $d' < d$, R' is called a *sub-global routing* of R. R is said to be *minimal* or an MGR if it does not contain sub-global routings. A (k, d)-GR is called *primitive* ((k, d)-PGR) if it does not contain two unequal nets of size one. A k-PMBGR stands for a k-way primitive minimal balanced global routing.

We represent the j-th terminal on the i-th side of a (k, W)-SB by a vertex $v_{i,j}$, and let $v_{i,j}v_{p,q}$ represent the switch joining the two terminals represented by the vertices $v_{i,j}$ and $v_{p,q}$, then the (k, W)-SB can be viewed as a k-partite graph $G = ((V_1, V_2, \cdots, V_k), E)$ where $V_i = \{v_{i,j} | j = 1, \ldots, W\}$ for $i = 1, \ldots, k$, and E is the set of switches. A *detailed routing* of a (k, W)-GR $\{N_i | i = 1, \ldots, v\}$ in G is a set of mutually vertex disjoint subgraphs $\{T(N_i) | i = 1 \ldots, v\}$ of G satisfying: (1) $T(N_i)$ is a tree of $|N_i|$ vertices, and (2) $|V_j \cap V(T(N_i))| = 1$ if $j \in N_i$, for $i = 1, \ldots, v$. $T(N_i)$ is called a *detailed routing* of N_i.

A (k, W)-SB is called *hyper universal* or a (k, W)-HUSB if it has a detailed routing for every (k, W)-GR.

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This is equivalent to that it has a detailed routing for every (k, W)-PBGR since a (k, W)-GR can be converted to a (k, W)-PBGR by adding in some singletons and combining different singletons.

Figure 2(a) shows a global routing and Figure 2(b) shows a detailed routing of the global routing (a) in the S-box H_4 (see Figure 4).

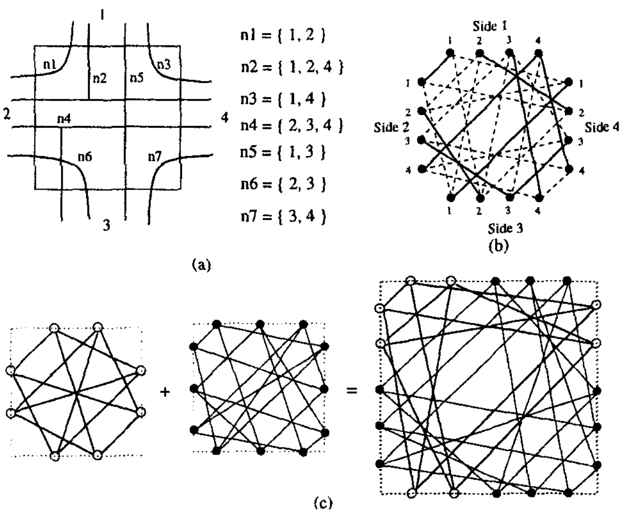

Figure 2. Examples of a global routing,a detailed routing and the disjoint union of two S-boxes

Let GR_1 and GR_2 be two k-way global routings and m a positive integer. The disjoint union of GR_1 and GR_2 as a multiple set is denoted by $GR_1 + GR_2$, and the disjoint union of m copies of GR_1 is denoted by mGR_1. Figure 2 (c) shows the disjoint union of a $(4,2)$-SB and a $(4,3)$-SB.

It is shown in [5] that a 4-PBGR can always be decomposed into 4-PMBGRs. Figure 3 provides the complete list of 4-PMBGRs. Let $\mathcal{PMBGR}$ denote the collection of all 4-PMBGRs in Figure 3.

In [5], we have designed the following $(4, W)$-HUSB $F(W)$.

$$F(W) = \begin{cases} hH_6 & \text{if } W = 6h, \\ (h-1)H_6 + H_7 & \text{if } W = 6h+1, \\ hH_6 + H_2 & \text{if } W = 6h+2, \\ hH_6 + H_3 & \text{if } W = 6h+3, \\ hH_6 + H_4 & \text{if } W = 6h+4, \\ hH_6 + H_5 & \text{if } W = 6h+5, \end{cases}$$

where the H_is are shown in Figure 4.

