A Direct Method to Derive All Generators of Solutions of a Matrix Equation in a Petri Net

— Extended Fourier-Motzkin Method —

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Abstract; In this paper, the old Fourier-Motzkin method (abbreviated as the old FM method from now on) is first modified to the form which can derive all minimal vectors as well as all minimal support vectors of nonnegative integer homogeneous solutions (i.e., T-invariants) for a matrix equation $Ax=b=0^{m\times1}$, $A\in Z^{m\times n}$ and $b\in Z^{m\times1}$, of a given Petri net, where the old FM method is a well-known and direct method that can obtain at least all minimal support solutions for $Ax=0^{m\times1}$ from the incidence matrix $A\in Z^{m\times n}$. Secondly, for $Ax=b\neq0^{m\times1}$, a new extended FM method is given; i.e., all nonnegative integer minimal vectors which contain all minimal support vectors of not only homogeneous but also inhomogeneous solutions are systematically obtained by applying the above modified FM method to the augmented incidence matrix $\tilde{A}=[A,-b]\in Z^{m\times(n+1)}$ s.t. $\tilde{A}\tilde{x}=0^{m\times1}$. However, note that for this extended FM method we need some criteria to obtain a minimal vector as well as a minimal support vector from both of nonnegative integer homogeneous and inhomogeneous solutions for $Ax=b$. Then those criteria are also discussed and given in this paper.

1. Introduction

Petri nets are one of promising models applicable to discrete event systems and concurrent systems. Many properties of systems modeled by Petri nets are obtained by solving a matrix equation $Ax=b$. An arbitrary non-negative integer solution $x\in Z_+^{n\times1}$ of a matrix equation $Ax=b$ is expressed or generated at level 4 (level 5, resp.) by each minimal support vector (each minimal vector, resp.) of both of nonnegative integer homogeneous and inhomogeneous solutions. The level is defined due to attitude of those generating vectors and the expansion coefficients. Those generating vectors are called the generators of an arbitrary solution. The analyses of Petri nets become efficient by executing of each generator of a solution or a set of solutions with certain characteristics. Therefore some systematic and efficient methods to obtain all generators of solutions at level 4 or level 5 are necessary.

Then the Fourier-Motzkin (abbreviated to FM) method is adopted as a direct method in this paper. However, the old FM method [1], [3] can directly generate at least all minimal support vectors, but not, in general, all minimal vectors of nonnegative integer homogeneous solutions for $Ax=b=0^{m\times1}$ from the incidence matrix $A\in Z^{m\times n}$, where the rank condition for a submatrix of the incidence matrix A is used to find only minimal support vectors. Therefore the old FM method has not been applied to derive all minimal vectors, i.e., all minimal T-invariants, of nonnegative integer homogeneous solutions for an arbitrary net $Ax=b=0^{m\times1}$. Another drawback of the old FM method is as follows: Even if the old FM method is applied to the augmented incidence matrix $\tilde{A}=[A,-b]$, i.e., to $\tilde{A}\tilde{x}=0^{m\times1}$, only a part, not all, of minimal support vectors of nonnegative integer inhomogeneous solutions for $Ax=b\neq0^{m\times1}$ are just obtained, in general. This is due to the fact that a part of nonnegative integer minimal support vectors (i.e., a part of particular solutions at level 4) of inhomogeneous solutions for $Ax=b\neq0^{m\times1}$ are corresponding to the nonnegative integer minimal, but not minimal support, solutions with the $(n+1)$ -th unity element on $\tilde{x}\in Z_+^{(n+1)\times1}$ for $\tilde{A}\tilde{x}=0^{m\times1}$ [4], [5]. Then, for some nets, it will happen that a part of nonnegative integer minimal support solutions at level 4 for $Ax=b\neq0^{m\times1}$ are not obtained by applying the old FM method to $\tilde{A}=[A,-b]$.

From the above facts, in this paper, we will give a new FM method which is doubly extended from the well-known and old FM method, where all minimal vectors (i.e., the level 5 generators) which contain all minimal support vectors (i.e., the level 4 generators) of not only nonnegative integer homogeneous solutions but also nonnegative integer inhomogeneous solutions of $Ax=b$ are systematically derived.

2. Preliminaries

The following definitions with respect to solutions of $Ax=b$ are used in this paper. [2]

① A homogeneous solution of $Ax=b$; an $n\times1$ matrix x is called a homogeneous solution or a T-invariant, where A is an $m\times n$ matrix and $b=0^{m\times1}$ is the $m\times1$ zero matrix.

