Investigation on the Analysis of Transmission Line with Frequency Dependent Lossy Term

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Abstract: The increaseing speeds are accompanied by decreases in pulse rise and fall time in VLSI circuits. These accenturate the high frequency spectral contents of the signals and cause the frequency dependent loss of the conductors which interconnect the various sub-circuits composing of VLSI circuit. The lossy effect is approximated by the square root of frequency dependence of the per unit length resistance. In the practical applications, several problems may arise along with this approximation, so we extend our investigation of the lossy effect by numerical Laplace inversion method.

1 Introduction

VLSI is the complicated system composed of the large number of linear/nonlinear lumped elements and interconnects. The increasing speed of the clock signal produces high frequency components in signal transients and accurate modeling of the interconnect by distributed transmission line is no longer avoidable. In the high frequency domain, the loss of the interconnect is caused by the skin effect of conductor which depends on the square root of frequency. In the practical applications, several problems may arise along with this approximation. In this paper we examine the validity of the approximation by numerical Laplace inversion method.

2 Computational model of lossy transmission line

2.1 Equations in time domain

Let us consider a multi-conductor uniform lossy transmission line with common return path depicted in Fig.1. In the time domain, voltage vector $\boldsymbol{v}(x,t)$ and current vector $\boldsymbol{i}(x,t)$ at the position x are subjected by the following telegrapher's equations[1],[2].

$$-\frac{\partial \boldsymbol{v}(x,t)}{\partial x} = \boldsymbol{L}\frac{\partial \boldsymbol{i}(x,t)}{\partial t} + \boldsymbol{Z}_i(t) * \boldsymbol{i}(x,t) \quad (1)$$

$$-\frac{\partial \boldsymbol{i}(x,t)}{\partial x} = \boldsymbol{C}\frac{\partial \boldsymbol{v}(x,t)}{\partial t} + \boldsymbol{G}\boldsymbol{v}(x,t) \quad (2)$$

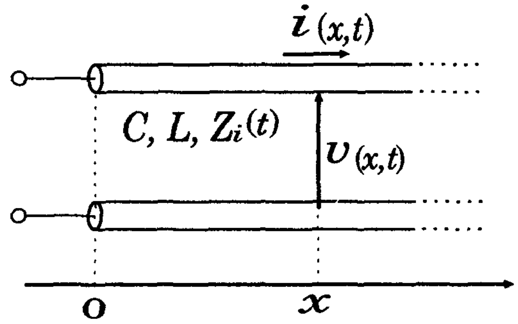

Fig.1 Lossy transmission line

$\boldsymbol{L}$, $\boldsymbol{C}$ and $\boldsymbol{G}$ are per unit length indactance matrix, capacitance martix and leakage matrix. $\boldsymbol{Z}_i(t)$ represents the per unit length lossy matrix whose elements depend on frequency and $*$ denotes the convolution integral.

2.2 Equations in Laplace domain

Since the equations (1) and (2) are linear it is possible to analyse them in s domain by Laplace transformation. When the initial distributions of voltage and current are zero, we obtain

$$-\frac{d\boldsymbol{V}(x,s)}{dx} = s\boldsymbol{L}\boldsymbol{I}(x,s) + \boldsymbol{Z}_i(s)\boldsymbol{I}(x,s) \quad (3)$$

$$-\frac{d\boldsymbol{I}(x,s)}{dx} = s\boldsymbol{C}\boldsymbol{V}(x,s) + \boldsymbol{G}\boldsymbol{V}(x,s) \quad (4)$$

where $\boldsymbol{V}(x,s)$ and $\boldsymbol{I}(x,s)$ are s functions of $\boldsymbol{v}(x,t)$ and $\boldsymbol{i}(x,t)$.

2.3 General solutions in Laplace domain

From equations (3) and (4) we obtain the following general solutions for $\boldsymbol{V}(x,s)$ and $\boldsymbol{I}(x,s)$

$$\boldsymbol{V}(x,s) = \exp[-\boldsymbol{Q}(s)x]\boldsymbol{K}^{+}(s) + \exp[\boldsymbol{Q}(s)x]\boldsymbol{K}^{-}(s) \tag{5}$$

$$\begin{aligned}\boldsymbol{I}(x,s) = \boldsymbol{P}(s)\{&\exp[-\boldsymbol{Q}(s)x]\boldsymbol{K}^{+}(s) \\ &- \exp[\boldsymbol{Q}(s)x]\boldsymbol{K}^{-}(s)\}\end{aligned} \tag{6}$$

where

$$\left.\begin{aligned}\boldsymbol{Q}^2(s) &= [s\boldsymbol{L} + \boldsymbol{Z}_i(s)][s\boldsymbol{C} + \boldsymbol{G}] \\ \boldsymbol{P}(s) &= [s\boldsymbol{L} + \boldsymbol{Z}_i(s)]^{-1}\boldsymbol{Q}(s)\end{aligned}\right\}. \tag{7}$$

$\boldsymbol{K}^{+}(s)$ and $\boldsymbol{K}^{-}(s)$ are arbitrary vectors determined by the conditions of both ends. For example, the step response of wave front is

$$\boldsymbol{V}(x,s) = \frac{1}{s}\exp[-\boldsymbol{Q}(s)x]. \tag{8}$$

After s functions $\boldsymbol{V}(x,s)$ and $\boldsymbol{I}(x,s)$ are determined, we can obtain the time responses of $\boldsymbol{v}(x,t)$ and $\boldsymbol{i}(x,t)$ by numerical Laplace inversion.

