FUZZY SLIDING MODE ITERATIVE LEARNING CONTROL OF A MANIPULATOR

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Abstract: In this paper, a new scheme of iterative learning control of a robot manipulator is presented. The proposed method uses a fuzzy sliding mode controller(FSMC), which is designed based on the similarity between the fuzzy logic control(FLC) and the sliding mode control(SMC), for the feedback. With this, the proposed method makes possible for fast iteration and has advantages that no linear approximation is used for the derivation of the learning law or in the stability proof. Full proof of the convergence of the fuzzy sliding base learning scheme is given.

Keywords: Learning control, iterative, manipulator, sliding mode, fuzzy logic control

1. Introduction

It is well known that the effects of unstructured uncertainty such as friction acting at the joints of most present-day manipulators are significant. Friction has a nonlinear dependence on joint velocity and position. Unfortunately, adaptive control schemes cannot achieve zero tracking error due to the unstructured uncertainties.

To reduce the trajectory following error, and to improve the performance of the device being controlled, a new concept of learning control has been proposed in [9]. The idea behind learning control is that uncertainties are compensated by an iterative correction term derived from the tracking error of previous trials with a suitable learning law. Thus, learning control is applicable only in cases where a certain motion is performed over and over repetitively. A practical observation is that most current robots are used in applications in which the trajectory is repeated, and a learning scheme is valuable. Because such schemes are advantageous when the system is difficult to model or has unknown structure, the learning control of robot manipulators has assumed considerable research significance.

A class of learning algorithms was proposed in [1,4,5]. One feature of these algorithms is that the learning term is based on tracking error only without consideration of the feedback control. To ensure stability, a small learning gain has to be used in these algorithms. In [2], the structure of the feedback controller was used to construct the learning term and a high gain was used to ensure the tracking error remained within a uniform bound. However, acceleration measurement is required by this algorithm. An improved scheme without the use of acceleration terms was presented in [6], involving linear approximations. However, the use of linear approximations presents a difficulty for robustness analysis and restricts the learning gain. Moreover, the convergent conditions for these algorithms are hard to test.

In this paper, a new scheme of iterative learning control algorithms for trajectory following control of robot manipulators is presented. Fuzzy sliding mode control is considered for the design of an iterative learning control scheme. The algorithm is globally convergent in the presence of disturbances and modelling uncertainties. With the robustness of the algorithm, a large gain for learning control can be used to achieve fast convergence of tracking errors.

The new learning algorithms are unique, with advantages in that: (i) a fuzzy sliding mode controller is used for the feedback control, (ii) no linear approximation is used for the derivation of the learning law or in the stability proof, (iii) measurement of manipulator accelerations is avoided, and only position error and velocity error measurements are required,

The structure of the control scheme proposed in this paper is similar to those in [2] and [6] in that a part of feedback control is used to construct the learning control. However, a fuzzy sliding mode controller is used for the feedback control, and thus no linear approximation is used for the derivation of the learning law. By partitioning the dynamics of the plant as an unknown but determinate term and a transient term, it is shown that the robustness of the learning scheme is guaranteed by the sliding mode control law, and a large learning gain can be used to achieve fast convergence for tracking errors.

Note that the stability of a learning control algorithm depends on the feedback controller. Since learning algorithms usually require tens of iterations to obtain reasonably small tracking errors, it is important to design a feedback controller which achieves reasonably good performance alone. Therefore, an advanced feedback controller which supplies a robust coarse control should be combined with a learning algorithm to achieve robust high performance control of robot manipulators. Thus in this paper, a fuzzy sliding mode algorithm of manipulators is used to achieve an efficient learning control scheme for robot manipulators.

2. Learning Control Scheme

Suppose that a finite-dimentional dynamical system is given, and consider a situation that given a desired ouput $y_d(t)$ over a finite-time duration $[0, T]$. Generally, the control problem is to find a control input $u^*(t)$, which exites the system and eventually produces an output $y^*(t)$ so that $y^*(t)$ must be coincident with $y_d(t)$ over $t \in [0, T]$. If a full description of the system is available, it may be possible to construct such a control input on the basis of the system. However, in practical situations, there exits structured and unstructured uncertainties on the modelling. In such a situation many researchers suggested so called an iterative learning control scheme. Figure 1 is shown the very basic scheme of an iterative learning suggested by Arimoto[1]. This learning control method can be described by a simple iterative rule of input modification defined as

$$u^{i+1}(t) = u^i(t) + \left(\phi + \Gamma\frac{d}{dt}\right)e^i(t) \qquad (1)$$

Convergency conditions on the rule (1) is show in detail in [1].

