

# Novel Design Conditions to Optimize Power Coupling in Optical Grating-Assisted Directional Couplers

Kwang-Chun Ho<sup>1</sup>, Hyung-Yun Kong<sup>2</sup>

<sup>1</sup>Department of Information and Communication, Hansung University  
389, Samsun-Dong 2-Ga, Sungbuk-Gu, Seoul 136-792, Korea  
Phone: +82-2-760-4253, Fax: +82-2-760-4435,  
kwangho@m.ieice.org

<sup>2</sup>Department of Electronic Engineering, University of Ulsan  
San 29, MuGeo-Dong, Nam-Gu, Ulsan City, Korea, 680-749  
Phone: +82-52-259-2194, Fax: +82-52-259-1685,  
hkong@uou.ulsan.ac.kr

**Abstract:** By defining a power distribution ratio (PDR) and coupling efficiency (CE) amenable to the rigorous analytical solutions of newly developed rigorous modal transmission-line theory (MTLT), we explicitly analyze the power coupling characteristics of TE modes propagating in GADCs. The numerical results reveal that the incident power is optimally coupled into the desired guiding channel if the powers of rigorous modes excited at the input boundary of grating-assisted coupler are equally partitioned.

## 1. Introduction

Stratified directional couplers with a periodic corrugation structure, which are called grating-assisted directional couplers (GADCs), are increasingly used for many applications in the area of photonics, especially, such optical power distribution devices as WDM and optical switching to implement wavelength selectivity. These couplers have so far been examined to explore the coupling efficiency between two rigorous modes in the context of a two-guide grating coupler [1], and proposed to improve the wavelength filtering performance only in the context of a three-guide grating coupler [2]. Although many applicable examples have adopted these configurations and have explored the modal characteristics, to our best knowledge, the optimal design characteristics of GADCs connecting their boundary interfaces with different guiding structures have excluded.

In this paper, thus we propose a novel criterion, which is called power distribution ratio (PDR), to search an optimal power coupling condition in such GADCs' geometry. As will be discussed in details later, we achieve this condition in terms of equivalent transmission-line network, composing nominally to couple incident power into a desired guiding channel. Furthermore, to verify the validity of our proposed criterion, we evaluate the power coupling efficiency for GADCs with two or three-guiding channels.

## 2. Equivalent Networks of GADCs

Typical grating-assisted configurations with two or three-guiding channels applicable to our proposed approach are illustrated in Fig. 1. As can see, the coupler has two or three-guides so that only two or three propagating rigorous modes have a significant meaning and dominate the power coupling of GADCs. Then, we can consist of

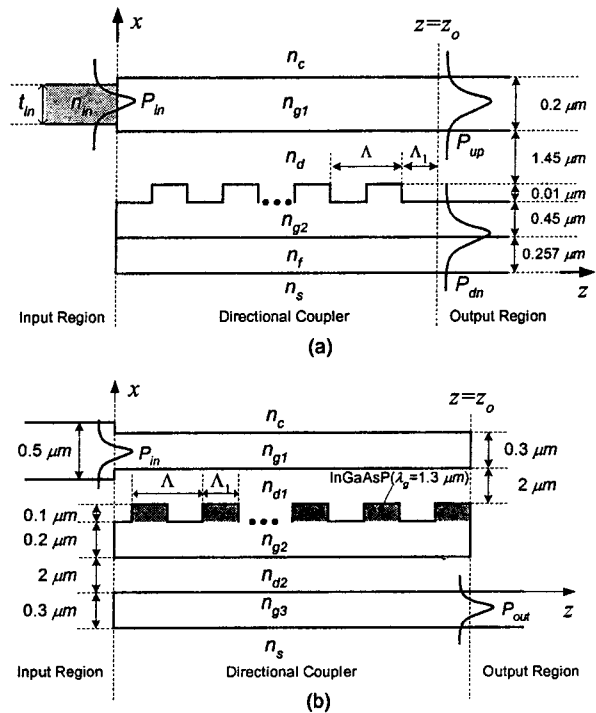


Fig. 1. Geometry of GADCs with (a) two-guiding channels, and (b) three-guiding channels.

the rigorous modes as equivalent network with two or three transmission-lines based on modal transmission-line theory (MTLT) [1, 3], and evaluate all the modal quantities of GADCs from those ones.