② An inhomogeneous solution of $Ax=b$; an $n\times1$ matrix x is called an inhomogeneous solution, where A is an $m\times n$ matrix and $b\neq0^{m\times1}$ is the nonzero $m\times1$ matrix.

③ A particular solution of $Ax=b$; at each level, an inhomogeneous solution x of $Ax=b$ is called a particular solution if and only if x is never expressed by the sum which contains at least a homogeneous solution.
④ A basic particular solution of $Ax=b$; a particular solution x of $Ax=b$ is called a basic particular solution if and only if x is never expressed by the sum together with the other particular solutions of $Ax=b$.
Although a basic particular solution is always a particular solution, the converse is not always true.
⑤ Support of $x \in Z_+^{n\times 1}$; the set of transitions (elements, resp.) corresponding to nonzero entities in a T-invariant (a solution, resp.) $x \geq 0^{n\times 1}$ is called the support of a T-invariant (a solution, resp.) $x \geq 0^{n\times 1}$ and is denoted $\mathrm{support}(x)$. A support is said to be minimal if no proper nonempty support of the support is also a support.
⑥Let $x(i)$ be the i-th element of $x \in Z_+^{n\times 1}$. Minimal vector; a vector (T-invariant, resp.) $x=(x(i)) \in Z_+^{n\times 1}$ is said to be minimal if there is no other vector (T-invariant, resp.) $x_1=(x_1(i)) \in Z_+^{n\times 1}$ such that $x_1(i) \leqq x(i)$ for all elements $i \in \{1,\cdots,n\}$ (all transitions $t_i \in T$, resp.).
See [2] for Petri net terminology.
⑦ A minimal support (i.e., an elementary) vector; given a minimal support of a vector, there exists a unique minimal vector corresponding to the minimal support. We call such a vector a minimal support (i.e., an elementary) vector.
Every minimal support (i.e., every elementary) vector is a minimal vector, but the converse is not always true.

3. Generators (U_L, V_L) for $L=3, 4, 5$

The level is divided into five according to the attitude of the generating vectors and the expansion coefficients. The generators at level 3,4, and 5 are as follows. [4]

Level 3: $u_i \in U_3 := \{ u_i \in Q_+^{n\times 1} \mid$ nonnegative rational number minimal support vectors of $Ax=0^{m\times 1}\}$, $v_j \in V_3 := \{ v_j \in Q_+^{n\times 1} \mid$ nonnegative rational number minimal support vectors of $Ax=b \neq 0^{m\times 1}\}$.

Level 4: $u_i \in U_4 := \{ u_i \in Z_+^{n\times 1} \mid$ nonnegative integer minimal support vectors (minimal support T-invariants) of $Ax=0^{m\times 1}\}$, $v_j \in V_4 := \{ v_j \in Z_+^{n\times 1} \mid$ nonnegative integer minimal support vectors (basic particular solutions) of $Ax=b \neq 0^{m\times 1}\}$.

Level 5: $u_i \in U_5 := \{ u_i \in Z_+^{n\times 1} \mid$ nonnegative integer minimal vectors (minimal T-invariants) of $Ax=0^{m\times 1}\}$, $v_j \in V_5 := \{ v_j \in Z_+^{n\times 1} \mid$ nonnegative integer minimal vectors (particular solutions) of $Ax=b \neq 0^{m\times 1}\}$.

Here, $u_i \in U_4$ is obtained by multiplying some non-negative integer from $u_i \in U_3$. However, $v_j \in V_4$ is not obtained from $v_j \in V_3$ by the same way.

It is known that a minimal, but not minimal support, T-invariant ($u_i \in U_5 \setminus U_4$) is expressed by nonnegative rational number weight linear combination of $u_i \in U_4$ and a particular, but not basic particular, solution ($v_j \in V_5 \setminus V_4$) is expressed by convex combination of $v_j \in V_4$. However, note that it is not easy in general to generate $U_5 \setminus U_4$ and $V_5 \setminus V_4$ from U_4 and V_4, respectively.