The detailed routing problem associated with $F(W)$ is:

Input: *A 4-GR* $R = \{N_i | i = 1, \ldots, v\}$ *and* W.
Problem: *Find a detailed routing of* R *in* $F(W)$.

3. The Detailed Routing Algorithm

The Algorithm

A Calculate the density of R and make it balanced and primitive.

$GR^1_1 = \{\{1,2,3,4\}\}$, $GR^1_{2,1} = \{\{1,2\},\{3,4\}\}$,
$GR^1_{2,2} = \{\{1,3\},\{2,4\}\}$, $GR^1_{2,3} = \{\{1,4\},\{2,3\}\}$,
$GR^1_{3,1} = \{\{1\},\{2,3,4\}\}$, $GR^1_{3,2} = \{\{2\},\{1,3,4\}\}$,
$GR^1_{3,3} = \{\{3\},\{1,2,4\}\}$, $GR^1_{3,4} = \{\{4\},\{1,2,3\}\}$,
$GR^2_{1,1} = \{\{1,2,3\},\{1,2,4\},\{3,4\}\}$,
$GR^2_{1,2} = \{\{1,2,3\},\{2,3,4\},\{1,4\}\}$,
$GR^2_{1,3} = \{\{1,2,4\},\{2,3,4\},\{1,3\}\}$,
$GR^2_{1,4} = \{\{1,3,4\},\{2,3,4\},\{1,2\}\}$,
$GR^2_{1,5} = \{\{1,2,3\},\{1,3,4\},\{2,4\}\}$,
$GR^2_{1,6} = \{\{1,2,4\},\{1,3,4\},\{2,3\}\}$,
$GR^2_{2,1} = \{\{1,2,3\},\{1,4\},\{2\},\{3,4\}\}$,
$GR^2_{2,2} = \{\{1,2,4\},\{1,3\},\{2\},\{3,4\}\}$,
$GR^2_{2,3} = \{\{2,3,4\},\{1,4\},\{2\},\{1,3\}\}$,
$GR^2_{2,4} = \{\{1,2,3\},\{1,4\},\{3\},\{2,4\}\}$,
$GR^2_{2,5} = \{\{1,4,3\},\{1,2\},\{3\},\{2,4\}\}$,
$GR^2_{2,6} = \{\{2,4,3\},\{1,2\},\{3\},\{1,4\}\}$,
$GR^2_{2,7} = \{\{2,4,3\},\{1,2\},\{4\},\{1,3\}\}$,
$GR^2_{2,8} = \{\{1,4,3\},\{1,2\},\{4\},\{2,3\}\}$,
$GR^2_{2,9} = \{\{1,2,4\},\{1,3\},\{4\},\{2,3\}\}$,
$GR^2_{2,10} = \{\{1,2,4\},\{4,3\},\{1\},\{2,3\}\}$,
$GR^2_{2,11} = \{\{1,4,3\},\{4,2\},\{1\},\{2,3\}\}$,
$GR^2_{2,12} = \{\{2,1,3\},\{4,2\},\{1\},\{4,3\}\}$,
$GR^2_{3,1} = \{\{1,2\},\{3,1\},\{2,3\},\{4\},\{4\}\}$,
$GR^2_{3,2} = \{\{1,2\},\{4,1\},\{2,4\},\{3\},\{3\}\}$,
$GR^2_{3,3} = \{\{1,3\},\{4,1\},\{3,4\},\{2\},\{2\}\}$,
$GR^2_{3,4} = \{\{3,2\},\{4,3\},\{2,4\},\{1\},\{1\}\}$,
$GR^3_1 = \{\{1,2,3\},\{1,2,4\},\{3,4,1\},\{2,3,4\}\}$,
$GR^3_{2,1} = \{\{1,2,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\}\}$,
$GR^3_{2,2} = \{\{2,3,4\},\{1,2\},\{1,3\},\{1,4\},\{2,3,4\}\}$,
$GR^3_{2,3} = \{\{3,4,1\},\{2,1\},\{2,3\},\{2,4\},\{3,4,1\}\}$,
$GR^3_{2,4} = \{\{4,1,2\},\{3,1\},\{3,2\},\{3,4\},\{4,1,2\}\}$.