For two-dimensional situations and electromagnetic fields with time dependence  $\exp(-i\omega t)$ , as will be assumed throughout this paper, all we have to do is to determine the unknown eigenvalue  $k_{z0}$ . It is calculated by applying transverse resonance condition of MTLT [3], and once determined the quantity  $k_{z0}$ , the propagation constant  $k_{zn}$  of arbitrary  $n$ -th space harmonic is given by

$$k_{zn} = k_{z0} + \frac{2n\pi}{\Lambda} \quad \text{with } k_{z0} = \beta + i\alpha, \quad (1)$$

and the electric and magnetic fields of propagating waves pertinent at a point  $(x, z)$  inside periodic interval  $0 \leq z \leq z_0$  can be precisely defined by modal descriptions [3].

To explore the optimal conditions on power coupling

of GADCs with the field solutions determined, we now assume that a wave is incident into upper guiding channel, as shown in Fig. 1. Then, for TE modes propagating in homogeneous stratified waveguides, the transverse electric  $E_y$  and magnetic  $H_x$  fields at the input ( $z < 0$ ) and the output ( $z > z_0$ ) regions are expressed as [4]

$$E_\xi(x, z) = V_\xi(z)e_\xi(x), \quad H_\xi(x, z) = -I_\xi(z)h_\xi(x), \quad (2)$$

where the modal voltage  $V_\xi$  and current  $I_\xi$  are related by

$$\frac{V_\xi(z)}{I_\xi(z)} = \frac{\omega\mu}{k_{z,\xi}}$$

with the propagation constant  $k_{z,\xi}$  designated  $\xi = in$  or  $out$  for the input or output region, respectively. Here,  $e_\xi(x)$  and  $h_\xi(x)$  denote the electric and magnetic modal functions in uniform stratified guides.

Consequently, the optical wave incident into  $z=0$  from upper guide generates rigorous modes, being guided along longitudinal  $z$ -direction of GADCs and satisfying field orthogonality conditions [1]. Then, the total field in coupling region ( $0 \leq z \leq z_0$ ) can be written by a linear superposition of propagating rigorous modes

$$E_c(x, z) = \sum_{m=1}^M \left\{ V_m(z) \sum_{n=-\infty}^{\infty} e_n^{(m)}(x) e^{i(2n\pi/\Lambda)z} \right\}, \quad (3)$$

where the modal voltage of  $m$ -th mode is  $V_m(z) = V_m(0)e^{ik_{z0,m}z}$ , for which  $k_{z0,m} = \beta_m + i\alpha_m$  designates the propagation constant of two ( $M=2$ ) or three ( $M=3$ ) lowest-order modes coupled within GADCs with two or three-guiding channels, respectively. Also,  $e_n^{(m)}(x)$  represents the spatial variation of  $n$ -th space harmonic traveling along  $x$ -direction.

Then, applying continuity condition to the junction interface between the input (homogeneous) and coupling (inhomogeneous) regions, we have

$$V_{in}(0)e_{in}(x) \cong \sum_{m=1}^M \left\{ V_m(0) \sum_{n=-\infty}^{\infty} e_n^{(m)}(x) \right\}. \quad (4)$$

Subsequently, performing cross-product in Eq. (5) with

$$\sum_{r=-\infty}^{\infty} k_{zr,m} e_r^{(m)}(x) \quad \text{for } k_{zr,m} = k_{z0,m} + \frac{2r\pi}{\Lambda},$$

and integrating over cross-section ( $cs$ ), the modal voltage of  $m$ -th rigorous mode satisfying field orthogonality condition [1] for TE modes is given by

$$V_m(0) = \frac{V_{in}(0)}{C_m} \int_{cs} \left\{ e_{in}(x) \sum_{r=-\infty}^{\infty} k_{zr,m} e_r^{(m)}(x) \right\} dx, \quad (5)$$

where  $C_m$  stands for the normalization constant dependent on the power conservation of equivalent network [1]. The power distribution ratio (PDR), referring to the amount of incident power distributing to each rigorous mode excited in input terminal of GADCs, is then defined as

