# Basic bifurcation by intermittently coupled capacitors

Junya Shimakawa, and Toshimichi Saito.
Department of Electronics and Electrical Computer Engineering, HOSEI Univ.
3-7-2, Kajino-cho, Koganei-shi, Tokyo, 184-8584, Japan.
TEL: +81-423-87-6204, FAX: +81-423-87-6122
E-mail : shimakawa@nonlinear.k.hosei.ac.jp

**Abstract**: This paper studies basic phenomena of intermittently coupled capacitors circuits. As an analysis tool, we introduce Hybrid return map of real and binary variables, and analyze bifurcation phenomena for three parameters. Co-existence of synchronous phenomena is also shown. Using a simple test circuit, typical phenomena are verified in the laboratory.

## 1. Introduction

Analysis of coupled oscillators is important not only as a fundamental nonlinear problem but also for engineering applications. The coupled oscillators exhibits rich synchronous and asynchronous phenomena, and various analysis results have been published ( see [1] [2] [3] and Refs. therein ). Also the coupled oscillators relate deeply to engineering applications including artificial neural networks [4] and beam scanning of array antennas [5]. In order to construct an interesting coupled oscillators, the method of the coupling is an important key. This paper presents intermittently coupled capacitors method that couples two relaxation oscillators and generates rich synchronous phenomena.

Fig.1 shows the coupled oscillators. Each oscillator consist of one capacitor and one hysteresis voltage-controlled current source (VCCS) $H_i$. The circuit dynamics is described by Equation (1).

$$\begin{cases} C_i \dfrac{dv_i}{dt} = H_i(v_i),\ Y_i = H_i(v_i), & \text{for } t \neq nT, \\ \bar{v}(t) = \dfrac{C_1}{C_1+C_2}v_1(t) + \dfrac{C_2}{C_1+C_2}v_2(t), & \text{for } t = nT, \end{cases}$$

$$H_i(v_i) = \begin{cases} J_i, & \text{for } v_i \leq E, \\ -J_i, & \text{for } v_i \geq -E, \end{cases} \qquad i = 1 \text{ or } 2, \tag{1}$$

The output of $H_i$ is switched from $J_i$ to $-J_i$ (respectively, from $-J_i$ to $J_i$), if $v_i$ hits the right threshold $E$ (respectively, the left threshold $-E$).

In this paper, as an analysis tool, we introduce Hybrid return map of real and binary variables, and analyze bifurcation phenomena for three parameters. Co-existence of synchronous phenomena is also shown. Using a simple test circuit, typical phenomena are verified in the laboratory. Discussion on the case $C_1 = C_2$ can be found in [6].

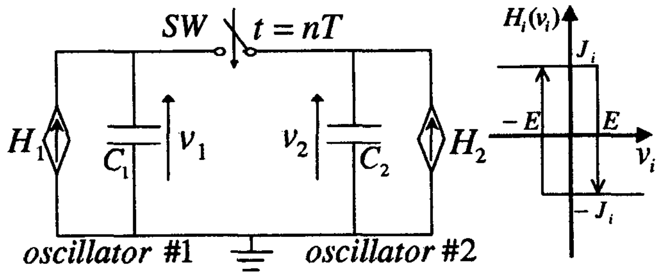



Figure 1. Intermittently coupled capacitors circuits

## 2. Intermittently coupled capacitors

Using the following dimensionless variables and parameters, the system dynamics is described by Equation(2).

$$\tau = \frac{t}{T},\ x_i = \frac{v_i}{E},\ s_i = \frac{TJ_i}{EC_i},\ \alpha = \frac{C_1}{C_1+C_2},$$
$$h(x_i) = \frac{1}{J_i}H_i(Ex_i),$$

$$\begin{cases} \dfrac{dx_i}{d\tau} = s_i y_i,\ y_i = h(x_i), & \text{for } \tau \neq n, \\ \bar{x}(\tau) = \alpha x_1(\tau) + (1-\alpha)x_2(\tau), & \text{for } \tau = n, \end{cases} \tag{2}$$

$$h_i(x_i) = \begin{cases} 1, & \text{for } x_i \leq 1, \\ -1, & \text{for } x_i \geq -1, \end{cases} \qquad i = 1 \text{ or } 2,$$