Figure 3. The complete list of 4-PMBGRs.

Find the number of occurrences $r_j = \sum_{i=1}^{l} |N_i \cap \{j\}|$ of k in R for $j = 1,2,3,4$.

Let $d = \max\{r_1, r_2, r_3, r_4\}$.

If $d > W$, then stop. R is not routable in $F(W)$; else for $k = 1,2,3,4$, if $(r_k < d)$, set $R' = R \cup \{(d - r_k)\{k\}\}$.

Combine the singletons into 2-pin nets. Let D be the set of all the 2-pin nets obtained by combining the singletons.

B Decompose R' into minimal global routings.

Let $R' = \{N_1, \ldots, N_l\}$.

Repeat Step 1 to Step 3 until $R' = \emptyset$ or $\mathcal{PMBGR} = \emptyset$.
Step 1 For any $M \in \mathcal{PMBGR}$, let $T_1, \ldots, T_m$ be all different nets in M. For $i = 1$ to m, let t_i be the number of replications of T_i in M and set $x_i = 0$. Set $\mathcal{M} = \emptyset$.
Step 2 For $i = 1$ to l, $j = 1$ to m, if $N_i = T_j$, set $x_j = x_j + 1$.
Step 3 If $x_j \geq t_j$ for all $j = 1$ to m, then set $\mathcal{M} = \mathcal{M} \cup M$, and set $GR' = GR' - M$. Else, set $\mathcal{PMGR} = \mathcal{PMGR} - M$.

C Group the sub-global routings in $\mathcal{M}$.

Let $d = 6h + r$, where r takes a value from $\{0,2,3,4,5,7\}$

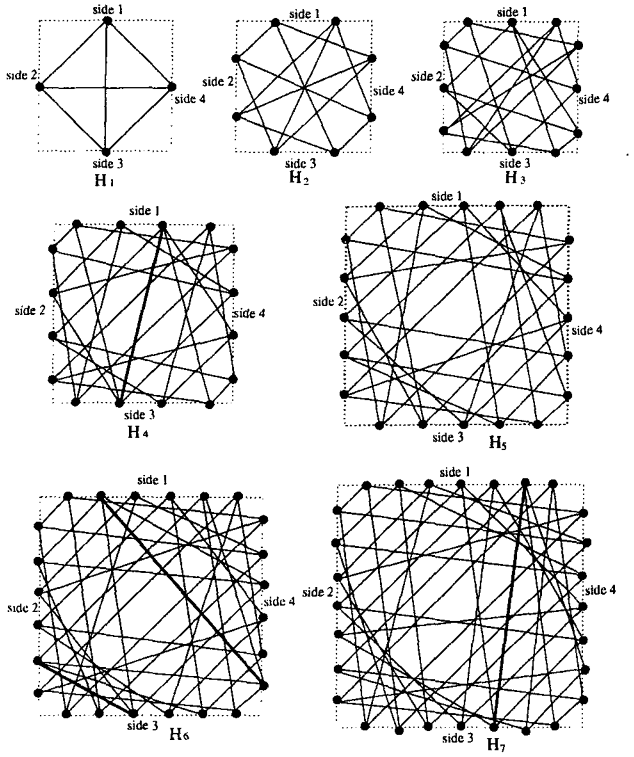

Figure 4. The elementary $(4, W)$-HUSBs.

if $d > 1$, and $r = 1$ otherwise.