4. Old and New Fourier-Motzkin Methods

4.1 The Usual and Old Fourier-Motzkin Method

The method for obtaining the set of T-invariants which include at least all minimal support T-invariants is called the usual and old Fourier-Motzkin method in this paper. However, it is noted that this method can not always obtain minimal, but not minimal support, T-invariants $u_i \in U_5 \setminus U_4$. The usual and old Fourier-Motzkin method is as follows. [1], [2], [3]

< **Old FM Method** >
Input; Incidence matrix $A \in Z^{m\times n}$,
Output; The set of T-invariants including all minimal support T-invariants.
Initialization; The matrix B is constructed by adjoining the identity matrix $E^{n\times n}$ to the bottom of the incidence matrix $A \in Z^{m\times n}$, with $B=[A^T E]^T \in Z^{(m+n)\times n}$.
Following operation a), b) are repeated from $i=1$ to $m=|P|$, where $|P|$ means the cardinality of the place set P.

a) Add to the matrix B all the columns which are linear combinations of pairs of columns of B and which annul the i-th row of B.
b) Eliminate from B the columns in which the i-th element is nonzero. ■

When this algorithm finished, each column of the submatrix $C \in Z_+^{n\times r}$ which is obtained by deleting the rows from the first to the m-th from the final outputted matrix $B \in Z_+^{(m+n)\times r}$ is a T-invariant. However, in general, this submatrix C includes also non-minimal-support T-invariants. Therefore if the following operation c) is added and applied to C, minimal support T-invariants are only obtained.

c) Each column vector $u_i \in Z_+^{n\times 1}$ which satisfies the rank condition $q(u_i) \geqq \mathrm{rank}(A'(u_i))+2$ is removed from the submatrix $C=[u_i] \in Z_+^{n\times r}$. Here, $q(u_i)$ is the number of nonzero elements of $u_i \in Z_+^{n\times 1}$ for $Au_i=0^{m\times 1}$ and $A'(u_i)$ is composed of the columns of A, of which columns are corresponding to nonzero elements of $u_i \in Z_+^{n\times 1}$.

4.2 Modified Fourier-Motzkin Method

Note that $u_i \in U_4$ is always obtained, but $u_i \in U_5 \setminus U_4$ can not be always obtained by applying the usual and old Fourier-Motzkin method to the incidence matrix $A \in Z^{m\times n}$. Therefore, we must modify the old Fourier-Motzkin method as follows to obtain always all of minimal T-invariants for $Ax = 0^{m\times 1}$.

< Modified FM Method >

Input; Incidence matrix $A \in Z^{m\times n}$,

Output; All of minimal T-invariants.

Initialization; The matrix B is constructed by adjoining the identity matrix $E^{n\times n}$ to the bottom of the incidence matrix $A \in Z^{m\times n}$, with $B = [A^T E]^T \in Z^{(m+n)\times n}$. Let the column index set of B be $I = \{1, \cdots, r\}$.

Step 0; $i = 1$, $j = 1$, $k = 1$, $r = n$.

Step 1; Find the i-th row of B. If the i-th row has no zero element, then go to Step 2. If the i-th row has at least one nonzero element, then go to Step 3.

Step 2; $i = i + 1$ and go to Step 3.

Step 3; If $i \leqq m$, then go to Step 4 else go to Step 14.

Step 4; If the i-th row of B has at least one pair of positive and negative elements, then go to Step 5 else go to Step 12.

Step 5; If the (i, j) element is negative, then go to Step 6 else go to Step 10.

Step 6; If the (i, k) element is positive, then go to Step 7 else go to Step 8.

Step 7; Let b^j and b^k be the j-th and k-th columns of B, respectively. Adjoin the new column $b^j + b^k$ to the matrix B and $r = r + 1$.

Step 8; $k = k + 1$ and go to Step 9.

Step 9; If $k \leqq r$, then go to Step 6 else go to Step 10.

Step 10; $j = j + 1$ and go to Step 11.

Step 11; If $j \leqq r$, then go to Step 5 else go to Step 12.

Step 12; Let $F = \{l \mid a_{il} = 0\}$. $|F|$ denotes the number of elements of F. Let the new matrix which has all columns of b^l for $l \in F$ be B. $r = |F|$ and go to Step 13.

Step 13; Non-minimal vectors are removed from B by comparing columns of the submatrix $C \in Z_+^{n\times r}$ which is obtained by deleting the rows from the first to the m-th from B. Let q be the number of non-minimal vectors for B. $r = |F| - q$ and go to Step 2.

