Neighborhood Reduction in Local Search based on Geospatial Relation for Multi Depot Vehicle Routing Problems

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Abstract: This paper proposes neighborhood reduction techniques in local search of the customer decomposition subproblem in the Multi Depots Vehicle Routing Problem with Time Windows (MDVRPTW) by using geospatial relation among depots and customers. The neighborhood of the customer decomposition subproblem can be simply and well defined but it should include excessively bad solution candidates. Our techniques find such candidates by using information of the problem domain, geographical relation. We use our techniques in Tabu Search and evaluate the effectiveness in computer experiment.

1. Introduction

The Vehicle Routing Problem (VRP) is a traditional optimization problem, characterized as NP-hard [1], [2]. In the VRP, the objectives are to minimize the number of vehicles and the total travel distance. The Multi-Depot Vehicle Routing Problems with Time Windows (MDVRPTW) are extended from the VRP by considering real life application, in which multiple depots are available and each customer can set a time window to be served[3].

Many approximate algorithms for the VRP have been developed to obtain approximate solutions within reasonable time. Specially, meta-heuristic approaches, such as simulated annealing, tabu search and genetic algorithms (GA), have succeeded to this purpose[4]. Tabu search and simulated annealing are based on local search while GAs are commonly combined with local searches in GA-based optimization, referred to as *hybrid GA*. Therefore, local search plays a very important roles in meta-heuristics. The computation time for local search is proportional to the size of the neighborhood set.

This paper proposes neighborhood reduction techniques in local search of the customer decomposition subproblem in the MDVRPTW by using geospatial relation among depots and customers. The neighborhood of the customer decomposition subproblem can be simply and well defined but it should include excessively bad solution candidates we can verify before searching. Our techniques find such candidates by using information of the problem domain, say geographical relation. We employ our techniques in Tabu search and evaluate the efficiency in computer experiment.

In Section 2, we describe the MDVRPTW definition and summarize simple idea of the classical heuristic algorithm of the VRP, called Saving Method. In Section 3 our routing procedure for the MDVRPTW is presented and in Section 4 reduction techniques are proposed. Section 5 evaluates the effectiveness by computer experiment. We conclude this paper in Section 6.

2. Preliminaries

2.1 Problem Definition

Let $V = \{v_1, v_2, \cdots, v_m\}$ be a set of vehicles, $C = \{c_1, c_2, \cdots, c_n\}$ be a set of customers and $D = \{d_1, d_2, \cdots, d_k\}$ be a set of depots. Each customer c_i demands delivery of load dm_i. The demand is met by a vehicle servicing it once per order. Each customer c_i has a time window $[et(i), lt(i)]$ such that a vehicle can visit and serve c_i after the time $et(i)$ and is required completion of the service before $lt(i)$. Time $st(i)$ is needed to serve at c_i. Each vehicle v_i can load up to the maximum capacity $cap(i)$ and starts and ends at the same depot.

The objective of the problem can be various. Usually we minimize the number of vehicles to serve all customers and also minimize the total travel distance, without violating the vehicle capacity constraint and the time window constraint for serving at each customer.

2.2 Saving Method

The saving method[5] is a heuristic algorithm to obtain an approximate solution to the VRP, but not to the MDVRPTW. The saving method firstly computes the difference of the total distance between two routing plans with respect to two customers c_i and c_j:

Plan1: two routes each of which includes c_i and c_j, respectively (See Fig. 1 (a)),

Plan2: a route including both c_i and c_j (See Fig. 1 (b)).

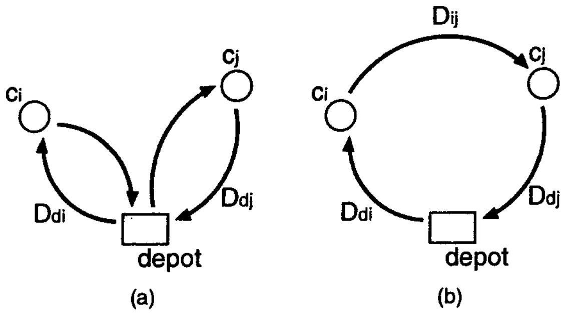
The difference is called *Saving Value*. Let $D_{d,i}$ be the distance between the depot and c_i and $D_{i,j}$ be the distance between c_i and c_j. The saving value SV can be computed as follows:

$$\begin{aligned} SV &= 2D_{d,i} + 2D_{d,j} - (D_{d,i} + D_{i,j} + D_{d,j}) \\ &= D_{d,i} + D_{d,j} - D_{i,j} \end{aligned}$$

That is, the saving value means the reduction length when two independent routes are combined.