Combine the minimal global routings in $\mathcal{M}$ to form h (4,6)-GRs and one $(4,r)$-GR if $r \neq 0$. Let $\mathcal{N} = \{GR_1, \dots, GR_h, GR_{h+1}\}$ be the output, where GR_i is a (4,6)-GR for $1 \leq i \leq h$ and GR_{h+1} is a $(4,r)$-GR if $r \neq 0$. $GR_{h+1} = \emptyset$ if $r = 0$.

D Find a detailed routing of R'.

There are five databases $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{D}_3$, $\mathcal{D}_4$, and $\mathcal{D}_6$. Each database $\mathcal{D}_i$ consists of some matrixes (actually incidence matrixes), and each such matrix represents a detailed routing of some global routing in H_i. Let A and B be two matrixes, the disjoint sum of A and B, $A \oplus B$ is the matrix

$$\begin{pmatrix} A & O \\ O & B \end{pmatrix}.$$

If a global routing is a disjoint union of a global routing of density p and a global routing of density q, we say the global routing is of the *form* "$p + q$".

$\mathcal{D}_1$ is the database for detailed routings in H_1 of density 1 primitive minimal global routings in Figure 3. There are 8 detailed routings in $\mathcal{D}_1$.

$\mathcal{D}_2$ is the database for detailed routings in H_2 of density 2 primitive global routings. There are 22 minimal global routings of density 2 and $\binom{8}{2}+8-\binom{4}{2}$ density 2 primitive global routings of the form "1 + 1". There are total 52 detailed routings (matrixes) in $\mathcal{D}_2$.

$\mathcal{D}_3$ is the database for detailed routings in H_3 of density 3 primitive minimal global routings in Figure 3. There are 5 detailed routings in $\mathcal{D}_3$.

$\mathcal{D}_4$ is the database for detailed routings in H_4 of density 4 primitive global routings which is a union of a density 3 and a density 1 minimal global routings in Figure 3. There are 40 detailed routings in $\mathcal{D}_4$.

$\mathcal{D}_6$ is the database for detailed routings in H_6 of density 6 primitive global routings which is a union of two density 3 minimal global routings in Figure 3. There are 25 detailed routings in $\mathcal{D}_6$.)

For each GR_i, find a detailed routing in G_6 if $i \leq h$:
Find a detailed routing DR_i of GR_i.
If $GR_i = GR^1 + GR^2 + GR^3$ is of the form "2 + 2 + 2", then search for $\mathcal{D}_2$ to find detailed routings DR^1, DR^2 and DR^3. Set $DR_i = DR^1 \oplus DR^2 \oplus DR^3$.
Else (GR_i is of the form "3 + 3"), then search for $\mathcal{D}_6$ to find a detailed routing DR_i.
If $r \neq 0$, find a detailed routing DR_{h+1} of GR_{h+1} in H_r.

(D1: r = 1) Search for $\mathcal{D}_1$ to find a detailed routing DR_{h+1} for GR_{h+1}.

(D2: r = 2) Search for $\mathcal{D}_2$ to find a detailed routing DR_{h+1} for GR_{h+1}.

(D3: r = 3) If $GR_{h+1} = GR^1 + GR^2$ is of the form "1+2", then search for $\mathcal{D}_1$ to find a detailed routing DR^1 for GR^1, and search for $\mathcal{D}_2$ to find a DR^2 for GR^2. Set $D_{h+1} = DR^1 \oplus DR^2$.
Otherwise, search for $\mathcal{D}_3$ to find a detailed routing DR_{h+1} of GR_{h+1}.

(D4: r = 4) If $GR_{h+1} = GR^1 + GR^2$ is of the form "2+2", then search for $\mathcal{D}_2$ to find detailed routings DR^1 and DR^2 for GR^1 and GR^2, respectively. Set $D_{h+1} = DR^1 \oplus DR^2$.
Otherwise, search for $\mathcal{D}_4$ to find a detailed routing DR_{h+1} for GR_{h+1}.

(D5: r = 5) We must have $GR_{h+1} = GR^1 + GR^2$ is of the form "2 + 3". Search for $\mathcal{D}_2$ to find a detailed routing DR^1 for GR^1, and apply the procedure (D3 r=3) to find a detailed routing DR^2 for GR^2. Set $DR_{h+1} = DR^1 \oplus DR^2$.