3. Fuzzy Sliding Mode Learning Control

In this section, we propose a fuzzy sliding mode learning control algorithm for robot manipulators in joint space.

3.1 Problem Formulation

Consider the rigid body dynamic n-link manipulator equations [3],

$$\begin{aligned} M(q)\ddot{q} + h(q, \dot{q}) + \tau_d(t) &= \tau(t) \\ h(q, \dot{q}) &= V(q, \dot{q})\dot{q} + F\dot{q} + G(q) \end{aligned} \qquad (2)$$

Generally, the parameters of robot manipulators are not known exactly, and are time varying. Let the nominal or assumed values be $\hat{M}(q)$, $\hat{h}(q, \dot{q})$. Then, we can write

$$\begin{aligned} \tilde{M}(q) &= M(q) - \hat{M}(q) \\ \tilde{h}(q, \dot{q}) &= h(q, \dot{q}) - \hat{h}(q, \dot{q}) \end{aligned} \qquad (3)$$

where $\tilde{(\cdot)}$ is the error between the true and the nominal (or assumed) value.

The control problem for robot manipulators is to synthesize a control law for the torques such that the joint output $q(t) \in R^n$, traces the desired trajectory, $q_d(t) \in R^n$, with a certain precision defined by

$$|q_d - q| \le \gamma_1, \ |\dot{q}_d - \dot{q}| \le \gamma_2, \ \gamma_1 > 0, \ \gamma_2 > 0 \qquad (4)$$

It is assumed that $q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ are well defined and bounded for all operational time t.

Define

$$e_q = q_d - q, z_q = \dot{e}_q + \wedge e_q \qquad (5)$$

with $\wedge = diag(\wedge_1, \wedge_2, \ldots, \wedge_n), \wedge_i > 0$.

Then, from (5),

$$M\dot{z}_q = M(\wedge\dot{e}_q + \ddot{q}_d) + h + \tau_d - \tau \qquad (7)$$

Now, we define following lemma for further development of algorithms.

Lemma 1 Suppose that $M > 0$ is a bounded differentiable matrix function of q and τ_d is bounded on $\dot{q}$. Then, the following statements hold.

(a) There exists constant $\eta > 0$ such that

$$z_q^T[M(\wedge\dot{e}_q + \ddot{q}_d) + h + \tau_d] + \frac{1}{2}z_q^T\dot{M}z_q \le \phi\eta|z_q|, \forall q, \dot{q} \qquad (8)$$

$$\text{with } \phi = 1 + \|z_q\| + |z_q\|^2 \qquad (9)$$

(b) There exist bounded nonlinear functions $\psi_1(t), \psi_2(t) \in R$ and differentiable functions $\theta(t) \in R^n$ such that

$$M(\wedge\dot{e}_q + \ddot{q}_d) + h + \tau_d = \theta(t) + \psi_1(t)z_q + \psi_2(t)\|z_q\|z_q, \forall q, \dot{q} \qquad (10)$$

Proof: see [7]

3.2 The Learning Algorithm

The learning controller is to compute the control input torque for the i-th iteration as follows:

$$\tau^i = u^i + \xi^i \qquad (11)$$

where τ^i is the input torque at the i-th iteration. ξ^i is the learning term generated by

$$\begin{aligned} \xi^{i+1} &= \xi^i + xu_*^i, \quad 0 < x < 2(1-\varepsilon), \ \xi^i(0) = 0, \ \forall i \\ u_* &= \text{Output of Fuzzy Sliding Mode Controller} \end{aligned} \qquad (12)$$

The structure of the proposed scheme is shown in Figure 1.

