A Consideration on Faster Convergence and Higher Reliability of The New Blind Equalization Algorithm using The Minimum Entropy Method

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ABSTRACT

The minimum entropy method is one of blind equalization method. A conventional algorithm using the minimum entropy method has two problems : slower convergence and lower reliability of recovered signals. We propose a new algorithm using the minimum entropy method for solving the two problems. Finally, we confirm the validity of the proposed algorithm through computer simulation.

1. INTRODUCTION

On communication system, blind equalization is a method of recovering transmitted signals from only received signals and a priori knowledge of transmitted signals. A conventional algorithm using minimum entropy method is one of blind equalization algorithms. The algorithm has two problems: slower convergence and lower reliability. In this paper, we propose a new algorithm using the minimum entropy method for solving the two problems above. The proposed algorithm is as follows.

1. It is based on the adaptive processing with only recived signals and recovered signals.
2. Kurtosis is estimated adaptively by recovered signals with each smple.
3. Cost function is decided by kurtosis.
4. Transmitted signals are recovered by received signals with each sample using decided cost function

The rest of this paper is organized as follows : In section 2, we present the conventional algorithm using the minimum entropy method. In section 3, we propose the new algorithm using the minimum entropy method. In section 4, we confirm the new algorithm though computer simulation. In section 5, we summarize our findings.

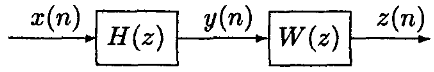

Fig.1 : Transmission System

2. A CONVENTIONAL ALGORITHM

In this section, we present the conventional algorithm using the minimum entropy method. First, we present a general transmission system of Fig.1.

In Fig.1, we assume the transmitted signals $x(n)$ are IID and that the noise is free, $y(n)$ are received signals, and $H(z)$ is a system of a channel. $z(n)$ are recovered signals, and $W(z)$ is a equalizer. In the minimum entropy method, the cost function C is as follows:

$$C = \frac{A}{E\left[|x(n)|^q\right]} \cdot E\left[\frac{1}{q}|z(n)|^q - \frac{1}{2}r_q z(n)^2\right] \quad (1)$$

where A is constant, q is constant, and

Table1 : The conventional algorithm

For time n,

1. Initialization,
   n=0
   $w_N^{(n)} = [0, \cdots, 0, 1, 0, \cdots, 0]^T$
   q is constant
2. Equalization algorithm
   $z(n) = w_N^{(n)T} y_N^{(n)}$
   $w_N^{(n+1)} = w_N^{(n)} - \alpha y_N^{(n)} z(n) \left[|z(n)|^{q-2} - r_q\right]$
   (q is constant)
   n=n+1
   go to 2.

Here, we call q degree number. The renewal equation for parameters vector w_N of $W(z)$ using eq.(1) is

Table2 The relation of degree number q and kurtosis S

Uniform	S	1.800					
Sub-Gaussian	q	3	4	5	6	7	8
	S	2.418	2.188	2.070	2.000	1.955	1.923
	q	9	10	11	12	13	14
	S	1.901	1.884	1.871	1.861	1.853	1.847
	q	15	16	17	18	19	20
	S	1.841	1.837	1.833	1.83	1.827	1.824
	q	21	22	23			
	S	1.822	1.821	1.820			

Table3 Degree number q_e and kurtosis S_e for recovered signals.

q_e	3	4	5	6
S_e	$2.418 < S_e$	$2.188 < S_e \leq 2.418$	$2.070 < S_e \leq 2.188$	$2.000 < S_e \leq 2.070$
q_e	7	8	9	10
S_e	$1.955 < S_e \leq 2.000$	$1.923 < S_e \leq 1.955$	$1.901 < S_e \leq 1.923$	$1.884 < S_e \leq 1.901$
q_e	11	12	13	14
S_e	$1.871 < S_e \leq 1.884$	$1.961 < S_e \leq 1.871$	$1.853 < S_e \leq 1.861$	$1.847 < S_e \leq 1.853$
q_e	15	16	17	18
S_e	$1.841 < S_e \leq 1.847$	$1.837 < S_e \leq 1.841$	$1.833 < S_e \leq 1.837$	$1.830 < S_e \leq 1.833$
q_e	19	20		
S_e	$1.827 < S_e \leq 1.830$	$S_e \leq 1.827$		

