Design method of interpolation kernel using piecewise n th polynomials

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Abstract: Sampling rate conversion widely used in subband coding, A/D and D/A transitions etc. is an important techniques. Nyquist filters and the filter banks have been used for the sampling converter. However, they need many memories and, whenever the sampling rate is changed, it is necessary to redesign filters. Then, we propose design method of the new interpolation kernel. Design method of the new interpolation kernel is approximated each piecewise of lowpass filter by n th polynomials. The proposed kernel is not redesigned, whenever the sampling rate is changed. The proposed kernel is a continuous function, the sampling rate of the rational number can be converted.

1. Introduction

Sampling rate conversion widely used in subband coding, A/D and D/A transitions etc: is an important techniques [1], [3]-[7]. In general, the sampling rate conversion is divided into the interpolation and sampling. The interpolation creates the continuous function from the discrete signal data. The sampling newly creates the discrete signal data from the continuous function. This is a convolution of the input signal and the interpolation kernel to create the continuous function from a discrete signal. An Impulse response of the ideal interpolation kernel is sinc function obtained by the Fourier transform of a square wave. However because this kernel is a polynomial with infinite length, it cannot be used as interpolation kernel.

Recently, the method of the sampling rate conversion by using the filter bank is proposed in the field of the digital signal processing [5]. However, to convert the fractionally sampling rate M/N, the input samples are first interpolated up by M and passed through a lowpass filter and then decimated down by N in this method. In the application of sampling rate conversion from CD to DAT the M and N are 160 and 147, respectively. As a result, computational complexity may become very large. Moreover, this method is necessary to redesign the filter, whenever the sampling rate is changed.

Then, we proposed an interpolation kernel approximated by using some quadratic functions for piecewise the sinc function [2]. This method is not necessary to redesign the kernel, whenever the sampling rate is changed. However, to obtain a large attenuation in the stopband, many quadratic functions are required.

In this paper, hence, we present a new interpolation kernel and its design method. The proposed kernel is approximated to the impulse response of lowpass filter by some n th polynomials. The kernel has a good stopband performance because it is designed by using the standard linear programming in the frequency domain. Moreover, the proposed kernel is unnecessary redesign, whenever the sampling rate is changed. In addition, a fractionally sampling rate can be converted, so that the proposed kernel is a continuous function. Finally, usefulness of the proposed kernel is verified through the examples.

2. Kernel

In general, a reconstruction of a piecewise continuous function from discrete data is taken to be a linear combination of input signal and a reconstruction kernel. For unit spaced samples, this is

$$f(x)=\sum_{i=-\infty}^{\infty} f_i y(x-i), \tag{1}$$

where, f_i is the sample value and $y(x)$ is the interpolation kernel.

The kernel approximated by the some quadratic functions is shown as follows:

$$y(x)=\begin{cases} a_{1,1}x^2+b_{1,1}x+c_{1,1} & \left(0\le|x|\le\frac{1}{N}\right) \\ \vdots & \\ a_{1,n}x^2+b_{1,n}x+c_{1,n} & \left(\frac{n-1}{N}\le|x|\le 1\right) \\ \vdots & \\ a_{s,n}x^2+b_{s,n}x+c_{s,n} & \left(s-1+\frac{n-1}{N}\le|x|\le s-1+\frac{n}{N}\right) \\ \vdots & \\ a_{S,N}x^2+b_{S,N}x+c_{S,N} & \left(S-1+\frac{N-1}{N}\le|x|\le S\right) \end{cases}, \tag{2}$$

where N and S are number of quadratic functions used in one piecewise and number of piecewise, respectively. $a_{s,n}$, $b_{s,n}$ and $c_{s,n}$ are coefficients of the quadratic function. Fig.1 shows an outline of the kernel using quadratic functions. However, large S and N are required so that kernel is realized large attenuation in the stopband. Therefore, the computational complexity becomes very large.

