A New Total Coloring Problem in Multi-hop Networks

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Abstract: New graph coloring problems are discussed as models of a multihop network in this report. We consider a total scheduling problem, and prove that this problem is NP-hard. We propose new scheduling models of a multi-hop network for CDMA system, and show the complexity results of the scheduling problems.

1. Introduction

In a multihop network, radio packets are employed for communication, and are often relayed through intermediate nodes (repeaters) in order to transfer messages. Each intermediate node can reuse channels/slots. A scheduling problem in multihop networks is the problem of finding an efficient assignment of channels/slots, and is often discussed theoretically using graph model [2], [4], [9]. Many researchers have studied link scheduling problem and the broadcast scheduling problem[9], [1], [10]. The problems have been discussed on a system employing frequency/time division multiple access (FDMA/TDMA). In this report, we propose new scheduling models for CDMA system, and show the complexity results on their scheduling.

Let $D = (V, A)$ be the digraph with a vertex set V, and an arc set A. The vertices of D denote stations in a multi-hop network. An arc (directed edge) (u, v) denotes that u can send packets to v, and u interfere for v with receiving packets. Let see link scheduling and the broadcast scheduling.

Link Scheduling

Link scheduling corresponds to one of arc-coloring $f : A \to Z^+$ such that if $f(a, b) = f(c, d)$ for any arcs (a, b) and (c, d), then the following conditions hold:

L1 : Vertices a, b, c and d are mutually distinct, and

L2 : $(a, d) \notin A$, and $(b, c) \notin A$.

Broadcast Scheduling

Broadcast scheduling corresponds to one of vertex-coloring $g : V \to Z^+$ such that, if $f(a) = f(b)$ for any different vertices a and b, then the following conditions hold:

B1 : $(a, b) \notin A$, and $(b, a) \notin A$, and

B2 : there is not any vertex c such that $(a, c) \in A$ and $(b, c) \in A$.

In this report, we propose new coloring problems based on these problems.

2. Total Scheduling

In this section, we introduce new scheduling (coloring). *Total scheduling* model is applied in situation when stations communicate one-to-one and broadcast simultaneously. An interpretation of such scheduling is one of total coloring of digraph such that all vertices and edges are colored. In this report, we define total scheduling as following:

Total Scheduling

Total scheduling is one of total coloring $h : V \cup A \to Z^+$ such that the following seven conditions hold.

T1 : If $f(a, b) = f(c, d)$ for any arcs (a, b) and (c, d), then Condition L1 holds,

T2 : if $f(a, b) = f(c, d)$ for any arcs (a, b) and (c, d), then Condition L2 holds,

T3 : if $f(a) = f(b)$ for any different vertices a and b, then Condition B1 holds,

T4 : if $f(a) = f(b)$ for any different vertices a and b, then Condition B2 holds,

T5 : if $(a, b) \in A$, then $f(a) \neq f(a, b)$ and $f(a, b) \neq f(b)$,

T6 : if $f(a, b) = f(c)$ for any arc (a, b), and any vertex $c(\neq a, b)$, then $(c, a) \notin A$ and $(c, b) \notin A$, and

T7 : if $f(a, b) = f(c)$ for any arc (a, b), and any vertex $c(\neq a, b)$, then there is not any vertex d such that $(a, d) \in A$ and $(c, d) \in A$.

We call $f(a)$ the color of a for any arc a. An total scheduling $f_{\min}$ is minimum if $|\{f_{\min}(a) : a \in A\}| = \min_f |\{f(a) : a \in A\}|$. Let $\chi_T(D)$ be the chromatic number of D on total scheduling, the number of colors in a minimum total scheduling of D. The total scheduling problem is defined as : Given a digraph D, find minimum total scheduling. In Sect.4, we prove the NP-hardness of the total scheduling problem.

3. Scheduling for CDMA System

In this section we deal with models of CDMA system.

Link scheduling for CDMA system

In this system, a station can receive (respectively, send) packets assigned the same frequency band by using different codes. This corresponds to the fact: Arcs $(a, c), (b, c)$ (respectively, $(c, a), (c, b)$) can be assigned the same color in link scheduling. We can drop Condition L2, and can allow that $a = c$ and $b = d$ in Condition L1. Hence the condition of link scheduling for CDMA system is defined as follows.