(D7: r = 7) We have $GR_{h+1} = GR^1 + GR^2$ is of the form "3 + 4". Apply the procedure (D3: r=3) to find a detailed routing DR^1 for GR^1, and apply the procedure (D4: r=4) to find a detailed routing DR^2 for GR^2. Set $D_{h+1} = DR^1 \oplus DR^2$.

Set $G = DR_1 \oplus \cdots \oplus DR_{h+1}$ if $r \neq 0$, and $G = DR_1 \oplus \cdots \oplus DR_h$ otherwise.

E Find a detailed routing of R.

For each 2-pin net $\{i, j\} \in D$, delete a detailed routing of $\{i, j\}$ in G.

Search the cells in G which represent the i-th and j-th sides. If find a cell (p_i, q_j) such that $g_{p_i,q_j} = 1$, and the p_i-th row and q_j-th column are all 0s except $g_{p_i,q_j} = 1$, then set $g_{p_i,q_j} = 0$.

F Output G.

We note that the output G is an incidence matrix of a graph which is a subgraph of $F(W)$.

4. Analysis of the algorithm

Part A: Given the input R and W, the algorithm calculates the density d of R in $O(W)$ basic operations. If

$d \leq W$, the algorithm also constructs a balanced primitive $(4, W)$-GR R' based on R in $O(W)$ operations.

Part B: This step decomposes R' into minimal sub-global routings. The algorithm compares the nets in R' with the nets of minimal global routing in $\mathcal{PMBGR}$. Since $\mathcal{PMBGR}$ contains only 35 elements, it takes $O(l) = O(W)$ comparisons.

Part C: The algorithm groups the decomposition of R' obtained in Part B into h $(4,6)$-GRs and one $(4,r)$-GR if $r \neq 0$. If we store the results in Part B using three vectors V_1, V_2 and V_3, such that V_i contains global routings of density i, then it is easy to get the desired groups if we start from V_3. We need a linear time, that is $O(W)$ to finish this task.

Part D: For each sub-global routing produced in Part C, we need search for certain data base(s) to find a detailed routing. As the data bases contains all detailed routings for primitive, minimal 4-way global routings, therefore, we can always find a detailed routing for each sub-global routing obtained in Part C. Combing all these detailed routings for the sub-global routings in Part C, we obtain a detailed routing of the global routing GR'. Upon deleting the detailed routings for the 2-pin nets in D, we obtain a detailed routing for the original input GR. Therefore, the algorithm produces a correct detailed routing for any input global routing GR. Note that in Part D, we need $O(W)$ basic operations.

Part E: In this step, we need search for $O(W^2)$ cells of G in the worst case.

Summing up, we have that the time complexity of the above detailed routing algorithm is $O(W^2)$.

5. Implementation of the algorithm

We develop a single switch box router to implement the above detailed routing algorithm in C++ with the output graphical user interface in JAVA. We use string of numbers $0, 1, 2, 3, 4$ as a data structure to represent global routings, where 0 is used as the separator of nets. For instance, "12302340234" represents $(4, 3)$-GR $\{\{1, 2, 3\}, \{2, 3, 4\}, \{2, 3, 4\}\}$. Figure 5 shows the detailed routing of "12302340234" in $F(3)$. In Figure 6, the channel density (W) is 4 and the Global Routing string is "1234012302340134". There are 4 nets where the first net connects all 4 different sides of the switch box.

6. Conclusions

An efficient detailed routing algorithm for the $(4, W)$-HUSB $F(W)$ is described in this paper. A detailed algorithmic analysis of the router is also included in the paper. From the theory and experiments we conducted, this router is able to complete any given routing topologies as long as the density is no more that W. Moreover, we conclude that if the S-boxes in the FPGA (shown in Figure 1) are replaced by $F(W)$, then with the detailed routing algorithm designed in this paper as a submodule, it is possible to design an efficient detailed router for the whole chip.