Step 14; Each column of the submatrix $C \in Z_+^{n\times r}$ which is obtained by deleting the rows from the first to the m-th from B is a minimal T-invariant. ■

The decision of minimal vectors in Step 13 is based upon the definition for a minimal vector which is given in ⑥ of section 2. Notice that $C \in Z_+^{n\times r}$ is a nonnegative integer matrix.

4.3 Extended Fourier-Motzkin Method

The modified Fourier-Motzkin method is essentially the method to obtain T-invariants of $Ax = 0^{m\times 1}$, but it can be also used to obtain particular solutions of $Ax = b$ by applying it to the augmented state equation $\tilde{A}\tilde{x} = 0^{m\times 1}$, where $\tilde{A} = [A, -b] \in Z^{m\times(n+1)}$. In this paper, let us call this new method which can generate $U_5 \supseteq U_4$ as well as $V_5 \supseteq V_4$ the extend FM method. This is briefly described as follows.

< Extended FM Method >

① Apply the modified FM method of section 4.2 to the augmented incidence matrix $\tilde{A} = [A, -b] \in Z^{m\times(n+1)}$ and obtain all minimal T-invariants $\tilde{u}_i \in Z_+^{(n+1)\times 1}$ of $\tilde{A}\tilde{x} = 0^{m\times 1}$.

② Now we can find the set of generators (U_5, V_5) at level 5 for $x \in Z_+^{n\times 1}$ in $Ax = b$ from $\tilde{u}_i \in \tilde{U}_5 := \{\tilde{u}_i \in Z_+^{(n+1)\times 1} \mid$ nonnegative integer minimal T-invariants of $\tilde{A}\tilde{x} = 0^{m\times 1}\}$.

(1) If $\tilde{u}_i(n+1) = 0^{1\times 1}$ on $\tilde{u}_i \in \tilde{U}_5$, then find $u_i := (\tilde{u}_i(1), \cdots, \tilde{u}_i(n))^T \in U_5$.

(2) If $\tilde{u}_j(n+1) = 1^{1\times 1}$ on $\tilde{u}_j \in \tilde{U}_5$, then find $v_j := (\tilde{u}_j(1), \cdots, \tilde{u}_j(n))^T \in V_5$.

(3) If $\tilde{u}_k(n+1) > 1^{1\times 1}$ on $\tilde{u}_k \in \tilde{U}_5$, then $x_j := (\tilde{u}_k(1), \cdots, \tilde{u}_k(n))^T$ is not a solution of $Ax = b$. ■

Remarks

(1) Since $|U_4| \leqq |U_5|$ and $|V_4| \leqq |V_5|$, we want to have (U_4, V_4) rather than (U_5, V_5) on occasion at the expense of difficulty to determine the expansion coefficients. Then we need to distinguish $U_5 \setminus U_4$ and U_4 as well as $V_5 \setminus V_4$ and V_4, where $U_5 \supseteq U_4$ and $V_5 \supseteq V_4$.

(2) The rank conditions are useful for (1), where q and $\mathrm{rank}(A')$ are defined in subsection 4.1. For $u_i \in U_4$, $q(u_i) = \mathrm{rank}(A'(u_i)) + 1$ and for $u_k \in U_5 \setminus U_4, q(u_k) \geqq \mathrm{rank}(A'(u_k)) + 2$. These have been given in [1]. For $v_j \in V_4$, $q(v_j) = \mathrm{rank}(A'(v_j))$ or $q(v_j) = \mathrm{rank}(A'(v_j)) + 1$. For $v_l \in V_5 \setminus V_4$, $q(v_l) \geqq \mathrm{rank}(A'(v_l)) + 2$. These have been proved in [5].

(3) Another properties for (1) are the support conditions which compare with two generators $u_i, u_k \in U_5 \supseteq U_4$ with respect to $\mathrm{support}(u_i)$ and $\mathrm{support}(u_k)$ as well as $v_j, v_l \in V_5 \supseteq V_4$. These are based on the facts that U_4 and V_4 are minimal support vectors and while $U_5 \setminus U_4$ and $V_5 \setminus V_4$ are minimal vectors. [4], [5] ■

[Example]