Thus, we approximated the proposed a new kernel by using n th polynomials for each piecewise. This kernel can be obtained as follows:

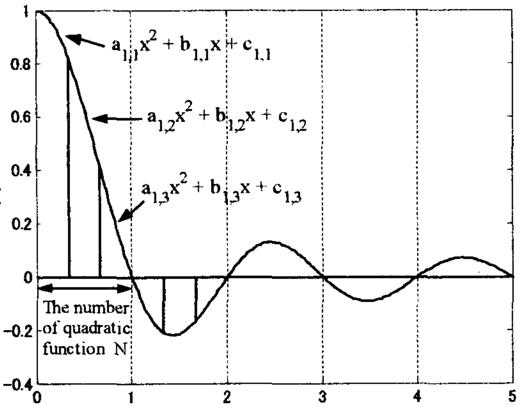

Fig.1 The outline of a kernel using quadratic functions

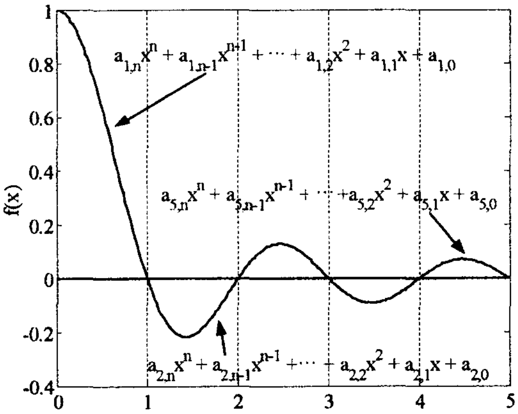

Fig.2 The outline of proposed a kernel (S=5)

$$f(x)=\begin{cases} f_1(x)=a_{0,n}x^n+a_{0,n-1}x^{n-1}+\cdots \\ \qquad +a_{0,2}x^2+a_{0,1}x+a_{0,0} \quad (0\le x\le 1) \\ \qquad \vdots \\ f_S(x)=a_{S-1,n}x^n+a_{S-1,n-1}x^{n-1}+\cdots \\ \qquad +a_{S-1,2}x^2+a_{S-1,1}x+a_{S-1,0} \quad (S-1\le x\le S) \end{cases}, \quad (3)$$

where, $a_{i,j}$ is j th coefficient of n th polynomials in the i th piecewise, and S is a total number of piecewise. Fig.2 shows an outline of the proposed kernel using n th polynomials.

Now to produce a useful interpolation kernel from eq. (3), it should be the condition of zero intersymbol interference. That is, it becomes $f(x)=0$ in integer value except $x=0$. Substituting the condition of zero intersymbol interference into eq. (3), we obtain

$$f(x)=\begin{cases} f_1(x)=a_{0,n}(x^n-1)+a_{0,n-1}(x^{n-1}-1)+\cdots \\ \qquad +a_{0,2}(x^2-1)+a_{0,1}(x-1) \quad (0\le x\le 1) \\ f_2(x)=a_{1,n}\{x^n+(1-2^n)x+(1-2^n)(-1)-1\} \\ \qquad +a_{1,n-1}\{x^{n-1}+(1-2^{n-1})x+(1-2^{n-1})(-1)-1\} \\ \qquad +\cdots+a_{1,2}(x^2-3x+2) \quad (1\le x\le 2) \\ \qquad \vdots \\ f_S(x)=a_{S-1,n}(x^n+A^nx+A^nB-C^n) \\ \qquad +a_{S-1,n-1}(x^{n-1}+A^{n-1}x+A^{n-1}B-C^{n-1}) \\ \qquad +\cdots+a_{S-1,2}(x^2+A^2x+A^2B-C^2) \quad (S-1\le x\le S) \end{cases}, \quad (4)$$

where, $A^n=(S-1)^n-S^n$, $B=-(S-1)$ and $C^n=(S-1)^n$.

3. Design method of kernel

In this section, the design method of the proposed kernel is shown.