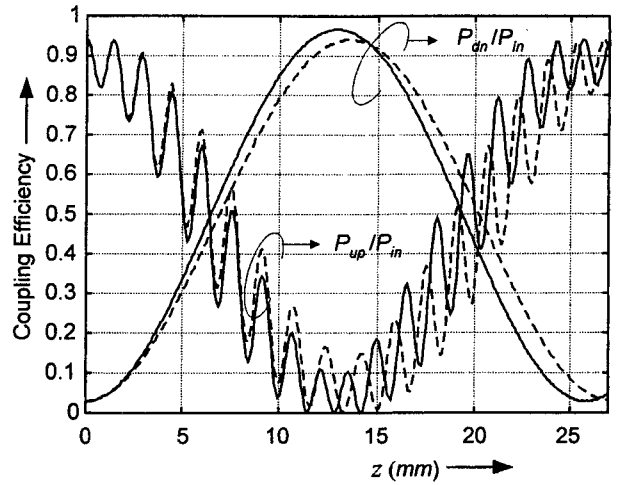


Fig. 2. Variation of coupling efficiency along propagation distance for the narrow-waist and equal PDR conditions.

$$PDR = \frac{P_m(z=0)}{P_{in}(z=0)} = \text{Re} \left( \frac{k_{z0,m}}{k_{z0,in}} \right) \left| \frac{V_m(z=0)}{V_{in}(z=0)} \right|^2. \quad (6)$$

As will be discussed in numerical analysis, it is a significant modal factor that should be evaluated to search an optimal power coupling condition at GADCs with discontinuity interface.

Consequently, the orthogonal rigorous modes excited at  $z=0$  propagate along longitudinal  $z$ -direction, and activate a modal voltage  $V_m(z_0)$  at an arbitrary accessible terminal  $z = z_0$ . Then, similarly to the boundary condition occurred at input interface, the continuity condition at output terminal  $z=z_0$  yields

$$V_{out}(z_0)e_{out}(x) \cong \sum_{m=1}^M \left\{ V_m(z_0) \sum_{n=-\infty}^{\infty} e_n^{(m)}(x) e^{i(2n\pi/\Lambda)z_0} \right\}, \quad (7)$$

and applying the field orthogonality condition [4] of output modal function  $e_{out}(x)$  to Eq. (8), we have output modal voltage  $V_{out}(z_0)$  emitting through one guiding channel desired such that

$$V_{out}(z_0) = \sum_{m=1}^M \left\{ V_m(0) A_m e^{ik_{z0,m}z_0} \right\}, \quad (8)$$

where the voltage transformation constant  $A_m$  at output interface is defined as

$$A_m = \int_{cs} \left\{ e_{out}(x) \sum_{n=-\infty}^{\infty} e_n^{(m)}(x) e^{i(2n\pi/\Lambda)z_0} \right\} dx.$$

Consequently, using the modal mechanism of equivalent network, we can define another convenient and powerful formalism to search an optimal condition on power coupling of TE modes, which is called coupling efficiency (CE). The coupling efficiency is the ratio of the output power ( $V_{out} I_{out}^*$ ) to the input power ( $V_{in} I_{in}^*$ ), which yields

