

# Performance Evaluation of a Mobile Stratospheric Communication System on Measured Rician Log-Normal Fading Channel Models

Byeong-Gwon Kang

Dept. of Information Technology Engineering, Soonchunhyang University  
646 Eupnae-ri, Shinchang-myun, Asan-si, Choongnam 336-745, Korea  
(Tel) +82-41-530-1348, (Fax) +82-41-530-1568  
Email : bgkang@sch.ac.kr

**Abstract :** In recent years, there are growing concerns about land mobile satellite (LMS) communication systems and mobile stratospheric communication systems (SCS) for the purpose of service upgrade of personal and mobile communications in near future. It is important to possess accurate channel model for prediction of the above system performance. Thus, in this paper, we evaluate the bit error rates of a coded BPSK system based on realistic channel model which can be applied to stratospheric communication systems. The channel data was made by fitting the parameters of probability distribution model to measured data. This approach was proposed by Corraza[1] and modified by You[2]. And also the effects of channel codings on the system performance are analyzed. As results, we can get the performance curve characteristics on realistic Rician log-normal fading channels with various communication environments.

## 1. Introduction

Recently, much concerns are devoted to land mobile satellite(LMS) communication systems and mobile stratospheric communication systems(SCS) for the purpose of service expansion of personal communications such as cellular, PCS and IMT-2000 systems. The channel models of LMS and SCS systems can be described as Rician fading and log-normal shadowing statistics. It is necessary to have more realistic channel models for more exact performance evaluation of the above systems. In this paper, we use a very realistic channel data obtained from a method provided by Corraza and Batalaro[1]. They considered LEO constellations and proposed an approach, in which the parameters of the probability distribution model are described by empirical formulas to fit measured data. The resulting empirical probability distribution model fits measured data over a wide range of  $\alpha$ , and it may be used for performance evaluation such as bit error probability. However, they limited their applications to rural tree shadowed circumstances. Thus, in this paper, we use a modified Corazza model suggested by M. H. You[2] et al. and they derived some parameters for open area, rural area and urban areas with modified Corazza equations.

We evaluate bit error rates of a coded BPSK system based on the measured channel models and we consider three kinds of areas and elevation angles. The selected channel codings are Hamming (7,4), BCH (15,7) block codes and convolutional code of constraint length  $K=3$  and rate  $R=1/2$ . As results, we assess the performance of a BPSK system over realistic Rician log-normal fading channel and analyze the effects of channel codings on system performance improvement.

## 2. Channel Model

The probability distribution function of channel statistics related with Rician fading and log-normal shadowing is briefly given in this section. First, the probability density function (p.d.f.) of received signal envelope,  $r$ , can be written as

$$p_r(r) = \int_0^\infty p(r|S)p_s(S)dS. \quad (1)$$

In equation (1),  $p(r|S)$  is a Rice p.d.f. conditioned on a certain shadowing,  $S$ :

$$p(r|S) = 2(K+1)\frac{r}{S^2} \exp\left[-(K+1)\frac{r^2}{S^2} - K\right] \cdot I_0\left(2\frac{r}{S}\sqrt{K(K+1)}\right) \quad (r \geq 0) \quad (2)$$

where  $I_0$  is the zero order modified Bessel function of the first kind, and  $K$  is the so called Rice factor. The shadowing,  $S$ , is lognormal with p.d.f.:

$$p_s(S) = \frac{1}{\sqrt{2\pi h\sigma S}} \exp\left[-\frac{1}{2}\left(\frac{\ln S - \mu}{h\sigma}\right)^2\right], \quad (S \geq 0) \quad (3)$$

where  $h=(\ln 10)/20$ ,  $\mu$  and  $(h\sigma)^2$  are the mean and the variance of the associated normal variate, respectively; in terrestrial channels  $\sigma$  is usually referred to as the "dB spread."

The signal envelope which meets the channel model (1)-(3) can be interpreted as the product of two independent processes, i.e.,  $r = RS$ , where  $R$  is a Rice process, and  $S$  is lognormal. Due to the independence between  $R$  and  $S$  we have [4]:

$$p_r(r) = \int_0^{\infty} \frac{1}{S} p_R\left(\frac{r}{S}\right) p_S(S) dS$$

$$= \int_0^{\infty} \frac{1}{R} p_S\left(\frac{r}{R}\right) p_R(R) dR \quad (4)$$

and by comparing (1) and (4) :

