The Achievable Performance of Unitary-ESPRIT Algorithm for DOA Estimation

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Abstract: In this paper, the accuracy of the direction-of-arrival (DOA) estimation of signal impinged on the uniform linear array (ULA) is investigated. The conventional beamformer and Capon's beamformer categorized in beamformaing techniques as well as MUSIC (MUltiple Signal Classification) and ESPRIT (Estimation of Signal Invariance Techniques) categorized in subspace-based methods are employed to estimate the DOAs. From the simulation result under uncorrelated environment, MUSIC can prominently distinguish the DOAs while the beamforming techniques cannot demonstrate the DOAs as clear as MUSIC does. Moreover, Unitary ESPRIT is employed to estimate the DOAs under uncorrelated signal conditions. By means of Unitary ESPRIT, the estimation has more accuracy with the computational-time reduction. In addition, it incorporates forward-backward averaging; thus Unitary ESPRIT can overcome the problem of the coherent signal condition.

1. Introduction

Currently, the demand for high performance in mobile and wireless communication has significantly been required. Adaptive or smart antenna introduced for the next generation for mobile communication has emerged as possible way to accomplish the requirement. By means of smart antenna, the increased capacity and quality of service are obtainable on account of reducing interference by spatial filter, sophisticated equalization as well as diversity techniques. Among smart antenna's benefits, the direction of arrival (DOA) estimation is exploited to improve performance of the antenna by controlling its directivity to decrease interference, delay spread and multipath fading [1].

Many DOA estimation algorithms have been studied and developed by a number of researchers [2-4]. However, one of the main categories of the parameter estimation techniques [5], spectral-based methods are well-receive by researchers. According to these techniques, some spectrum-like functions of the DOAs, which are located the highest peaks of the function correspond to the DOAs of the signals impinged on the antenna array, is formed. Spectral-based approaches can be classified by Krim and Viberg into beamforming techniques and subspace-based methods.

In this paper, the DOA estimation of signal impinged on the uniform linear array (ULA) is investigated. Two beam forming techniques, the conventional beamformer and Capon's beamformer, and two subspace-based methods, MUSIC and ESPRIT, are employed to estimate the DOAs under the uncorrelated signal environment. Nevertheless, ESPRIT is much emphasized in this paper because of its robustness and computational efficiency, especially Unitary-ESPRIT [6]. For using Unitary-ESPRIT, it can reduce the computational complexity of the standard ESPRIT algorithm by employing real-valued computations from the beginning to the end. Furthermore, since Unitary ESPRIT incorporates forward-backward averaging, it conquers the problem of coherent signal sources. In addition, it can be used to estimate the number of sources impinged on the antenna array as well.

2. Signal Model

For a uniform linear array (ULA) geometry, if there are M signals impinged on an L-element array ($M < L$) from individual DOAs θ_1, θ_2, ..., θ_M defined counter-clockwise relative the x-axis, the output vector of the array antenna takes the form [5]

$$\mathbf{x}(t) = \sum_{m=1}^{M} \mathbf{a}(\theta_m) s_m(t), \tag{1}$$

where $s_m(t)$, $m = 1, 2, \ldots, M$ denotes the baseband signal waveforms and $a(\theta_m)$ is the steering vector of a single signal at DOA θ_m. The ULA steering vector can be written as

$$\mathbf{a}(\theta) = [1 \;\; e^{-jkd\cos\theta} \ldots \;\; e^{-j(L-1)kd\cos\theta}]^T, \tag{2}$$

where k is the wave number and can write $k = 2\pi/\lambda$, λ is the wavelength and d is the inter-element distance.

Assume that there are N snapshots $\mathbf{x}(1)$, $\mathbf{x}(2)$, ..., $\mathbf{x}(N)$. The compact form of the output vector including an additive noise can be written as [5], [7]

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t), \; t = 1, 2, \ldots, N, \tag{3}$$

where $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_M)]_{L\times M}$ is a steering matrix, $\mathbf{s}(t) = [s_1(t), \ldots, s_M(t)]^T$ is an M-signal vector, $\mathbf{n}(t) = [n_1(t), \ldots, n_L(t)]^T$ is an additive noise vector.