where $x_i \in \mathbf{R}$ and $y_i \in \mathbf{B} \equiv \{1, -1\}$ are real and binary states of the i-th oscillator, respectively. We show typical laboratory measurements in Fig.2. As shown in the Fig.2(a), the hysteresis VCCS is implemented using two operational transconductance amplifiers (OTAs, NM 13600). Before the coupling, each oscillators exhibits periodic waveform as shown in Fig.2(b)(c). If $C_1 = C_2$ ($\alpha = 0.5$), the equalized voltage $\bar{v}(t)$ is the average of $v_1$ and $v_2$ as shown in Fig.2(d). If $C_1 \neq C_2$ ($\alpha \neq 0.5$), $\bar{v}(t)$ is the weighted sum of $v_1$ and $v_2$ as shown in Fig.2(e). $Y_i$ shows binary states of the hysteresis VCCS in Fig.2(d)(e). This system has three parameters $(s_1, s_2, \alpha)$. We consider following parameters region for simplicity.

$$0 \leq \alpha \leq 0.5,\ 0 \leq s_1 \leq 0.5,\ 0 \leq s_2 \leq 0.5, \tag{3}$$

## 3. Hybrid return map

In order to introduce hybrid return map (HRM), we note that the following initial real state can be used without loss of generality because the two real states are equalized instantaneously at every switching moment:

$$\bar{x}(0) = x_1(0) = x_2(0) \tag{4}$$

As the domain of HRM, we define $\mathbf{D_H}$ which consists of one real variable $\bar{x}(n)$ and two binary variables $y_1(n) and y_2(n)$.

$$\mathbf{D_H} \equiv \{(\bar{x}, y_1, y_1) : |\bar{x}| < 1, y_1, y_1 \in \mathbf{B}\}, \tag{5}$$

As an orbit starts from $\mathbf{D_H}$ at $\tau = n$, this orbit returns to $\mathbf{D_H}$ at $\tau = n+1$. Then we can define the following HRM.

$$\mathbf{F} : \mathbf{D_H} \to \mathbf{D_H}, \tag{6}$$

The HRM can be described as the following.

$$\begin{cases} \bar{x}(n+1) = \alpha f(x_1(n), y_1(n), 1 - s_1) \\ \qquad\qquad +(1-\alpha) f(x_2(n), y_2(n), 1 - s_2), \\ y_1(n+1) = g(\bar{x}(n), y_1(n), 1 - s_1), \\ y_2(n+1) = g(\bar{x}(n), y_2(n), 1 - s_2), \end{cases} \tag{7}$$

where

$$\begin{aligned} & f(x_i(n), y_i(n), 1 - s_i) \\ & \quad = y_i(n) + y_i(n)\{x_i(n) - y_i(n)(1 - s_i)\} \\ & \qquad \times \mathrm{sgn}\{y_i(n)(1 - s_i) - x_i(n)\}, \\ & g(x_i(n), y_i(n), 1 - s_i) \\ & \quad = \mathrm{sgn}\{y_i(n)(1 - s_i) - x_i(n)\}, \end{aligned} \tag{8}$$

We show an example of HRM in Fig.3. A, B, C and D are codes by combination of $\{y_1, y_2\}$ as the table in Fig.4. Real state $\bar{x} \equiv [-1, 1]$ is assigned to each code.

Next, we consider synchronous phenomena. Let $\mathbf{X} = (\bar{x}, y_1, y_2)$. The system is said to generate synchronization with period $k$ if the following equation is satisfied in the steady state.

$$\mathbf{X}(n+p) = \mathbf{X}(n),\ \mathbf{X}(n+k) \neq \mathbf{X}(n),\ \text{for } k < p, \tag{9}$$

where $k$ is a positive integer and $p$ is a measure of $k$. Let $\mathbf{S}(x)$ be an $\bar{x}$-directional slope of HRM at point $\mathbf{x} \equiv (\bar{x}, y_1, y_2)$ :

$$\mathbf{S}(x) = \alpha \frac{d}{d\bar{x}} f(\bar{x}, y_1, 1 - s_1) + (1-\alpha) \frac{d}{d\bar{x}} f(\bar{x}, y_2, 1 - s_2), \tag{10}$$

$|\mathbf{S}(x)|$ can be either 1 or $1 - 2\alpha$.

## 4. Bifurcation phenomena

First, we show an algorithm to find the period of synchronous phenomena. For an initial point $d_0 \in \mathbf{D_H}$, we calculate the iteration.

$$d_{n+1} = \mathbf{F}(d_n), \tag{11}$$

If the following is satisfied, we declare that period is $T$.

$$|d_n - \mathbf{F}^T(d_n)| < \epsilon, \tag{12}$$

where $\epsilon$ is sufficiently small value. Left column of Fig.6 shows bifurcation of $T_s$, where the tone is to be dark as the period increases.