given as follows:

$$r_q = \frac{E[|x(n)|^q]}{E[x(n)^2]} \tag{2}$$

$$w_N^{(n+1)} = w_N^{(n)} - \alpha y_N^{(n)} z(n)(|z(n)|^{q-2} - r_q) \tag{3}$$

where α is step gain, $y_N^{(n)}$ received signals vector. Here, the vectors of $y_N^{(n)}$, $w_N^{(n)}$ are defined as

$$y_N^{(n)} = [y(n), y(n-1), \cdots, y(n-N+1)]^T \tag{4}$$

$$w_N^{(n)} = \left[w(0)^{(n)}, w(1)^{(n)}, \cdots, w(N-1)^{(n)}\right]^T \tag{5}$$

Table1 is the conventional algorithm. But the conventional algorithm has two problems : slower convergence and lower reliability of recovered signals.

3. A PROPOSAL OF THE ALGORITHM

In this section, we propose a new algorithm using the minimum entropy method for solving two problem (slower convergence and lower reliability) of the conventional algorithm. It is known that one is able to solve these problems by controlling degree number q of eq.(3) for a change of the probability density function of recovered signals by sample time(References[1]). But its algorithm is not realized because it is difficult to estimate the probability density function of recovered signals by sample time. In this paper, the probability density function of recovered signals is not used. The idea of the proposed algorithm is as follows:

First, kurtosis is estimated adaptively by each sample of recovered signals.

Second, degree number q is decided by kurtosis.

Last, transmitted signals are recovered by received signals with each sample using the renewal equation including estimated q .

For the estimation of q, kurtosis is used because the relation of kurtosis and probability density function is one-to-one, when the probability density function of recovered signals $P(z(n))$ is assumed by

$$P(z(n)) = K \exp\left[-\left|\frac{z(n)}{\mu}\right|^q\right] \tag{6}$$

Table4 The proposed algorithm

For time n

1. Initialization

   For $n = -m+1$ to 0

   $w_N^{(n)} = [0, \cdots, 0, 1, 0, \cdots, 0]^T$

   $z(n) = w_N^{(n)^T} y_N^{(n)}$

   next n

   $$S_4^{(0)} = \frac{1}{m} \sum_{n=-m+1}^{0} z(n)^4$$

   $$S_2^{(0)} = \frac{1}{m} \sum_{n=-m+1}^{0} z(n)^2$$

   $$S_e^{(0)} = \frac{S_4^{(0)}}{[S_2^{(0)}]^2}$$

2. Equalization algorithm

   $q = q_e \Leftarrow S_e^{(n)}$ (Look Table3)

   $w_N^{(n+1)} =$

   $w_N^{(n)} - \alpha y_N^{(n)} z(n) \left[|z(n)|^{q-2} - r_q\right]$

   $n = n+1$

   $z(n) = w_N^{(n)^T} y_N^{(n)}$

   $S_4^{(n)} = (1-d)S_4^{(n-1)} + dz(n)^4$

   $S_2^{(n)} = (1-d)S_2^{(n-1)} + dz(n)^2$

   $S_e^{(n)} = \frac{S_4^{(n)}}{[S_2^{(n)}]^2}$

   go to [2. Equalization algorithm]

where,

Super Gaussian distribution	:	$1 \leqq q < 2$
Gaussian distribution	:	$q = 2$
Sub Gaussian distribution	:	$2 < q < \infty$
uniform distribution	:	$q = \infty$

K is assumed constant.

μ is assumed constant.