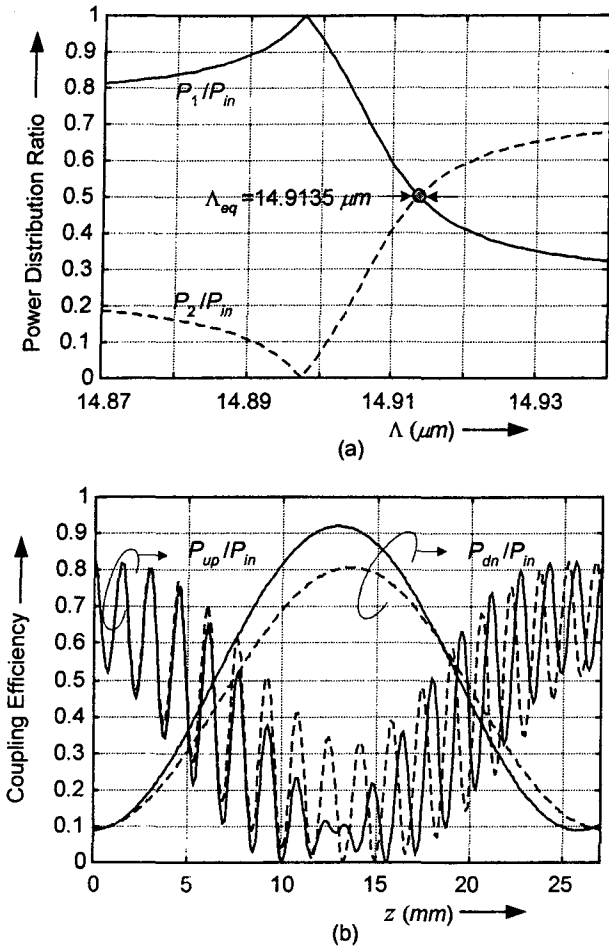


Fig. 3. (a) PDR for two-channel GADC with discontinuity of strong type. (b) Variation of coupling efficiency along propagation distance for the narrow-waist and equal PDR conditions.

$$CE = \frac{P_{out}(z_0)}{P_{in}(0)} = \text{Re} \left( \frac{k_{z,out}}{k_{z,in}} \right) \left| \frac{V_{out}(z_0)}{V_{in}(0)} \right|^2 \quad (9)$$

in terms of Eqs. (5) and (8), where  $k_{z,in}$  and  $k_{z,out}$  are the propagation constants at the input and output regions, respectively. Although the modal characteristics of TM modes are not mentioned here, their quantitative properties may be a little bit different from TE modes in many integrated-optical applications. The readers interesting in the explicit formulation and physical meaning for TM modes refer to Ref. [1].

### 3. Numerical Results and Discussions

Recently, the coupling efficiency of power transfer is improved by adopting the narrowest gap of rigorous leaky-wave modes instead of phase-matching condition of surface-wave modes [1]. Of course, this criterion is true if GADCs have coupling region of infinite length and does not connect with any other outer devices such as planar waveguides with different type or optical fibers. However, if the boundary interfaces of GADCs connect with different guiding structures, we cannot adopt anymore the condition and need to develop a newly unified condition

called power distribution ratio (PDR) in this paper. The novel criterion is that optimal condition on power coupling of GADCs mainly depend on whether the incident power is equally distributed into each power of rigorous modes generated at the input boundary of GADCs or not.

To verify the validity of our proposed condition, we evaluate optimal power coupling of GADC with two guiding channels, which was originally analyzed by Marcuse [5]. As shown in Fig. 1(a), GADC is made by using different compositions of InGaAsP in different layers, and the refractive indexes translated by band-edge wavelength  $\lambda_g$  are:  $n_c = n_d = n_s = 3.18$  ( $\lambda_g = 0.93 \mu\text{m}$ ),  $n_{g1} = n_{g2} = 3.282$  ( $\lambda_g = 1.1 \mu\text{m}$ ), and  $n_f = 3.45$  ( $\lambda_g = 1.38 \mu\text{m}$ ). The operating wavelength is then held fixed at  $\lambda = 1.55 \mu\text{m}$ .