Next, we explain co-existence phenomena. Plural synchronous states can co-exist for initial state. In right column of Fig.5, the black region represents the co-existence of synchronous states with different periods. In such a case, the shortest period represents the tone in left column of Fig.5. Also, if the HRM has an asymmetric attractor, its symmetric one must exist because the HRM is symmetric with respect to the center of HRM. As $\alpha$ varies, we have confirmed the following properties:

- The bifurcation diagram is distorted and the symmetry is broken for $\alpha < 0.5$.
- In the border of each parameters for $\alpha < 0.5$, tone is darkend. It means generation of synchronization with longer period.
- As $\alpha$ decreases, regions of longer period tends to be extended.
- Co-existence phenomena of different periods are impossible for $\alpha = 0.5$. As $\alpha$ decreases, co-existence region is to be extended.

## 5. Conclusion

We have considered bifurcation phenomena of intermittently coupled capacitors circuit. Using the HRM, we derived basic bifurcation diagrams for three parameters. Using a simple test circuit, typical phenomena was verified in the laboratory.

### References


[1] T. Endo and S. Mori, "Mode analysis of a ring of a large number of mutually coupled van der Pol oscillators," IEEE Trans. CAS, 25, 1, pp. 7-8 (1978)

[2] T. Saito, "On a coupled relaxation oscillator," IEEE Trans. CAS, 35, 9, pp. 1147-1155 (1988)

[3] T.Matushita, T.Saito and H.Torikai, "Basic synchronous phenomena by intermittently coupled capacitors," IEEE Trans. CAS-I, 47, 7, pp. 1105-1109 (2000)

[4] L. O. Chua, M. Hasler, G. S. Moschytz and J. Neirynck, "Autonomous Cellular Neural Networks: A unified paradigm for pattern formulation and active wave propagation," IEEE Trans. CAS-I, 42, 10, pp. 559-577 (1995)

[5] R. J. Ram, R. Sporer, H. -R. Blank, R. A. York, "Chaotic dynamics in coupled microwave oscillators," IEEE trans. on MIcrowave Theory and Techniques, Vol. 48, no. 1, pp. 1909-1916 (2000)

[6] F. Komatsu, H. Torikai and T. Saito, " Various superstable synchronous phenomena in switch-coupled relaxation capacitors," in Proc. IEEE/ISCAS, Sydney,III, pp. 696-699 (2001)