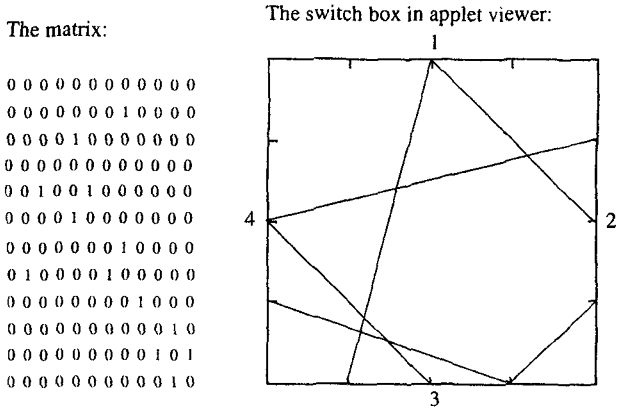

Figure 5. Example of Channel density (W) = 3

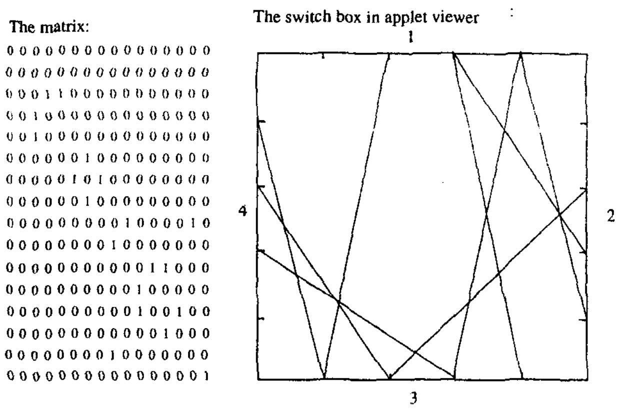

Figure 6. Example of Channel density (W) = 4

References

[1] M. J. Alexander and G. Robins. "New performance FPGA routing algorithms". *Proceedings DAC*, pages 562–567, 1995.

[2] S. Brown, R. J. Francise, J. Rose, and Z. G. Vranesic. *Field-Programmable Gate Arrays*. Kluwer-Academic Publisher, Boston MA, 1992.

[3] S. Brown, J. Rose, and Z. G. Vranesic. "A detailed router for field-programmable gate arrays". *IEEE Trans. on Computer-Aided Design*, 11:620–628, May 1992.

[4] Y. W. Chang, D. F. Wong, and C. K. Wong. "Universal switch modules for FPGA design". *ACM Trans. on Design Automation of Electronic Systems*, 1(1):80–101, January 1996.

[5] H. Fan, J. Liu, and Y. Wu. "General Models and a Reduction Design Technique for FPGA Switch Box Designs". *IEEE Transactions on Computers*, to appear.

[6] H. Fan, J. Liu, Y. L. Wu, and C. C. Cheung. "On Optimum Switch Box Designs for 2-D FPGAs". *Proc. IEEE/ACM Design Automation Conference (DAC)*, Jun. 2001.

[7] Y. S. Lee and C. H. Wu. "A performance and routability driven router for FPGAs considering path delays". *Proceedings DAC*, pages 557–561, 1995.

[8] J. F. Pan, Y. L. Wu, G. Yan, and C. K. Wong. "On the optimal four-way switch box routing structures of FPGA greedy routing architectures". *Integration, the VLSI Journal*, 25:137–159, 1998.

[9] J. Rose and S. Brown. "Flexibility of interconnection structures for field-programmable gate arrays". *IEEE J. Solid-State Circuits*, 26(3):277–282, 1991.

[10] Y. L. Wu and M. Marek-Sadowska. "Routing for array type FPGAs". *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, 16(5):506–518, May 1997.

[11] Y. L. Wu, S. Tsukiyama, and M. Marek-Sadowska. "Graph based analysis of 2-D FPGA routing". *IEEE Trans. on Computer-Aided Design*, 15(1):33–44, 1996.