3 Formulation of frequency dependent lossy term

3.1 Skin effect loss of cylindrical conductor

At high frequency, current flows concenturatedly nearby the surface of the conductor and increases the per unit length resistance. For the cylindrical conductor with radius R we obtain the theoretical formula of per unit length resistance

$$Z_i(s) = \frac{1}{2\pi R}\sqrt{\frac{s\mu}{\sigma}}\frac{I_0(\sqrt{s\sigma\mu}R)}{I_1(\sqrt{s\sigma\mu}R)} \tag{9}$$

where $I_0(x)$ and $I_1(x)$ are modified Bessel functions.

By the asymptotic expansion, we obtain

$$\hat{Z}_i(s) \simeq \frac{1}{\sigma\pi R^2} + \frac{1}{2\pi R}\sqrt{\frac{s\mu}{\sigma}} \tag{10}$$

and this means the approximate square root formula of frequency dependence.

3.2 Examination for validity of square root approximation

In the practical application, it is necessary to examine the validity of square root approximation. For this purpose we calculate $Z_i(s), \hat{Z}_i(s)$ and

$$\Lambda(z,s) = \left|\frac{\exp\{-\sqrt{[sL + \hat{Z}_i(s)]sCz}\}}{\exp\{-\sqrt{[sL + Z_i(s)]sCz}\}}\right| \tag{11}$$

vs. f $(s = i2\pi f)$. Obtained results are shown in Fig.2 and Fig.3 where conductor is copper and per unit length parameters are $L = 1.0\mu$H/m, C=100.0pF/m and G=0.

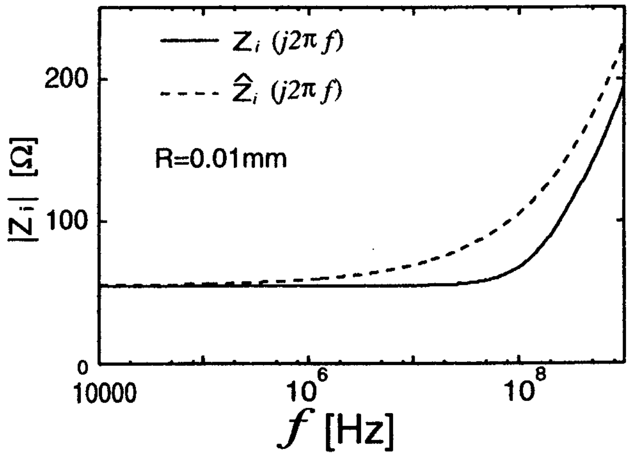

Fig.2 $|Z_i(s)|, |\hat{Z}_i(s)|$ vs. f

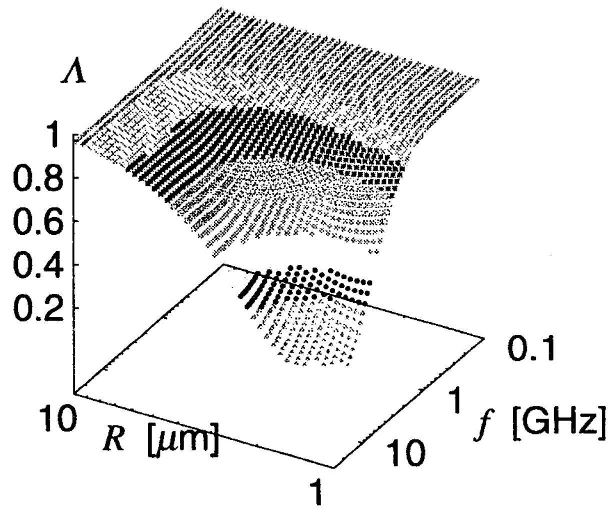

Fig.3 $\Lambda(0.2,s)$ vs. f and R

From these two examples, it may be concluded that square root approximation becomes worse as f (absolute value of s) increases.

4 Single lossy transmission line terminated with nonlinear element

When the lossy transmission line is terminated by nonlinear element, we do not analyse it only in the s domain and main problem is the treatment of the term

$$H(s) = Z_i(s)I(x,s) \tag{12}$$

contained in the right hand side of eq. (3) in the time domain.