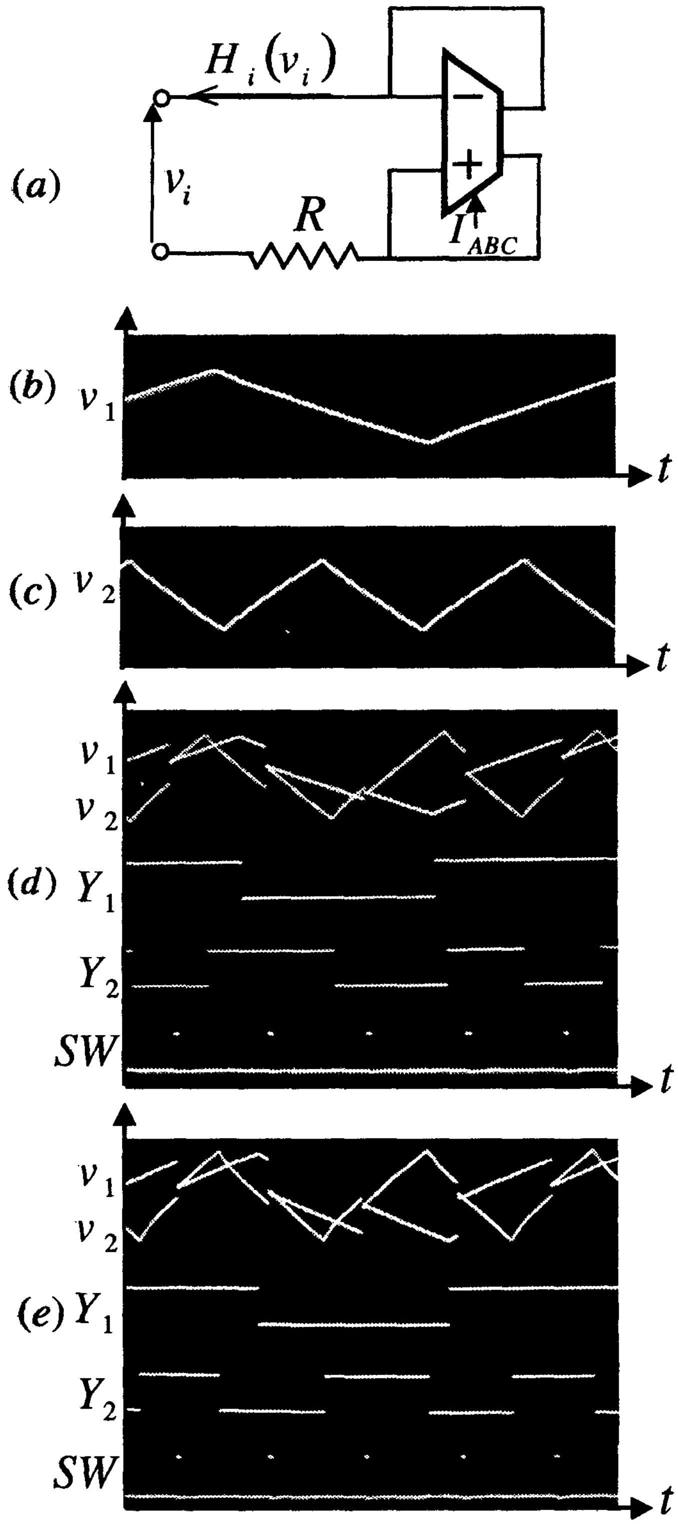
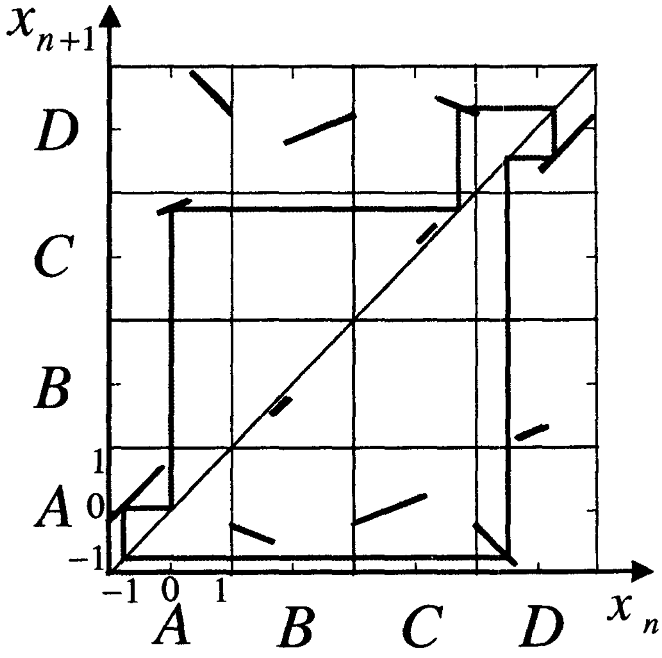



Figure 2. Basic laboratory measurement ($E \simeq 0.5[V], T \simeq 100[\mu sec], 0.5V/div, 0.5msec/div.$)
(a)Hysteresis VCCS.
(b)$Oscillator\#1$ $J_1 = 1[\mu A]$.
(c)$Oscillator\#2$ $J_2 = 2[\mu A]$.
(d)$C_1 = 0.01[\mu F]$, $C_2 = 0.01[\mu F]$.
(e)$C_1 = 0.01[\mu F]$, $C_2 = 0.033[\mu F]$.




| | $y_1$ | $y_2$ |
|---|---|---|
| A | 1 | 1 |
| B | 1 | −1 |
| C | −1 | 1 |
| D | −1 | −1 |

Figure 3. Hybrid return map

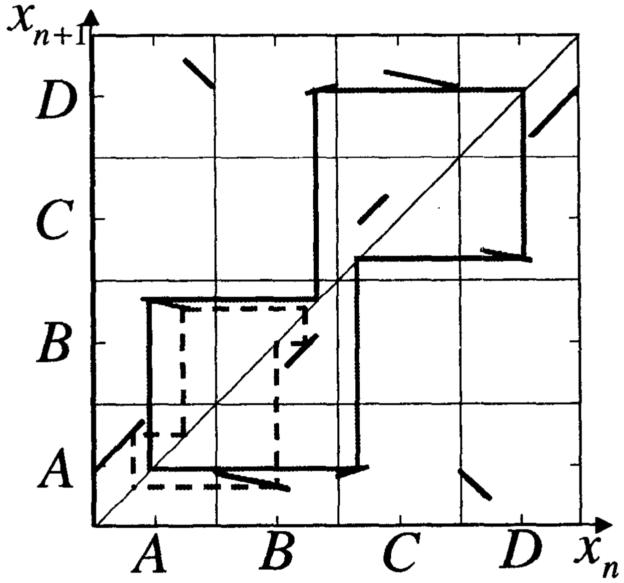



Figure 4. Co-existence ($s_1 = 0.4, s_2 = 1.185, \alpha = 0.3$)