And if q of eq.(6) is used for q of eq.(1) by sample time, the two problems of the conventional algorithm are solved. Next, Table.2 provides the relation of kurtosis S and degree number q. In the proposed algorithm, we give the Table3 based on Table2, and Table3 is used for the estimation of q. Here, q_e is the estimated degree number q for probability density function of recovered signals by sample time, and the estimated kurtosis $S_e^{(n)}$ of the recovered signals by sample time is defined as

$$S_e^{(n)} = \frac{\frac{1}{m} \sum_{i=n}^{n-m+1} z(i)^4}{\left[\frac{1}{m} \sum_{i=n}^{n-m+1} z(n)^2\right]^2} \quad (7)$$

where m is data length for estimation of $S_e^{(n)}$. Here,we give eq.(8),eq.(9) and eq(10) for adaptive estimation of $S_e^{(n)}$ based on eq.(7). $S_e^{(n)}$ia as follows:

$$S_e^{(n)} = \frac{S_4^{(n)}}{[S_2^{(n)}]^2} \quad (8)$$

where $n \geqq 0$

$S_4^{(n)}$ia as follows:

$$S_4^{(n)} = (1-d)S_4^{(n-1)} + dz(n)^4 \quad (9)$$

where

$$S_4^{(0)} = \frac{1}{m} \sum_{n=-m+1}^{0} z(n)^4$$

$$0 < d << 1$$

$S_2^{(n)}$ia as follows:

$$S_2^{(n)} = (1-d)S_2^{(n-1)} + dz(n)^2 \quad (10)$$

where

$$S_2^{(0)} = \frac{1}{m} \sum_{n=-m+1}^{0} z(n)^2$$

$$0 < d << 1$$

Table4 presents the proposed algorithm.

4. SIMULATION

In this section, we confirm the validity of the new algorithm through computer simulation. First, probability density function of transmitted signals series is assumed to have a uniform distribution. The channel is 5 degrees FIR type, and these parameters are presented in Table5. MSE is defined as:

$$MSE = 10\log_{10} \frac{\frac{1}{100} \sum_{n=100k}^{100k+99} (x(n) - z(n))^2}{\frac{1}{100} \sum_{n=0}^{99} (x(n) - z(n))^2} \quad (11)$$

where k is block number for calculation of MSE. Here, the computer simulation condition are as follows:

1. The conventional algorithm (q=4)
2. The conventional algorithm (q=9)
3. The proposed algorithm (d=0.01,m=1)

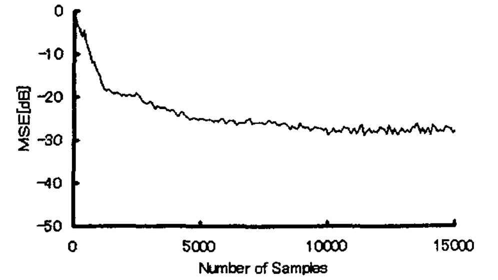

Fig.2 The conventional algorithm (q=4)

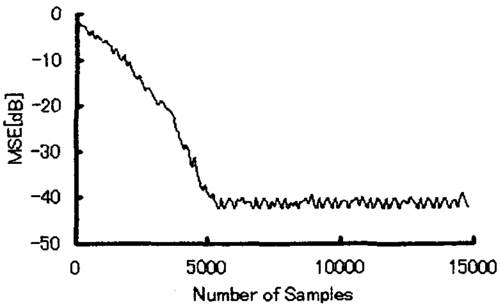

Fig.3 The conventional algorithm (q=9)

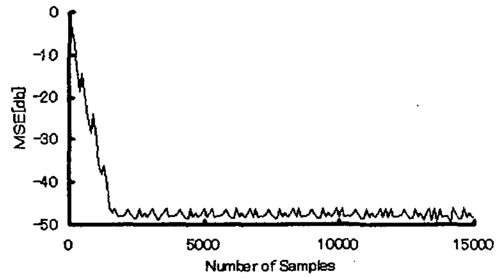

Fig.4 The proposal algorithm (d=0.01)

In Fig.2, Fig.3, and Fig.4, we confirm the validity of the proposed algorithm. Thus, the proposed algorithm solves the two problems (slower convergence and lower reliability) of the conventional algorithm.

Table5 Constants of channel.

h(0)	h(1)	h(2)
+0.1	-0.3	+1.0
h(3)	h(4)	
+0.4	-0.1	

5. CONCLUSION

We have proposed the new blind equalization algorithm using the minimum entropy method. As a result, we can confirm the validity of the new algorithm through computer simulation. The new algorithm solves the two problems of the conventional algorithm.

6. REFERENCES

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