In order to determine the coefficients $a_{i,j}$ with $1\le i\le S$ and $1\le j\le K$. We consider the approximation in the time domain to the sinc function. However, the sinc function has infinite length polynomials. Thus we consider to optimize $a_{i,j}$ in the frequency domain. With the condition of phase characteristics, eq. (4) can be written as

$$F(\omega)=\sum_{j=1}^{S}\sum_{i=(j-1)K}^{jK} f_j(iT)\cos(i\omega T), \quad (5)$$

where, K is a sampling point in one piecewise.

Next, let the ideal frequency characteristic be

$$D(\omega)\begin{cases}1 & passband \\ 1 & stopband\end{cases}, \quad (6)$$

and weighting function be

$$W(\omega)\begin{cases}1 & passband \\ 1 & stopband\end{cases}. \quad (7)$$

In order to determine the coefficients $a_{i,j}$, the weighted error of approximation $E(\omega)$ is,

$$E(\omega)=W(\omega)[D(\omega)-F(\omega)]. \quad (8)$$

The optimal kernel is the one for which the maximum error $E(\omega)$ is minimized over all ω. Letting δ represent the maximum error, a set of linear inequalities can be written to describe this minimax problem,

$$-\delta\le W(\omega_i)[D(\omega_i)-F(\omega_i)],\ with\ \Omega, \quad (9)$$

where Ω is a dense grid of frequency in the bands over which the approximation is being made. Eq. (9) can formally be written as linear program;

Minimize δ

$$\begin{aligned} -W(\omega_i)\sum_{j=1}^{S}\sum_{i=(j-1)K}^{jK} f_j(iT)\cos(i\omega_iT)-\delta &\le -W(\omega_i)D(\omega_i) \\ W(\omega_i)\sum_{j=1}^{S}\sum_{i=(j-1)K}^{jK} f_j(iT)\cos(i\omega_iT)-\delta &\le W(\omega_i)D(\omega_i) \end{aligned}. \quad (10)$$

Eq. (10) can be solved by using the standard linear programming techniques.

4. Examples

To show the proposed kernel effectiveness, we consider about the following examples.

4.1. Example 1

We think about the kernel design of the following specification.

[Specification]

Order of polynomials N : 4

Number of amplitudes S : 6
Sampling point K : 9
Rolling off rate R : 0.25
Weight $W(\omega)$: 1(All the frequencies)

The obtained kernel and its frequency characteristics are shown in figs. 3 and 4, respectively. It's clear from fig.3 that the proposed kernel is the zero intersymbol interference because $f(x)$ is zero value for x of integer number except $x=0$. And it's clear from fig.4 that the obtained maximum stopband attenuation is –54 [dB]. In fig. 4, the dotted line indicates the frequency characteristics of the kernel to be upsampling from 6 to 9, and the one point dot-dashed line indicates the frequency characteristics of the kernel to be downsampling from 12 to 9. It's clear from fig. 4 that the frequency characteristic of the kernel does not change even if the sampling rate is changed. That is, the proposed kernel is robustness to change the sampling rate.

Next if we design the kernel with the same attenuation in the stopband, the number of the polynomial coefficient of the proposed kernel is less than one of the previous kernel by using ref. [2]. That is the number of the polynomial coefficient of the proposed kernel in 19 though one of previous kernel in 61.

4.2.Example 2

It is time when this example changed the order of 5 and amplitudes into 9 as for the number of polynomials.

[Specification]

Order of polynomials N : 5
Number of amplitudes S : 9
Sampling point K : 9
Rolling off rate R : 0.25
Weight $W(\omega)$: 1(All the frequencies)

The obtained kernel and its frequency characteristics are shown in figs. 5 and 6, respectively. Moreover, the frequency characteristics of the obtained kernel to be upsampling from 6 to 9 is shown by fig. 6 the dotted line. And, the frequency characteristics of kernel to be downsampling from 12 to 9 is also shown by fig. 6 the one point dot-dashed line. It's clear from fig.3 that proposed kernel is the zero intersymbol interference becomes $f(x)$ is zero value for x of integer number except $x=0$ as well as 4th polynomial. And it's clear from fig.4 that the obtained maximum stopband attenuation is –75 [dB]. A large attenuation in the stopband is obtain by enlarging N and S.