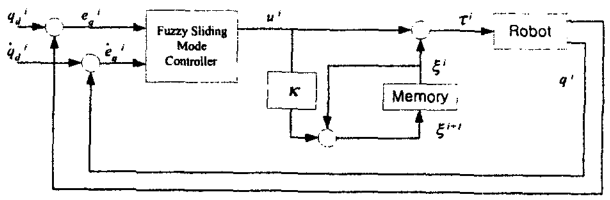


Figure 1. Structure of Fuzzy learining control

3.3 The Fuzzy Sliding Mode Algorithm

The output of FSMC of i-th iteration, u_*^i, in (12) is obtained from a sliding mode based fuzzy logic *controller to calculate control input,* τ, *to solve the* control problem (2). A possible sliding mode control(SMC) law to calculate control input, τ, to solve the control problem (2) can be defined by following sliding mode control law [7]

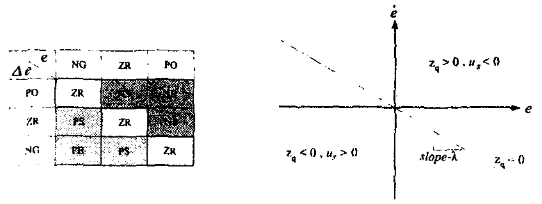
$$u=\begin{cases}\phi\eta_k\frac{z_q}{\|z_q\|}, & \text{if}\|z_q\|>\varepsilon \\ & ;\eta_k>0,\ 1>\varepsilon>0 \\ \phi\eta_k\frac{z_q}{\varepsilon}, & \text{otherwis}\end{cases} \tag{13}$$

with ϕ defined by (9).

We see that the basic concept of the SMC rule of (13) is: "*If the error is negative, push hard enough in the positive direction (and conversely)*". Therefore, the following properties on the FLC can be inferred qualitatively in the error-phase plane:

$$u_*=\begin{cases}u_{fuzzy}(e,\dot{e})_{s=0} & \approx 0 \\ u_{fuzzy}(e,\dot{e})_{s>0} & <0 \\ u_{fuzzy}(e,\dot{e})_{s<0} & >0\end{cases} \tag{14}$$

Then control rules are designed to assign a fuzzy set of the control input u for each combination of fuzzy sets of the e and Δe depending on $z_q=\Delta e+\lambda e$. Fig. 5(a) shows one of possible control rules. The similarity between the FLC and the SMC is shown in Figure2(b).



(a) (b)

Figure2. (a) The rule base of FLC (b) The output characteristics of a SMC.

3.4 Convergency Rule of the Learning Control

Consider the following performance index for an operating time interval $t\in[0,t_f]$

$$V^i(t):=\int_0^t z_q^{i\top}M^{-1}z_q^i d\tau,\ \forall t\in[0,t_f] \tag{15}$$

Theorem 1: Consider the control law (11) and the learning law (12). Suppose that $e^i(0)=0,\ \forall i.$ If η_k and ε are chosen so that $\eta_k>\eta$ and $0<\varepsilon<1$, then we have that

(a) $V^{i+1}(t)\le V^i(t)$,

(b) $\lim_{t\to\infty} z_q^i(t)=0$ and

(c) $\lim_{t\to\infty}\xi^i(t)=\theta(t),\ \forall t\in[0,t_f]$

Proof: From (7), we have that

$$M\dot{z}_q=M(\wedge\dot{e}_q+\ddot{q}_d)+h+\tau_d-u^i-\xi^i \tag{16}$$

From Lemma 1, and with the given sliding mode control law, it is easily checked that $\|z^i\|\le\varepsilon,\ \forall t\in[0,t_f]$, with the initial error $e^i(0)=0$.

Thus, in this case, from (13), we have that

$$u^i=\phi\eta_k\frac{1}{\varepsilon}z_q^i=(\phi\eta_k\frac{1-\varepsilon}{\varepsilon}+\phi\eta_k)z_q^i \tag{18}$$

Let,

$$\hat{u}=\phi\eta_k\frac{1-\varepsilon}{\varepsilon},\quad \bar{u}=\hat{u}+\phi\eta_k \tag{19}$$

Then from (13) and (16), we see that

$$u^i=(\hat{u}+\phi\eta_k)z_q^i=\bar{u}z_q^i \tag{20}$$

and $\hat{u}>0,\ \bar{u}>0$.