3. The ESPRIT Algorithm

ESPRIT [4] is a computationally efficient and robust method for DOA estimation. In this method, the L elements of the receiving array are divided into two identical overlapping subarrays, each of which consists of the $L - 1$ elements sensor doublets displaced by a known constant displacement vector Δ that sets the reference direction and all angles are measured with reference to this vector. The magnitude of Δ is given by Δ_0 measured in wavelengths. Let the output of each subarray is denoted by $\mathbf{x}(t)$ and $\mathbf{y}(t)$. These two outputs can be written as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n_x}(t), \quad (4)$$
$$\mathbf{y}(t) = \mathbf{A}\mathbf{\Phi}\mathbf{s}(t) + \mathbf{n_y}(t), \quad (5)$$

where $\mathbf{s}(t)$ denotes the M source signals observed at a reference element; $\mathbf{n_x}(t)$ and $\mathbf{n_y}(t)$ denote the noise induced on the elements of the two subarrays and $\mathbf{A}$ denotes a $(L-1) \times M$ matrix, with its columns denoting the M steering vectors corresponding to M directional sources associated with the first subarray. The steering vectors corresponding to M directional sources associated with the second subarray are given by $\mathbf{A\Phi}$ where $\mathbf{\Phi}$ is an $M \times M$ diagonal matrix, with its m th diagonal element given by $e^{j2\pi\Delta_0 \cos\theta_m}$, $m = 1, \ldots, M$.

Let $\mathbf{U_x}$ and $\mathbf{U_y}$ denote two $(L-1) \times M$ matrixes with their columns denoting the M eigenvectors corresponding to the largest eigenvalues of the two array covariance matrixes $\mathbf{R_{xx}}$ and $\mathbf{R_{yy}}$, respectively. As these two sets of eigenvectors span the same M-dimensional signal space, it follows that these two matrixes $\mathbf{U_x}$ and $\mathbf{U_y}$ are related by a unique nonsingular transformation matrix $\mathbf{\Psi}$, we get

$$\mathbf{U_y} = \mathbf{U_x}\mathbf{\Psi}. \quad (6)$$

Similarly, these matrixes are related to steering vector matrixes $\mathbf{A}$ and $\mathbf{A\Phi}$ by a unique nonsingular transformation matrix , that is

$$\mathbf{U_x} = \mathbf{AT}, \quad (7)$$
$$\mathbf{U_y} = \mathbf{A\Phi T}. \quad (8)$$

Substituting for $\mathbf{U_x}$ and $\mathbf{U_y}$, and the fact that $\mathbf{A}$ is of full rank yields

$$\mathbf{T\Psi T^{-1}} = \mathbf{\Phi}, \quad (9)$$

which states that the eigenvalues of $\mathbf{\Psi}$ are equal to the diagonal elements of $\mathbf{\Phi}$ and that the columns of $\mathbf{T}$ are eigenvectors of $\mathbf{\Psi}$. This is the main relationship in the development of ESPRIT.

An eigendecomposition of $\mathbf{\Psi}$ gives its eigenvalues, and by equating them to $\mathbf{\Phi}$ leads to the DOA estimation, so we get

$$\theta_m = \arccos\left\{\frac{\arg(\lambda_m)}{2\pi\Delta_0}\right\},\ m = 1, \ldots, M. \quad (10)$$

The ESPRIT algorithm has appeared in two versions in the literature. The first is termed least squares (LS) ESPRIT and was introduced in [8]. And the second is termed total least squares (TLS) ESPRIT [9].

4. The Unitary ESPRIT Algorithm

Since the standard ESPRIT algorithm operates on complex baseband data, complex computation has to be executed all over the entire algorithm. In order to reduce the computational complexity, Unitary ESPRIT [6] algorithm has been developed. It transforms the input data matrix $\mathbf{X}$ to a real-valued representation. Therefore, all computations are real-valued.

This transformation $\mathcal{T}$ is applicable to any centro-symmetric antenna array configuration. Then $\mathbf{X}$ becomes centro-hermitian and by means of mapping $\mathbf{X}$ to its real valued equivalent Z with the help of so-called left $\mathbf{\Pi}$-real transformation matrices $\mathbf{Q}$, the real-valued data matrix can be written as

$$\mathcal{T}(\mathbf{X}) = \mathbf{Z} = \mathbf{Q}_L^H[\mathbf{X}\Pi_L\bar{\mathbf{X}}\Pi_N]\mathbf{Q}_{2N} \in \mathbf{R}^{L\times 2N}, \quad (11)$$

where Π_L is the $L \times L$ matrix with ones on its antidiagonal and zeros elsewhere,