First, when the dimensions of input guiding structure are similar to those of GADC such as  $t_{in} = t_{g1}$  and  $n_{in} = n_{g1}$  for the grating depth of  $0.01 \mu\text{m}$ , we analyze the power coupling at condition addressing in published papers and our proposed condition. For the equal PDR condition  $\Lambda_{eq} = 14.91 \mu\text{m}$ , the solid curve in Fig. 2 shows that the coupling efficiency (CE) from upper guide to lower guide is almost 97% at coupling length  $z_0 = 12.9 \text{ mm}$ . On the contrary, the dashed curve for conventional condition  $\Lambda_{min} = \Lambda_{ph} = 14.907 \mu\text{m}$  presents that CE is only 94% at coupling length  $z_0 = 13.5 \text{ mm}$ . Thus, those results imply that the conventional regime does not always yield optimal power coupling between two grating-coupled guides.

On inspecting the power transfer of GADC connected with different input geometry in which  $t_{in} = 0.05 \mu\text{m}$  and  $n_{in} = 3.45$ , we find that CE can be improved if an equal PDR condition is chosen instead of the conventional condition independent on outer guiding structures connected. The solid curve of Fig. 3(b) shows that CE is almost 91% at coupling length  $z_0 = 12.9 \text{ mm}$  for a grating period  $\Lambda_{eq} = 14.9135 \mu\text{m}$ , in which PDR of two rigorous modes intersects each other as shown in Fig. 3(a). However, the dashed curve for conventional condition  $\Lambda_{min} = \Lambda_{ph} = 14.907 \mu\text{m}$  shows that CE at coupling length  $z_0 = 13.6 \text{ mm}$  is about 80% being 11% less value than that for equal PDR condition. As a result, we know that PDR of rigorous modes rather than modal characteristic of propagation constants supplies a unified criterion to determine optimal condition on power coupling of GADCs, no matter what they connect.

Finally, we evaluate another applicable example to announce our proposed optimal condition, that is, the power coupling in three-guide GADCs with discontinuity input interface as shown in Fig. 1(b). In general, an interesting feature for this kind of optical device is complete power transfer between outside guides. However, unfortunately the dispersion curve of three rigorous modes provides neither deterministic information nor an

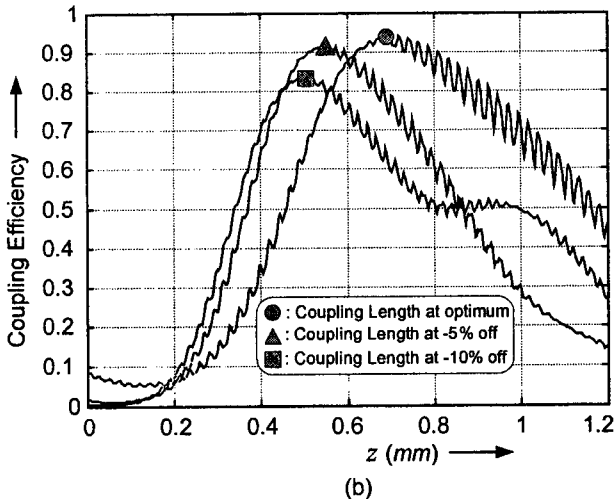
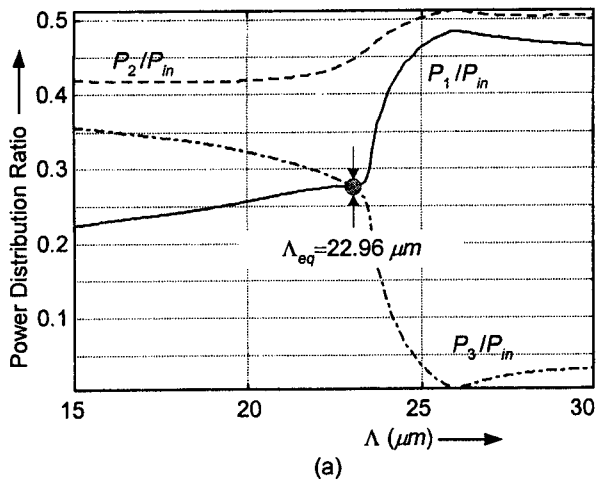


Fig. 4. (a) PDR for three-channel GADC given in Fig. 1(b). (b) Variation of coupling efficiency along propagation distance at the intersection point of PDR,  $-5\%$ , and  $-10\%$  walk-off from the point.

excellent criterion to evaluate such power property of three-guide GADCs considered here. Thus, an alternative method has to be chosen, and this can be achieved by PDR of 1<sup>st</sup>-order and 3<sup>rd</sup>-order rigorous modes, which dominate the power coupling of outside guides.