From (16) and (20), it follows that

$$\begin{aligned}M\dot{z}_q^i &= \psi z_q^i+\theta-\phi\eta_k\frac{1}{\varepsilon}z_q^i-\xi^i \\ &= (\psi-\phi\eta_k)z_q^i-\phi\eta_k\frac{1-\varepsilon}{\varepsilon}z_q^i-\xi^i+\theta \\ &= -(b+\hat{u})z_q^i-\xi^i+\theta\end{aligned} \tag{21}$$

where $b=\phi\eta_k-\psi$. Note that $\eta_k>\eta$. From Lemma 1, it can easily be seen that $\phi\eta\ge\psi$. Therefore, we see that $\phi\eta_k>\phi\eta\ge\psi$, hence b > 0 .

Denote $\delta z_q^i=z_q^{i+1}-z_q^i$. From (21) and the learning law (12), we have

$$M(\dot{z}_q^{i+1}-\dot{z}_q^i)=-(b+\hat{u})(z_q^{i+1}-z_q^i)-(\xi^{i+1}-\xi^i) \tag{22}$$

Or, $M\delta\dot{z}_q^i=-(b+\hat{u})\delta z_q^i-xu^i$

From, $u^i=\bar{u}z^i$, it follows that

$$z_q^i=\frac{-[M\delta\dot{z}_q^i+(b+\hat{u})\delta z_q^i]}{x\bar{u}} \tag{23}$$

Now, we obtain

$$\begin{aligned}\Delta V^{i(t)} &= V^{i+1}(t)-V^i(t) \\ &= \int_0^t(z_q^{i+1\top}M^{-1}z_q^{i+1}-z_q^{i\top}M^{-1}z_q^i)d\tau \\ &= \int_0^t(\delta z_q^{i\top}M^{-1}\delta z_q^i-2\delta z_q^{i\top}M^{-1}z_q^i)d\tau\end{aligned} \tag{24}$$

Substitution of z_q^i and integration by parts yields

$$\begin{aligned}\Delta V^i &= \int_0^t\delta z_q^{i\top}M^{-1}\delta z_q^i d\tau \\ &\quad -\int_0^t\frac{2}{x\bar{u}}\delta z_q^{i\top}M^{-1}[M\delta\dot{z}_q^i+(b+\hat{u})\delta z_q^i]d\tau \\ &= -\int_0^t(\frac{2b^i}{x\bar{u}}+\frac{2(1-\varepsilon)}{x}-1)\delta z_q^{i\top}M^{-1}\delta z_q^i d\tau \\ &\quad -\frac{1}{x\bar{u}}\delta z_q^{i\top}\delta z_q^i mid^{t_0}\end{aligned} \tag{25}$$

Since $0<x<2(1-\varepsilon)$, $\Delta V^i(t)\le 0$ for any $\|z_q^i(t)\|\ge\|z_q^i(0)\|$. Note that $M^{-1}>0,\ b>0,\ \hat{u}>0$ and $\bar{u}>0$. Now, (a) follows and the equality holds only when $\delta z_q^i=0,\ \forall t.$ Finally, Delta $V^i(t)\le 0$ implies that δz_q^i vanishes as $i\to\infty\,\forall t\in[0,t_f]$ and, thus, $\lim_{i\to\infty}\delta\dot{z}_i(t)=0,\ \forall t.$ This, in turn, implies that $\lim_{i\to\infty}\dot{z}_q^i=\lim_{i\to\infty}z_q^i=0,\ \forall t$. Thus, (b) holds and (c) follows from (21).