4.1 Square root approximation by $Z_i(s)$

For $\hat{Z}_i(s) = \sqrt{s}$, we obtain

$$h(t) = \frac{1}{\sqrt{\pi}} \int_0^t \frac{1}{\sqrt{\tau}} f(t-\tau) d\tau \qquad (13)$$

in the time domain where

$$f(t-\tau) = \frac{\partial}{\partial(t-\tau)} i(x, t-\tau). \qquad (14)$$

Discretization by time step Δt yields the following approximate formula[3]

$$\begin{aligned} h[(n+1)\Delta t] &= \frac{1}{\sqrt{\pi}} \int_0^{(n+1)\Delta t} \frac{1}{\sqrt{\tau}} f[(n+1)\Delta t - \tau] d\tau \\ &\simeq \sqrt{\frac{\Delta t}{\pi}} \sum_{m=0}^{n} f^{n+1-m} Z_0(m) \end{aligned} \qquad (15)$$

where

$$Z_0(m) = \int_m^{m+1} \frac{1}{\sqrt{\tau}} d\tau = 2(\sqrt{m+1} - \sqrt{m}). \qquad (16)$$

4.2 Numerical Laplace inversion of $Z_i(s)$

In the time domain we obtain

$$h(t) = \frac{d}{dt} \int_0^t Z_0(t-\tau) i(x,\tau) d\tau \qquad (17)$$

for eq. (12). By discretization we obtain

$$h(m) = \sum_{p=0}^{m} Z_0(m-p) i(x,p) - \sum_{p=0}^{m-1} Z_0(m-1-p) i(x,p) \qquad (18)$$

where almost accurate time sequence $Z_0(p)$ $p = 0, 1, \cdots, m$ can be obtained by numerical Laplace inversion of s function

$$Z_0(s) = \frac{1}{s} Z_i(s). \qquad (19)$$

5 Calculated example

We calculate transient voltage response of $v_2(t)$ of the model shown in Fig.4 where $L = 0.277\mu$H/m, C=40.07pF/m and G=0. Characteristic of nonlinear diode is given by

$$i_d(t) = i_0 \left[\exp\left(\frac{v_d(t)}{v_T} \right) - 1 \right] \qquad (20)$$

where v_T=25mV and i_0=1.0pA.

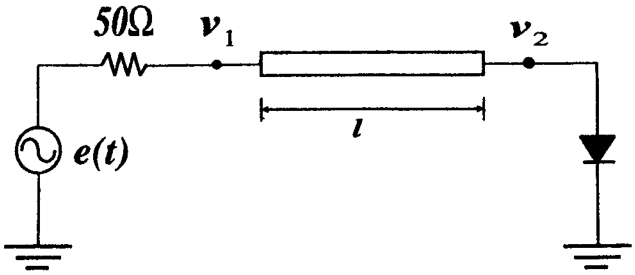

Fig.4 Lossy transmission line terminated by nonlinear diode

Waveform of source voltage is shown in Fig.5.

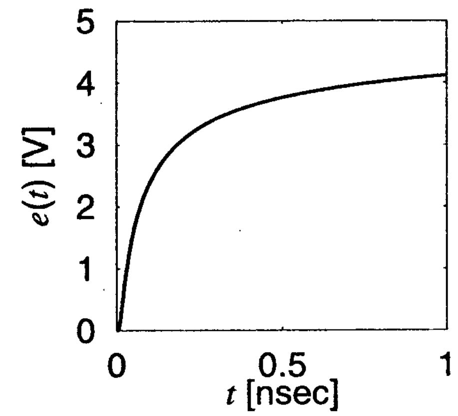

Fig.5 Waveform of source voltage

Partial differential equations governing the whole circuit are discretized and solved in the time domain by FDTD method. Fig.6 shows the calculated results. From the previous examination it may be concluded that result by using $Z_i(s)$ is more accurate.

6 Conclusion

In the previous works approximate square root formula of s has been derived as stated in **3.1**. Strictly speaking, this formula seems to have weak theoretical ground. In this work we investigate the validity of square root formula by applying the numerical Laplace inversion method.

The points of discussion are essentially summarized as follows:

1. to clarify the problem of approximation by eq. (10)
2. to clarify the problem of approximation by eq. (15).

In all cases we use eq. (9) as the exact frequency dependent lossy resistance.

References

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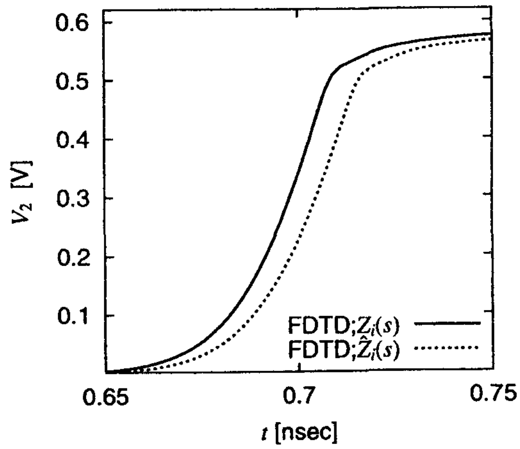

Fig.6 Calculated wave front of $v_2(t)$