L'1 : If $(a,b) \in A$ and $(b,c) \in A$, then $f(a,b) \neq f(b,c)$.

This scheduling is equivalent to arc coloring of digraphs[5], [8]. Let $\chi'(D)$ be the arc-chromatic number of D, which is the minimum number of colors in arc-colorings of D. It is NP-hard to find an arc coloring of $\chi'(D)$[5]. Hence it is NP-hard to find an optimum link scheduling for CDMA system.

Broadcast scheduling for CDMA system

Similarly we can define the condition of broadcast scheduling for CDMA system as follows.

B'1 : If $(a,b) \in A$ then $f(a) \neq f(b)$.

This scheduling is equivalent to vertex coloring of digraphs. Let $\chi(D)$ be the arc-chromatic number of D, which is the minimum number of colors in vertex-colorings of D. It is NP-hard to find a vertex coloring of $\chi(D)$[3]. Thus it is NP-hard to find an optimum broadcast scheduling for CDMA system.

Total scheduling for CDMA system

For total scheduling, dropping Condition T2, T4, and T7, we can define the conditions for CDMA system as follows.

T'1 : If $(a,b) \in A$ and $(b,c) \in A$, then $f(a,b) \neq f(b,c)$,

T'2 : if $(a,b) \in A$ then $f(a) \neq f(b)$,

T'3 : if $(a,b) \in A$, then $f(b) \neq f(a,b)$, and

T'4 : if $f(a,b) = f(c)$ for any arc (a,b), and any vertex $c (\neq a,b)$, then $(c,a) \notin A$.

Let $\chi_{T'}(D)$ be the chromatic number of D under such a total coloring of D, which is the minimum number of colors associated vertices and arcs of D. Since the conditions of the total coloring involve one of ordinary vertex-coloring, we have $\chi(D) \leq \chi_{T'}(D)$. If we know a vertex coloring f of D, we can obtain a total coloring by associating the color $f(a)$ to each arc (a,b) of a digraph with a vertex coloring f. Hence $\chi(D) = \chi_{T'}(D)$. Therefore it is NP-hard to find an optimum total scheduling for CDMA system.

h-Link Scheduling

We provide a more practical graph model than the link scheduling model in the previous section for CDMA system. This scheduling is generalized from link scheduling for CDMA. Let $D = (V,A)$ be the *multiple* digraph with a vertex set V and an arc multi-set A. A vertex of V corresponds to a node in a radio packet network, and an arc (u,v) of A corresponds to the unidirectional communication link from u to v. Let f be a labeling on positive integers of the arcs of A. The value of $f(u,v)$ means a frequency band (or a time slot for CDMA/TDMA hybrid system) assigned on the link from u to v. In a multihop network, each node often intermediates radio packets. We assume that each node can receive and send packets simultaneously, and that can not use a frequency band common to reception and transmission. But the CDMA nature allows that an identical frequency band is simultaneously associated with some received (respectively, sent) packets. We assume that every node can simultaneously receive (send respectively) at most h packets, where h is a positive integer. These assumptions indicate the following condition:

1)If $(u,v) \in A$ and $(v,w) \in A$, then $f(u,v) \neq f(v,w)$.

2)Let h be a fixed positive integer. For any vertex v of V, and for any label l of f,

$$|u \in N^+(v) : f(v,u) = l| \leq h,$$

and

$$|u \in N^-(v) : f(u,v) = l| \leq h.$$

(In this paper, let $N^+(x)$ (respectively, $N^-(x)$) denote the set of vertices adjacent from (to respectively) x, and call $|N^+(x)|$ ($|N^-(x)|$ respectively) the outdegree (respectively, indegree) of a vertex x.)