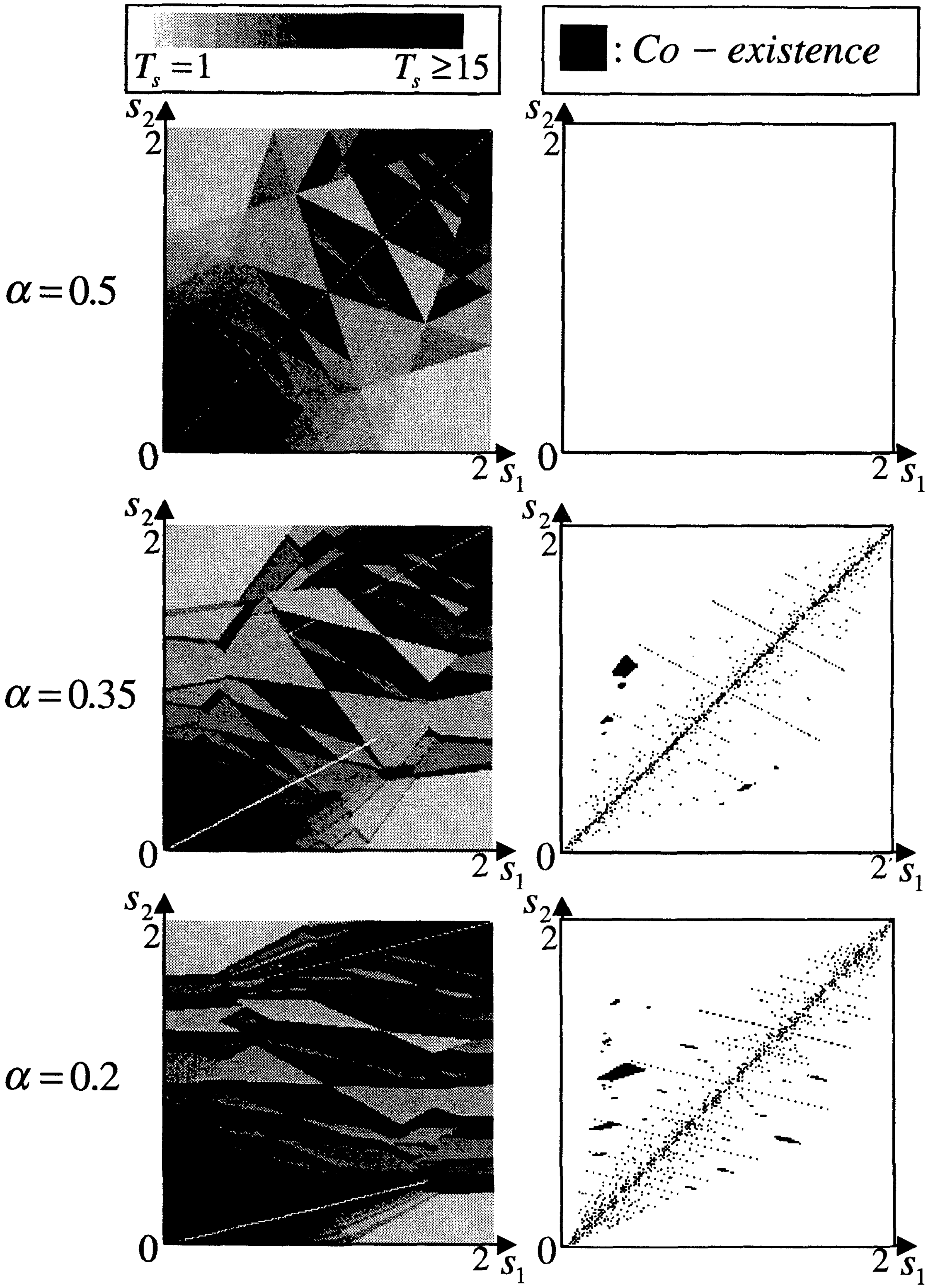



Figure 5. Left: Bifurcation diagrams,
Right: Region of Co-existence phenomena of synchronous with different periods.