At the beginning of the route planning, we assume that there exist n_d independent routes, where n_d is the number of customers assigned to the depot. That is, each route includes just one customer.

The second step of the saving method tries to combine routes as small as possible total cast. In the saving method, starting from one route, a route with higher saving value are combined step by step with considering the capacity constraint. This is a kind of *greedy algorithm* and obtains an approximate solution.



$$SV_{ij} = 2D_{di} + 2D_{dj} - (D_{di} + D_{ij} + D_{dj}) = D_{di} + D_{dj} - D_{ij}$$

Figure 1. Saving Value

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procedure MultiDepotRouting;
begin
  { N: the number of iterations}
  { f(x): the value of the objective function}
  generate x = {D_1, D_2, ..., D_k} by any manner;
  x_best := x;
  repeat
    construct routing plan by the ESM and
      compute f(x') for each x' ∈ N(x);
    select the new decomposition x̂ from N(x)
      according to the TS strategy;
    if f(x) > f(x̂) then x_best := x̂;
    x := x̂;
  until the number of iterations reaches N;
  return x_best;
end;
```

Figure 2. Multi Depot Routing

3. Multi Depot Routing

3.1 Procedure

To solve the MDVRTW, the following two steps are needed:

1. assign all customers to depots (that is, decomposition of the customer set into $|D|$ subsets),
2. construct a routing plan of the assigned customers for each depot.

Step 1 should influence the route planing but obtaining the optimum customer decomposition for the best route planning is very difficult, say NP-hard. Moreover, it is also NP-hard to obtain the optimum routing plan for the assigned customers at each depot since it is a single depot VRP. Therefore, we employ tabu search for the customer decomposition (Step 1) and a traditional heuristic algorithm, called *saving method*, for the route planning after the decomposition (Step 2). Note that we need to extend the original saving method since it can not allow the time window constraint.

For the decomposition problem, we define the neighborhood structure of solution (decomposition) $x = \{D_1, D_2, \cdots, D_k\}$ where D_i represents a set of customers assigned to depot d_i. Neighborhood $\mathcal{N}(x)$ of x is defined as a set of decomposition x' where x' is generated from x by moving one customer from the assigned depot in x to another one. This is called *one-movement neighborhood.*

Figure 2 shows the procedure to solve MDVRTW problems in which ESM represents the extended saving method.

3.2 Extended Saving Method

The original saving method described in Section 2.2 cannot be applied to the MDVRPTW since the time window at each customer is not considered. Therefore we present an extended saving method which introduces a new saving value.

Because of the time window constraint, we need to check the starting time constraint of the customers on the route when we combine a route in the saving method. Even if the starting time constraint is feasible, some vehicle may need to wait for the starting time of the customer on the route. So we need to consider the waiting time for better quality of a approximate solution.

Let $W_{d,i}$ be the waiting time at c_i on the route including only c_i from the depot and $W_{i,j}$ be the total waiting time on the route including c_i and c_j.