$$p(r|S) = \frac{1}{S} p_R\left(\frac{r}{S}\right) \equiv \frac{r}{\sigma_R^2 S^2} \cdot \exp\left[-\frac{1}{2}\left(\frac{r^2}{S^2 \sigma_R^2} + 2K\right)\right]$$

$$\bullet I_0\left(\frac{r}{S\sigma_R} \sqrt{2K}\right), (r \geq 0) \quad (5)$$

which implies  $\sigma_R^2 = 1/2(K+1)$ . Equation (4) allows further observations: when  $K \rightarrow \infty$ ,  $p_R(R)$  tends to a Dirac pulse located at  $R=1$  and  $p_r(r)$  tends to  $p_S(r)$  i.e., the channel is lognormal. When  $K \rightarrow \infty$  and  $\sigma \rightarrow \infty$  fading is absent. The cumulative distribution function (c.d.f.) of the envelope can be obtained as equation (6).

$$P_r(r_0) \equiv \Pr\{r < r_0\}$$

$$= \int_0^{r_0} \int_0^{\infty} \frac{p_S(S)}{S} p_R\left(\frac{r}{S}\right) dS dr$$

$$= 1 - E_s \left\{ Q\left(\sqrt{2K}, \frac{r_0}{S} \sqrt{2(K+1)}\right) \right\} \quad (6)$$

Here,  $E_s\{\}$  means the average with respect to  $S$  and  $Q$  is the Marcum  $Q$  function[6].

### 3. Empirical Formulas and System Model

#### 3.1 Empirical Formulas

The channel model proposed by Corazza[1] was validated with respect to measurement data available in his paper. As stated in the above, empirical formulas should be derived to fit measured data over a wide range of elevation angle of  $\square$ . Data fitting was conditioned on the following intuitive indications: the greater is  $\alpha$ , the larger is  $K$  and the smaller is  $\sigma$ . However, Corazza used some data limited to rural tree-shadowed environment. Thus we use a more general data and formulas modified by You[2] *et al.* They derived parameter values based on the measured data suggested by ICO, Iridium and Globalstar. These LEO and MEO satellite systems provided fading margin curves for calculation of link availability and link margin.

The resulting empirical formulas allow interpolation for any  $\alpha$  in the range  $20^\circ < \alpha < 80^\circ$ :

$$K(\alpha) = K_0 + K_1\alpha + K_2\alpha^2 + K_3\alpha^3 + K_4\alpha^4 \quad (7)$$

$$\mu(\alpha) = \mu_0 + \mu_1\alpha + \mu_2\alpha^2 + \mu_3\alpha^3 + \mu_4\alpha^4 \quad (8)$$

$$\sigma(\alpha) = \sigma_0 + \sigma_1\alpha \quad (9)$$

The parameter calculation results suggested by You *et al.* are given in Table 1 and these values are validated in [2].

Table 1.

Parameter values  $K$ ,  $\mu$ , and  $\sigma$  for three kinds of areas with elevation angles  $\alpha = 20^\circ, 45^\circ$  and  $80^\circ$

| Elevation Angles    | Open area                                 | Rural area                                       | Urban area                                      |
|---------------------|---|--|---|
| $\alpha = 20^\circ$ | $K = 19.983$<br>$\mu = 0$<br>$\sigma = 0$ | $K = 1.64$<br>$\mu = -0.719$<br>$\sigma = 3.5$   | $K = 0.548$<br>$\mu = -1.43$<br>$\sigma = 5.25$ |
| $\alpha = 45^\circ$ | $K = 34.663$<br>$\mu = 0$<br>$\sigma = 0$ | $K = 3.62$<br>$\mu = -0.0804$<br>$\sigma = 2.25$ | $K = 1.21$<br>$\mu = -1.09$<br>$\sigma = 3.375$ |
| $\alpha = 80^\circ$ | $K = 44.925$<br>$\mu = 0$<br>$\sigma = 0$ | $K = 13.2$<br>$\mu = -0.017$<br>$\sigma = 0.5$   | $K = 4.41$<br>$\mu = -0.707$<br>$\sigma = 0.75$ |

Table 1 is the calculation results by substituting the coefficient values of Table 1 to equations (7) to (9). Using these values we can derive the average bit error rate(BER) performance of BPSK systems. The resulted BER equation can be given as equation (10). Here,  $Q$  function means the BPSK bit error rates conditioned on faded signal envelope  $r$  and signal to noise ratio  $E_b/N_0$  which is given in [6].

$$Pe = \int_0^{\infty} \int_0^{\infty} Q\left(\sqrt{\frac{2r^2 E_b}{N_0}}\right) \cdot 2(K+1) \frac{r}{S^2} \exp\left[-(K+1) \frac{r^2}{S^2} - K\right]$$

$$\cdot I_0\left(\frac{2r}{S} \sqrt{K(K+1)}\right) \cdot \frac{1}{\sqrt{2\pi h\alpha S}} \exp\left[-\frac{1}{2} \left(\frac{\ln S - \mu}{h\sigma}\right)^2\right] dS dr \quad (10)$$

And  $I_0$  is the first kind zero order modified Bessel function included in Rician probability density function.