$$\mathbf{Q}_{2n} = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_n & j\mathbf{I}_n \\ \Pi_n & -j\Pi_n\end{bmatrix} \quad (12)$$

and

$$\mathbf{Q}_{2n+1} = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_n & 0 & j\mathbf{I}_n \\ 0^T & \sqrt{2} & 0^T \\ \Pi_n & 0 & -j\Pi_n\end{bmatrix} \quad (13)$$

are, for instance, the left Π-real unitary matrices of even and odd order, respecively. $\mathbf{I}_n$ is the $n \times n$ identity matrix, $(\cdot)^H$ and $(\bar{\cdot})$ denote the Hermitian transpose and complex conjugate, respectively.

Furthermore, an incorporated forward-backward averaging effectively doubles the number of columns and therefore improves resolution capability. As a result, two coherent or highly correlated signals can be resolved.

Similar to standard ESPRIT, the real-valued signal subspace can be computed via an eigen-decomposition of the real-valued covariance matrix estimation. Therefore, the $\mathbf{E}_S \in \mathbf{R}^{L\times M}$ denotes the M principal eigenvector of

$$\mathcal{T}(\mathbf{X})\mathcal{T}(\mathbf{X})^H \in \mathbf{R}^{L\times L}. \quad (14)$$

The Unitary ESPRIT algorithm is based on the solution of the real-valued representation of he invariance equation

$$\mathbf{K_1 E}_S\mathbf{\Upsilon} \approx \mathbf{K_2 E}_S \in \mathbf{R}^{L-1\times M}, \quad (15)$$

by means of least squares (or total lease squares) techniques. The selection matrices $\mathbf{K_1}$ and $\mathbf{K_2}$ can be written as

$$\mathbf{K_1} = \mathbf{Q}_l^H(\mathbf{J_1} + \Pi_l\mathbf{J_1}\Pi_L)\mathbf{Q_L}, \quad (16)$$
$$\mathbf{K_2} = \mathbf{Q}_l^H j(\mathbf{J_1} - \Pi_L\mathbf{J_1}\Pi_L)\mathbf{Q_L}, \quad (17)$$

where $\mathbf{J_1}$ is the $l \times L$ matrix selecting the first l $(l < L)$ rows of an arbitrary matrix with the vertical dimension L.

The eigen-decomposition of the obtained real-valued matrix Υ can be written as

$$\mathbf{\Upsilon} = \mathbf{T\Omega T}^{-1} \in \mathbf{R}^{M\times M}, \quad (18)$$

where Ω is the $M \times M$ diagonal matrix of eigenvalues

$$\Omega = \text{diag}\{\omega_i\}_{i=1}^{M}. \tag{19}$$

Then, the directions of arrival can be obtained from

$$\mu_i = 2\arctan\omega_i,\ i = 1,\ 2,\ \ldots,\ M. \tag{20}$$

Namely, Υ and Ω are related through a transformation and the desired DOAs can be computed straightforward from the eigenvalues of Υ.

5. Simulation Results

A comparison between conventional beamformer, Capon's beamformer and MUSIC algorithm is shown in Fig. 1. Considered under the condition of two uncorrelated signals of equal power with SNRs of 10 dB impinged on the 8-element uniform linear array at -10 and 10 degrees, the simulations illustrate that MUSIC can clearly distinguish the peaks that locate the DOAs compared with the other two approaches.

Fig. 2 shows the DOAs compared between the standard ESPRIT and Unitary ESPRIT when two uncorrelated signals of equal power with SNRs of 10 dB impinge on the 8-element uniform linear array at 20 and 30 degrees. Although both methods can find the peaks locating the DOAs, Unitary ESPRIT reduces the computational time consumption and its results are more accurate than the standard ESPRIT.

Fig. 3 demonstrates the DOAs compared between the standard ESPRIT and Unitary ESPRIT under the coherent signal environment. In this case, two coherent signals of equal power with SNRs of 10 dB arrive at the 8-element uniform linear array at 20 and 30 degrees. From Fig. 3, it is found that the coherent signals can be clearly distinguish by using Unitary ESPRIT, while we cannot find the exact peaks of each signal by using the standard ESPRIT.