Such a behavior is illustrated in Fig. 4. Here the refractive indexes of GADC structure composed by InP-InGaAsP materials are chosen as 3.413 for InGaAsP ( $\lambda_g = 1.3 \mu\text{m}$ ),  $n_c = n_{d1} = n_{d2} = n_s = 3.155$  for InP, and  $n_{g1} = n_{g2} = n_{g3} = 3.282$  for InGaAsP ( $\lambda_g = 1.1 \mu\text{m}$ ) at the free-space optical wavelength  $\lambda = 1.55 \mu\text{m}$ . As expected, PDR of 1<sup>st</sup>-order and 3<sup>rd</sup>-order modes intersects each other at  $\Lambda_{eq} = 22.96 \mu\text{m}$ , and CE is almost 94% at coupling length  $z_0 = 717 \mu\text{m}$  for the grating period.

Subsequently, we explore CE for the cases for which the grating period is  $-5\%$  and  $-10\%$  off from its optimum value  $\Lambda_{eq} = 22.96 \mu\text{m}$ . Figure 4(b) shows that the CE between two outside guides becomes about 91% at coupling length  $z_0 = 525 \mu\text{m}$  for  $-5\%$  walk-off. Furthermore, the power coupling of about 83% occurs at

the coupling length  $585 \mu\text{m}$  for  $-10\%$  walk-off. It means that the 10% change from the value of optimal grating period reduces the power coupling by 11%. However, if the discontinuity between the input and coupling regions is of a strong type rather than a weak type as the application example considered here, the reduction of power coupling along the increase of offset value of optimal grating period will be remarkably presented.

#### 4. Conclusion

We have analyzed a unified optimal condition on power coupling of grating-assisted direction couplers (GADCs) with two or three-guiding channels, which serve as optical power dividers with filtering property. Unlike such conventional conditions as the phase-matching or narrow-waist condition between rigorous modes, we have found that the power coupling of GADCs connected with outer devices of different type can be optimized by power distribution ratio (PDR) of rigorous modes generated at the interface of coupling region.

In the case of GADC with two-guiding channels, the coupling efficiency (CE) has dramatically enhanced when an intersection point of PDR is chosen and the discontinuity between the input and coupling regions is strong type. Furthermore, the numerical results for three-channel GADC have revealed that our proposed condition can easily and generally take up into multi-channel GADCs, no matter whether optical directional couplers are periodic or non-periodic geometry.

#### References

- [1] K. C. Ho and Y. K. Kim, "On Leaky-Wave Approach of Rigorous Modes Coupled in Multilayered Periodic Waveguides," *IEICE Trans. Electron.*, Vol. E84-C, No. 1, pp. 84-95, 2001.
- [2] K. Watanabe and K. Yasumoto, "Coupled-mode analysis of waveguide filtering in a grating-assisted asymmetric three-waveguide directional coupler," *J. Opt. Soc. Amer.*, Vol. 14, pp. 2994-3000, 1997.
- [3] T. Tamir and S. Zhang, "Modal Transmission-Line Theory of Multilayered Grating Structures," *J. Lightwave Technol.*, Vol. 14, pp. 914-927, 1996.
- [4] K. C. Ho, G. Griffel and T. Tamir, "Polarization Splitting in Lossy/Gainy MQW Directional Couplers," *J. Lightwave Technol.* Vol. 15, pp. 1233-1240, 1997.
- [5] D. Marcuse, "Directional Couplers made of Nonidentical Asymmetrical Slabs. Part II: Grating-Assisted Couplers," *J. Lightwave Technol.*, Vol. 7, pp. 268-273, 1987.