If a labeling f satisfies these conditions, then we call f an h-link scheduling of D. We define the size of f as follows:

$$|\{f(a) : a \in A\}|.$$

An h-link scheduling is minimum if the size of it is equal to $c_h(D) = \min_f |\{f(a) \mid a \in A\}|$. The minimum h-link scheduling problem is the problem of finding a minimum h-link scheduling. When h is greater than the maximum of indegrees and outdegrees, this problem is equivalent to the arc coloring problem that Harner and Entringer discussed in Ref. [5]. The authors proved that the h-link scheduling problem is NP-hard[11].

h-Broadcast Scheduling

We provide a more practical graph model than the broadcast scheduling model in the previous section for CDMA system. This scheduling is generalized from broadcast scheduling for CDMA. Let $D = (V,A)$ be the simple digraph with a vertex set V and an arc set A. Let f be a labeling on positive integers of the vertices of V. The value of $f(v)$ means a frequency band (or a time slot for CDMA/TDMA-hybrid system) assigned on the node v. In the broadcast scheduling problem, we suppose the following condition:

1)If $(u,v) \in A$, then $f(u) \neq f(v)$.

2)Let h be a fixed positive integer. For any vertex v, and for any label l of f,

$$|u \in N^+(v) : f(v) = l| \leq h,$$

and

$$|u \in N^-(v) : f(v) = l| \leq h.$$

In this paper, if a labeling f satisfies these conditions, then we call f an h-broadcast scheduling of D. We define the size of f as follows:

$$|\{f(v) : v \in V\}|.$$

An h-broadcast scheduling is minimum if the size of it is equal to $d_h(D) = \min_f |\{f(v) \mid v \in V\}|$. The minimum h-broadcast scheduling problem is the problem of finding a minimum h-broadcast scheduling. The authors proved that the h-link scheduling problem is NP-hard[11].

4. NP-completeness Result

In this section, we prove that the total scheduling problem is NP-hard. Let $\chi_L(D)$ be the chromatic number of D, the number of colors in a minimum link scheduling of D. We define the following decision problems corresponding to the total and link scheduling problem as follow.

TS
Instance A digraph D, and a positive integer k.
Question $\chi_T(D) \leq k$?
LS
Instance A digraph D, and a positive integer k.
Question $\chi_L(D) \leq k$?
The problem LS is known to be NP-complete[6], [7] We prove the following theorem.

Theorem 1: The problem TS is NP-complete.
proof: This problem belongs to NP clearly, since we can check that a given coloring is a total coloring and know the number of its colors in polynomial time. To prove the theorem, we reduce the LS to TS. Let $D = (V, A)$ and k be a digraph and a positive integer in the instance of LS respectively. Let s be a new vertex. We add the vertex s, and the arc (v, s) for any vertex v of V to D. Let $D = (V', A')$ be the obtained digraph, and let $k' = k+2|V|+1$. We can accomplish this transformation in polynomial time.

We must show that D has a link scheduling with k colors if and only if D' has a total scheduling with k' colors. Assume that there exists a link scheduling f with k colors in D. We suppose that $1 \leq f(x) \leq k$ for any $x \in V \cup A$ without loss of generality. We write each vertex of V as $v_1, \cdots, v_{|V|}$. We associate a color $g(x)$ to each vertex and arc of D' as follows: for any arc x of A',

$$g(x) = \begin{cases} k+i & \text{if } x = \{(v_i, s)\} \text{ for some } i \\ f(x) & \text{otherwise} \end{cases},$$

and for any vertex x of V',

$$g(x) = \begin{cases} 2k+i & \text{if } x = v_i \text{ for some } i \\ k' & \text{otherwise} \end{cases}.$$

The coloring g consists of k' colors, and is a total coloring of D.

Conversely We suppose that there exists a total scheduling g with k' colors in D'. Since the conditions of a total scheduling include ones of total scheduling, the constriction of g on A' is a link scheduling. Hence the constriction f of g on A is also a link scheduling. The following propositions hold:

- The color of each vertex is disjoint (from the conditions T4 of total scheduling).
- There is not any arc with the same color as a vertex of D' (from the conditions T5, T6 and T7 of total scheduling).
- There is not any arc of A with the same color as an arc of $\{(v, s) : v \in V\}$ (from the conditions T2 of total scheduling).

Hence the number of colors in f is at most $k'-2|V|-1 = k$. We have proved this theorem. □

5. Conclusion

In this report, we proposed link, broadcast, and total scheduling for CDMA system. We prove that the total scheduling problem is NP-hard.

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