Next if we design the kernel with the same attenuation in the stopband, the number of the polynomial coefficient of the proposed kernel is less than one of the previous kernel by using ref. [2]. That is the number of the polynomial coefficient of the proposed kernel in 37 though one of previous kernel in 181.

4.3.Example 3

In this example, it is shown to convert the fractional sampling rate.

The basic specification is the same as example 2.

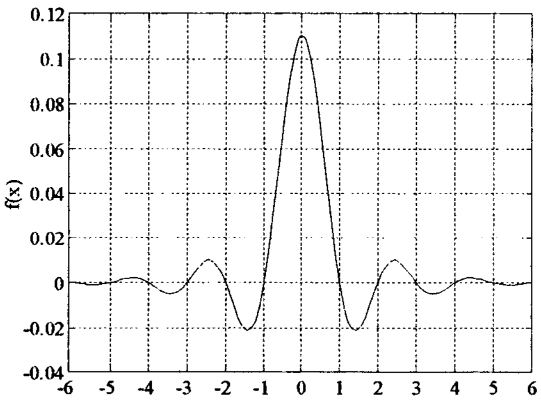

Fig.3 The kernel of the 4th polynomials

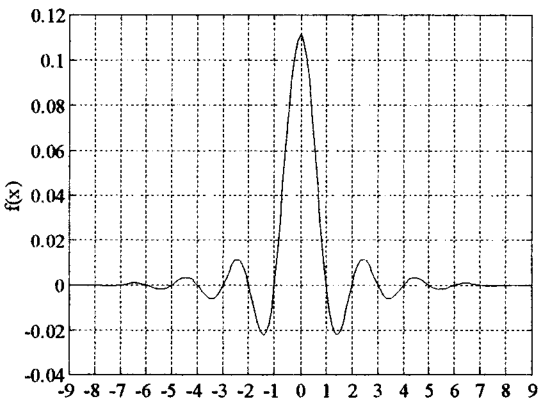

Fig.5 The kernel of the 5th polynomials

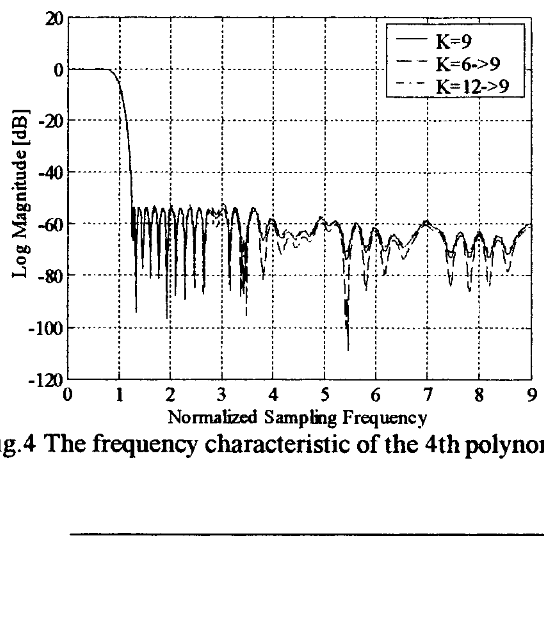

Fig.4 The frequency characteristic of the 4th polynomials

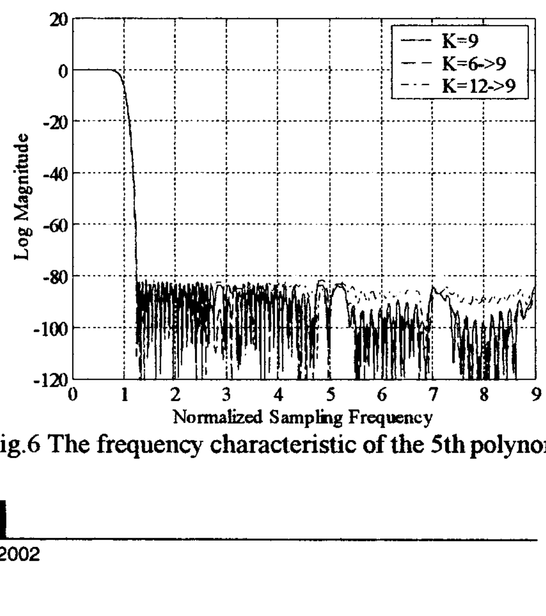

Fig.6 The frequency characteristic of the 5th polynomials

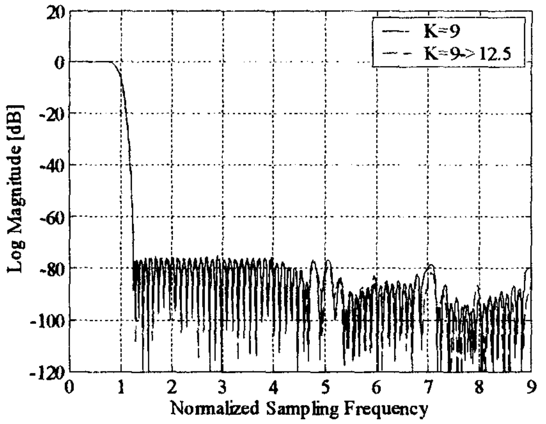

Fig.7 The frequency characteristic of the 5th polynomials

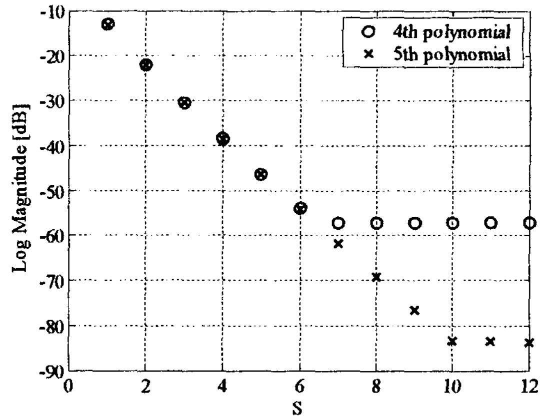

Fig.8 Limit of maximum of attenuation

[Specification]

Order of polynomials N : 5

Number of amplitudes S : 9

Sampling point K : 9

Rolling off rate R : 0.25

Weight $W(\omega)$: 1(All the frequencies)

In fig. 7, the frequency response of the original kernel is shown by the solid line and the obtained kernel to be upsampling from 9to 12.5 is shown by the dotted line. It is clear from fig. 7 that the frequency characteristic of kernel does not change even if the sampling rate is changed into the rational number.

5. Limit of maximum of attenuation

In this section, the relation between the order of polynomial and the maximum attenuation in the stopband is described.

[Specification]

Sampling point K : 9

Rolling off rate R : 0.25

Weight $W(\omega)$: 1(All the frequencies)

It's clear from fig. 8 that the maximum attenuation in the stopband increases for 4th order polynomial and 5th order polynomial if the number of piecewise is increased. However, the maximum attenuation in the stopband for the kernel with 4th order polynomial does not change even if the number of piecewise S is increased more than 7. Therefore, the limit in the stopband attenuation of the kernel with 4th polynomials in the stopband attenuation is -57 [dB]. Similarly, the attenuation in the stopband for the kernel with 5th order polynomial is –84 [dB].

6. Conclusion

In this paper, we proposed the interpolation kernel by using n th polynomials and its design method. The proposed kernel is approximated to the impulse response of lowpass filter by some n th polynomials. The kernel has a good stopband performance because it is designed by using the standard linear programming in the frequency domain. Moreover, the proposed kernel is unnecessary redesign, whenever the sampling rate is changed. In addition, a fractionally sampling rate can be converted, so that the proposed kernel is a continuous function. Finally, usefulness of the proposed kernel is verified through the examples.

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