#### 3.2 The BER with FEC Codings

For a comparison between coded and uncoded systems, the total energy used for transmitting  $k$  uncoded symbols is assumed to be equal to that used for transmitting  $n$  coded symbols. We consider two kinds of simple block channel codes: the (7,4) Hamming and (15,7) BCH code.

The former corrects one channel error while the latter corrects two channel errors. For a block code using hard decision decoding, the bit error probability  $P_B$  can be written as,

$$P_B = \frac{1}{n} \sum_{i=t+1}^n i \binom{n}{i} P_e^i (1-P_e)^{n-i} \quad (11)$$

where  $P_e$  is the symbol error rate,  $n$  is the coded block length and  $t$  is the error recovery capability.

While in a convolutional code with constraint length  $K=3$  and code rate  $R=1/2$ , the bit error probability has an upper bound as follows[6]:

$$P_c \leq \frac{\{2[P_e(1-P_e)]^{1/2}\}^5}{\{1-4[P_e(1-P_e)]^{1/2}\}^2} \quad (12)$$

where  $P_c$  means the BER with convolutional encoding and  $P_e$  is the symbol error rate.

#### 4. Performance Results

In this section, we present the performances of the system for three kinds of areas discussed above with three cases of elevation angles  $\alpha = 20^\circ$ ,  $45^\circ$ , and  $80^\circ$ . The bit error probabilities are assessed by equation (10) ~ (12) considering each area with different parameter values of Table 2. Fig. 1 to Fig. 3 show the BER curves of uncoded and coded BPSK modulation for open areas with different elevation angles. These figures show very low BER's due to no obstruction and no interferences, and there are not much performance differences among the figures. In coded case, we can get the BER of  $10^{-6}$  with 6 to 9 dB of SNR and the performance improvement of 0.5 dB is obtained as the elevation angle is larged. Also in uncoded case, the performance is improved about 0.5 dB as the elevation angle is increased from 20 degree to 80 degree and in each elevation angle, coding gain of 4 dB is achieved.

Fig. 4 to Fig. 6 are for rural areas with same elevation angles as in Fig.1 to Fig.3. We can see that the performances are worse than those of open areas due to more obstructions and interferences. And as elevation angle is increased, better performance is achieved because of more direct signal component on Rician fading. In rural areas, coded performances are varied on 6~22 dB of SNR at BER of  $10^{-5}$  as the variation of  $\alpha$ .

For example, the BER of  $10^{-5}$  is obtained on 19 ~ 22 dB at  $\alpha = 20^\circ$  and on 11 ~ 14.5 dB at  $\alpha = 45^\circ$ . And also at  $\alpha = 80^\circ$ , this performance is given on 5 ~ 6 dB. These differences are larger than those in open areas. Also the coding gains are increased and channel codings become more effective.

Finally, the performance curves are shown in Fig. 7 to Fig. 9 for urban areas. The performances are worse than those of open and rural areas due to decreased line of sight components.

When the elevation angle is  $20^\circ$ , voice communication is possible by only BCH channel coding with SNR of 25 dB. And the coded systems with the other channel codings and uncoded system need more signal energy for proper operations. We can get BER of  $10^{-5}$  with SNR distribution over 21 ~ 24 dB and 14 ~ 16.5 dB for  $\alpha = 45^\circ$  and  $80^\circ$ , respectively, and we can see the performance difference of about 7 dB. Also the coding gain of 5dB is obtained in elevation angle of  $\alpha = 80^\circ$  and we can not estimate the coding gains in another elevation angle cases in urban environment.

#### 5. Conclusions

In this paper, we evaluate the bit error rates of a coded BPSK system based on realistic channel model which can be applied to land mobile satellite and stratospheric communication systems. The realistic channel data was obtained by fitting the parameters of probability distribution model to measured data. It is shown that the performances of the open area are better than those of rural and urban areas because of relatively less shadowing and more specular signal components. The coding gains are obtained about 5 ~ 10 dB in all areas and elevation angles. These performance results could be used in system design and link budget calculations for land mobile satellite and stratospheric communication systems.

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