6. Conclusions

In this paper, we propose the direction finding of signals impinged on the uniform linear array antenna using beamforming techniques and subspace-based methods under uncorrelated and coherent signal conditions. Nevertheless, we have much focused on Unitary ESPRIT algorithm that is greatly effective in terms of the computational time consumption and the estimation accuracy. From the simulation results, we can see that under the uncorrelated source environment, MUSIC can prominently distinguish the DOAs compared with the beamforming techniques. Moreover, Unitary ESPRIT is employed to estimate the DOAs under uncorrelated and coherent signal conditions. By means of Unitary ESPRIT, the estimation is more accurate with the computational-time reduction. In addition, it incorporates forward-backward averaging; thus Unitary ESPRIT can overcome the problem of the coherent signal condition.

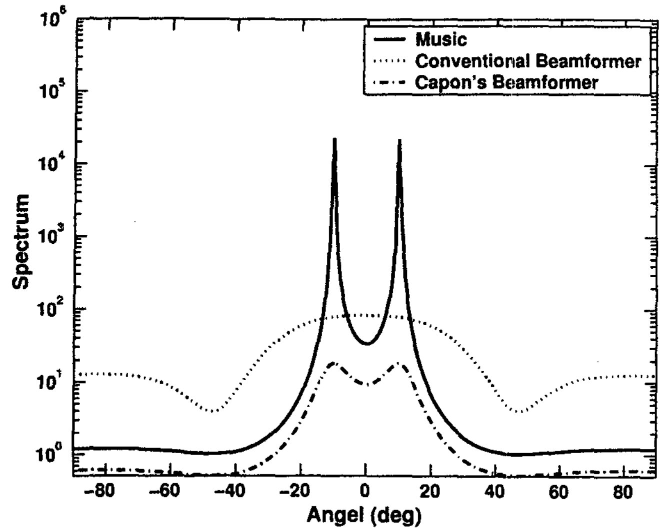

Figure 1. Spatial spectrum compared between conventional beamformer, Capon's beamformer and MUSIC under uncorrelated signal environment.

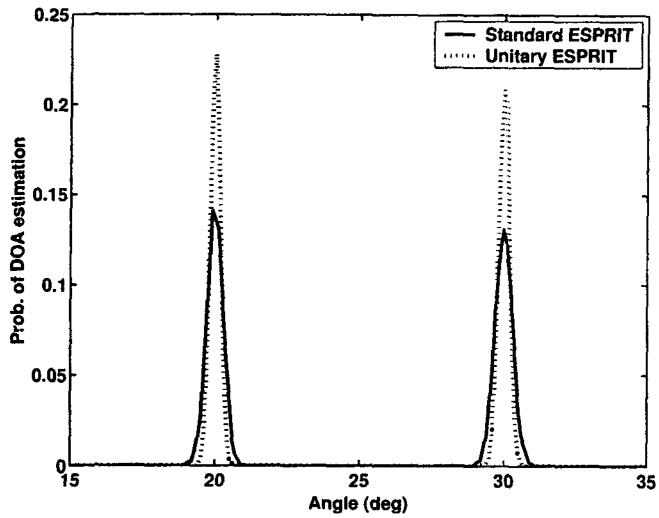

Figure 2. The DOA estimation using the standard ESPRIT and Unitary ESPRIT under uncorrelated signal environment.

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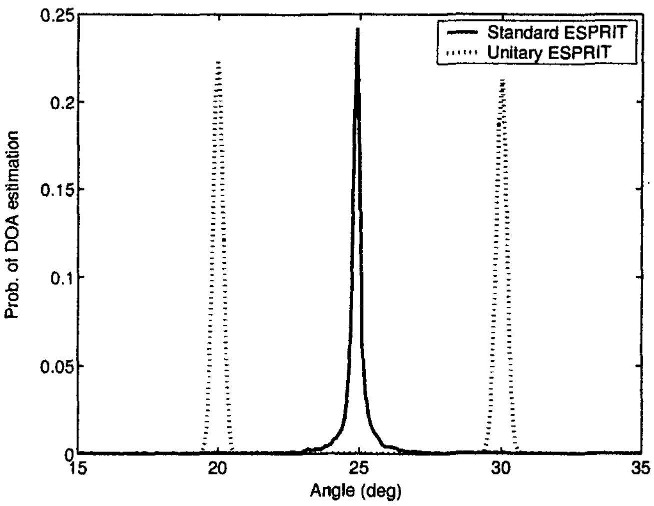

Figure 3. The DOA estimation using the standard ESPRIT and Unitary ESPRIT under coherent signal environment.

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