The saving value SV_W of waiting time can be computed as follows:

$$\begin{aligned} SV_W &= W_{d,i} + W_{d,j} - (W_{d,i} + W_{i,j}) \\ &= W_{d,j} - W_{i,j} \end{aligned}$$

That is, the saving value of waiting time means the reduction of waiting time when two independent routes are combined.

We refer to the original saving value described in Section 2.2 as SV_D. So now we introduce new saving value as follows:

$$SV = \alpha \cdot SV_D + \beta \cdot SV_W$$

The tradeoff parameters α and β are determined by considering various conditions. By using the new saving value we employ the saving method to the MDVRPTW.

4. Neighborhood Reduction

The size of $\mathcal{N}(x)$ in the one-movement neighborhood is calculated as follows:

$$
\begin{aligned}
|\mathcal{N}(x)| &= (|C| - |D_1|) + (|C| - |D_2|) \\
&\quad + \cdots + (|C| - |D_k|) \\
&= k \cdot |C| - (|D_1| + |D_2| + \cdots |D_k|) \\
&= k \cdot |C| - |C| \\
&= |C|(k-1)
\end{aligned}
$$

where k is the number of depots.

The neighborhood structure of the customer decomposition subproblem is simple and well-defined but it should include excessively bad solution candidates and we can verify such candidates by using problem domain information before searching. That is, our approach is to reduce neighborhood solution candidates with high probability of extremely bad objective value.

The reduction is performed by using geospatial relation between depots and customers. In the one-movement neighborhood, all cases of one customer movement is defined but in the reduced one-movement neighborhood the movement of customer c_h to depot d_i from d_j is allowed only when there exits the geospatial relation among customers and/or depots.

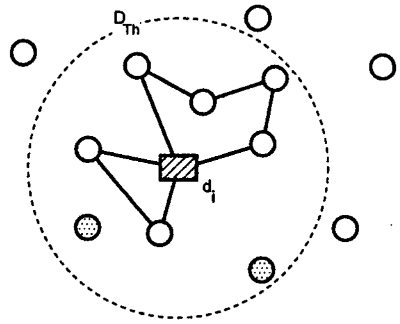
What is the geospatial relation? The geospatial relation can be defined in various ways. In this paper we propose two conditions for the relation. We define the reduced one-movement neighborhood such that the movement of customer c_h to depot d_i from d_j is allowed when

NR1: distance between d_i and c_h is shorter than threshold D_{Th}.

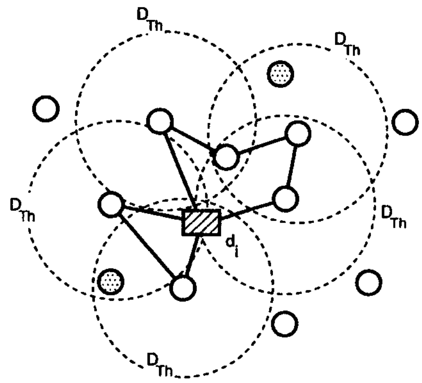
NR2: distance between c_h and any customer assigned to d_i is shorter than threshold D_{Th}.

Figure 3 explains these two conditions. In the figures, polygons show depots and (small) circles customers. The (small) circles connected by lines represent the current assigned customers to the depot shown in the figures while the others are assigned to another depot not shown in the figures. The dotted line circles depict D_{Th}: In (a), we consider the distance between d_i and c_h and in (b) the distance between c_h and all the customers assigned to d_i. The shaded (small) circles in the both figures are available of the reduced one-movement neighborhood. That is, the non-shaded (small) circles are reduced from the neighborhood definition.

We understand that both conditions can reduce drastically the size of the neighborhood comparing with the original one-movement neighborhood and removed candidates (decompositions) have very few possibility to generate better routing plan than the current decomposition.



(a) NR1



(b) NR2

Figure 3. Geospatial Relation

5. Experimental Evaluation