4. Simulation Results

A simple two-link robot manipulator described in [3] has been simulated to test the proposed learning control law. The manipulator was modeled as a set of nonlinear coupled differential equations given as

$$\begin{aligned}\tau_1 &= m_2 l_2^2(\ddot{q}_1+\ddot{q}_2)+m_2 l_1 l_2 c_2(2\ddot{q}_1+\ddot{q}_2)+\\ &\quad (m_1+m_2)l_2^1\ddot{q}_1-m_2 l_1 l_2 s_2\dot{q}_2^2-2m_2 l_1 l_2 s_2\dot{q}_1\dot{q}_2\\ &\quad +m_2 l_2 g s_{12}+(m_1+m_2)l_1 g s_1\\ \tau_2 &= m_2 l_2^2(\ddot{q}_1+\ddot{q}_2)+m_2 l_1 l_2 c_2\ddot{q}_1+m_2 l_1 l_2 s_2\dot{q}_1^2+m_2 l_2 g s_{12}\end{aligned} \tag{26}$$

where $c_1 := \cos(q_1)$, $s_{12} := \sin(q_1+q_2)$.

The fuzzy sliding mode learning algorithm of Theorem 1 was used for this simulation. The desired trajectories, disturbance τ_d, and the values of the manipulator parameters were same as in Section ref{sec:adx}, they were

$$\begin{bmatrix} q_{d1} \\ q_{d2} \end{bmatrix} = \begin{bmatrix} 1+0.2\sin(\pi t) \\ 1-0.2\cos(\pi t) \end{bmatrix}, \quad \tau_d = 5\begin{bmatrix} \sin(4\pi t) \\ \sin(4\pi t) \end{bmatrix}, \quad \text{for } t\in[0,3]$$

$l_1=l_2=1m$, $m_1=1kg$ and m_2 was changed from $1kg$ to $3kg$ at $t=1\mathrm{sec}$. The sampling time is set to be 10^{-3}sec. In this simulaiton, for fuzzification, the number of input membership function for error e and Δe are chosen three respectively, and the ouput membership function is chosen five. The input fuzzy set is designed as $e\in[-0.2, 0.2]$ and $\Delta e\in[0, 50]$, and for the output fuzzy set $u_*\in[0, 50]$.

The plant initial states were set as

$$q_{d1}=1, q_{d2}=0.8, \dot{q}_{d1}=0.2\times\pi, \dot{q}_{d2}=0$$

Figure 3 show position errors under the FSMLC. In the first iteration, large position errors (Figure 3(a)) were seen. In the second iteration, Figure 3(b), it can be seen that both position errors were decreased to $|e|<1.5\times10^{-3}, \forall t$. In the third iteration, Figure 3(c), the tracking errors have further reduced and achieved $|e|<2.5\times10^{-4}, \forall t$. From the simulation results, we can see that position were reduced rapidly for all t in repeated iterations, which demonstrated that the proposed algorithm worked effectively when large parameter uncertainties and disturbances exist.

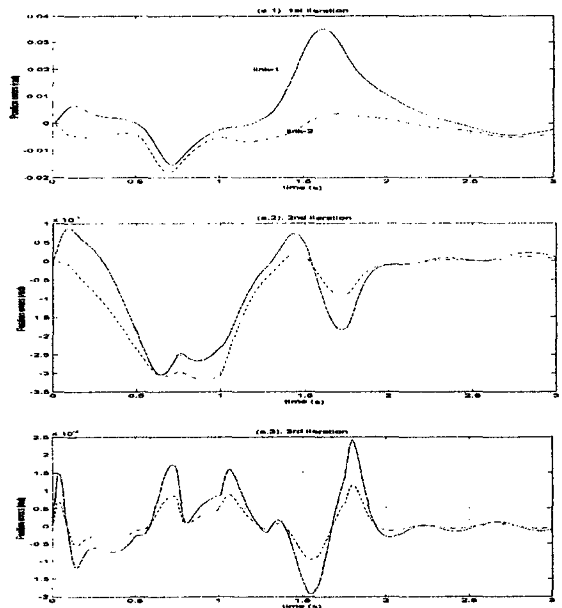


Figure 3. Simulation results under fuzzy sliding mode learning control.

5. Conclusions

In this paper, fuzzy sliding mode iterative learning control algorithms have been presented for the tracking control of manipulators in joint space. The iterative learning control algorithms were based on a sliding mode controller. The developed learning control algorithms were able to achieve nearly zero tracking error in the presence of disturbances and modelling uncertainties. The algorithms also have the advantages of modest computational requirements and fast convergence. Simulation results show robustness and fast convergence where large parameter uncertainties and disturbances exist.

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