To evaluate the reduction technique, we apply our method to problem instances obtained from *OR-Library(http://mscmga.ms.ic.ac.uk/info.html)*. Table 1 shows the size ratio of the one-movement neighborhood and our reduction method (in $|\mathcal{N}_{NR}|/|\mathcal{N}_{TS}|$), computation time, and solution quality for an instance, *pr03*(4 depots and 144 customers). "TS" represents the traditional tabu search without any neighborhood reduction and "NR1" and "NR2" are our proposed method with Tabu Search. We can observe our reduction technique can contribute to reduce the computation time without losing the solution quality. Figures 4 and 5 show the comparison of quality improvement between TS without any reduction and our methods NR1 and NR2, respectively. Figures 6 and 7 dipict the reduction rate vs. the solution quality and the reduction rate vs. the computation time, respectively.

From these results, we can confirm that our approaches are very effective. NR1 allow us to reduce the neighborhood up to 20% while NR2 up to 10% but NR2 requires more computation time than NR1.

6. Concluding Remarks

In this paper, we proposed neighborhood reduction techniques in local search of the customer decomposition subproblem in the Multi Depots Vehicle Routing Problem with Time Windows (MDVRPTW) by using geospatial relation among depots and customers. The neighborhood of the customer decomposition subproblem can be simply and well defined but it should include excessively bad solution candidates. Our techniques find such candidates by using information of the problem domain, geographical relation. We employed our tech-

Table 1. Evaluation Results (pr03 (4,144))

	TS	NR1	NR2
$\lvert\mathcal{N}_{NR}\rvert/\lvert\mathcal{N}_{TS}\rvert$	-	80%	
comp. time[s]	755.9	604.7	611.7
$f(x_{best})$	9578.0	9578.0	9578.0

	TS	NR1	NR2
$\lvert\mathcal{N}_{NR}\rvert/\lvert\mathcal{N}_{TS}\rvert$	-	60%	
comp. time[s]	755.9	454.4	456.5
$f(x_{best})$	9578.0	9578.0	9578.0

	TS	NR1	NR2
$\lvert\mathcal{N}_{NR}\rvert/\lvert\mathcal{N}_{TS}\rvert$	-	40%	
comp. time[s]	755.9	298.1	298.1
$f(x_{best})$	9578.0	9578.0	9578.0

	TS	NR1	NR2
$\lvert\mathcal{N}_{NR}\rvert/\lvert\mathcal{N}_{TS}\rvert$	-	30%	
comp. time[s]	755.9	223.6	226.6
$f(x_{best})$	9578.0	9560.1	9578.0

	TS	NR1	NR2
$\lvert\mathcal{N}_{NR}\rvert/\lvert\mathcal{N}_{TS}\rvert$	-	20%	
comp. time[s]	755.9	140.3	148.9
$f(x_{best})$	9578.0	9695.8	9578.0

	TS	NR1	NR2
$\lvert\mathcal{N}_{NR}\rvert/\lvert\mathcal{N}_{TS}\rvert$	-	10%	
comp. time[s]	755.9	66.1	72.3
$f(x_{best})$	9578.0	9701.1	9571.6

	TS	NR1	NR2
$\lvert\mathcal{N}_{NR}\rvert/\lvert\mathcal{N}_{TS}\rvert$	-	5%	
comp. time[s]	755.9	29.6	32.9
$f(x_{best})$	9578.0	9732.1	9644.6

niques in Tabu Search and showed the effectiveness by experimental evaluation.

As future works, we need to analyze in detail the neighborhood structure to explain the effectiveness theoretically and also to experiment with lots of large scale benchmark problems.

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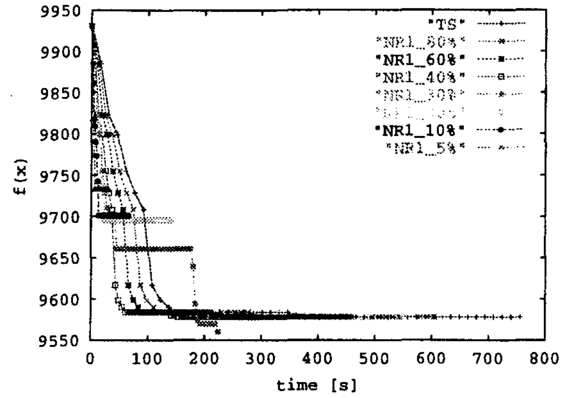

Figure 4. TS vs. NR1

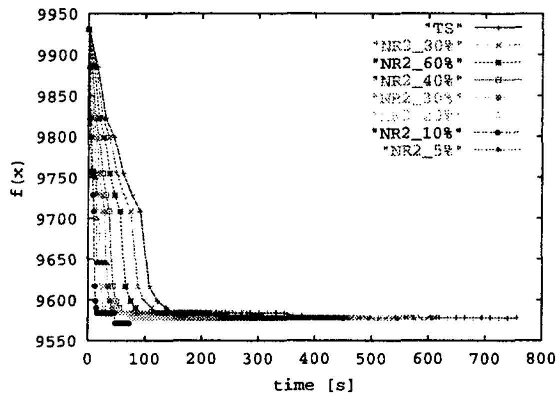

Figure 5. TS vs. NR2

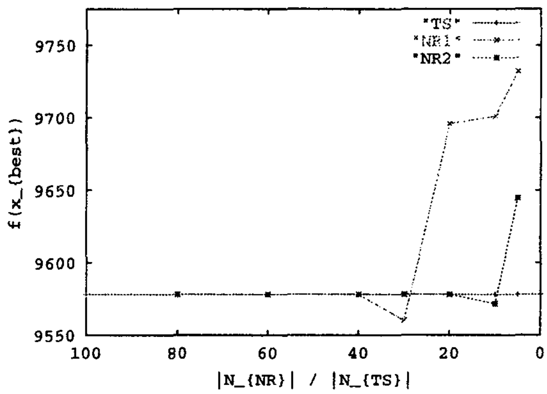

Figure 6. Relation between Reduction Rate and $f(x_{best})$

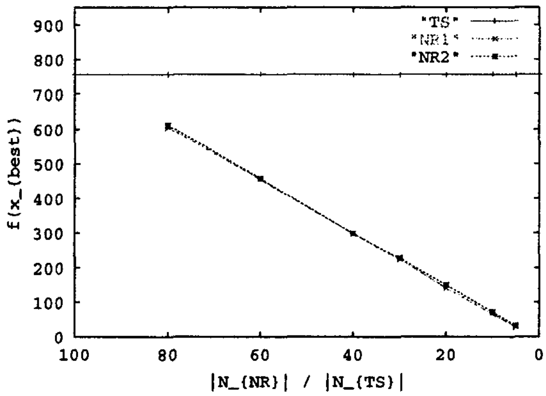

Figure 7. Relation between Reduction Rate and Comp. Time