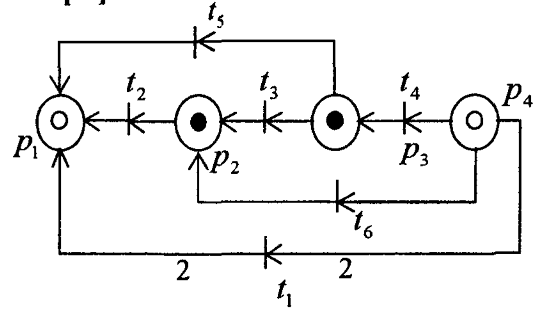

Figure 1 A simple example of a Petri net

We have the incidence matrix A and the marking difference vector b for Figure 1 as, where ● (○ , resp.) on a place is the initial (destination, resp.) marking;

$$A=\begin{bmatrix}-2 & 1 & 0 & 0 & 1 & 0\\ 0 & -1 & 1 & 0 & 0 & 1\\ 0 & 0 & -1 & 1 & -1 & 0\\ 2 & 0 & 0 & -1 & 0 & -1\end{bmatrix},\ b=\begin{bmatrix}1\\ -1\\ -1\\ 1\end{bmatrix}.$$

If the old FM method without the operation c) in subsection 4.1 is applied to $A \in Z^{4\times6}$, generators for $Ax = 0^{4\times1}$ are obtained as follows; $U_4 = \{u_1, u_2, u_3\}$, where $u_1 = (1\,2\,0\,0\,0\,2)^T$, $u_2 = (1\,0\,0\,2\,2\,0)^T$, and $u_3 = (1\,2\,2\,2\,0\,0)^T$.

If the modified FM method in subsection 4.2 is applied to $A \in Z^{4\times6}$, generators for $Ax = 0^{4\times1}$ are shown as follows by referring (2) of Remarks; $U_5 = \{U_4, U_5 \backslash U_4\}$, $U_5 \backslash U_4 = \{u_4, u_5, u_6\}$, where $u_4 = (1\,1\,0\,1\,1\,1)^T = (1/2)u_1 + (1/2)u_2$, $u_5 = (1\,2\,1\,1\,0\,1)^T = (1/2)u_1 + (1/2)u_3$, and $u_6 = (1\,1\,1\,2\,1\,0)^T = (1/2)u_2 + (1/2)u_3$.

Moreover, we apply the modified FM method of subsection 4.2 to the augmented incidence matrix $\widetilde{A} = [A, -b] \in Z^{4\times7}$ (i.e., we use the extended FM method of subsection 4.3 for $\widetilde{A}$) and obtain $\widetilde{u}_i \in \widetilde{U}_5 = \{\widetilde{u}_i \in Z_+^{7\times1} \mid$ all minimal T- invariants of $\widetilde{A}\widetilde{x} = 0^{4\times1}$ at level 5 for the augmented system $\widetilde{A}\widetilde{x} = 0^{4\times1}\}$ as follows.

$$\widetilde{U}_5 = \left\{\begin{matrix}1&1&1&1&1&1&1&1&1&1&1&1&1&1\\ 2&1&2&2&3&0&1&2&1&2&3&2&3&4\\ 0&0&0&1&1&0&0&0&1&1&1&2&2&2\\ 0&1&0&1&0&2&1&0&2&1&0&2&1&0\\ 0&1&1&0&0&2&2&2&1&1&1&0&0&0\\ 2&1&1&1&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&1&0&1&2&0&1&2&0&1&2\end{matrix}\right\}$$

(1) For $\widetilde{U}_5 = \{\widetilde{u}_i\} \in Z_+^{7\times14}$, if each set of vectors $\widetilde{u}_i \in \widetilde{U}_5$ has the seventh element with zero, we have a minimal T-invariant $u_i \in Z_+^{6\times1}$ of $Au_i = 0^{4\times1}$ deleting the seventh zero element from $\widetilde{u}_k \in Z_+^{7\times1}$, k= 1, 2, 4, 6, 9, 12. Then we have the same $U_5 = \{u_1, \cdots, u_6\}$ as the above U_5 obtained by the modified FM method.

(2) For $\widetilde{U}_5 = \{\widetilde{u}_i\} \in Z_+^{7\times14}$, if each set of vectors $\widetilde{u}_i \in \widetilde{U}_5$ has the seventh element with unity, we have a non-negative integer particular solution $v_j \in Z_+^{6\times1}$ of $Ax = b$ deleting the seventh unity element from $\widetilde{u}_k \in Z_+^{7\times1}$, k= 3, 5, 7, 10, 13. Then we have V_5, the set of all nonnegative integer particular solutions of this example as follows. $V_5 = \{v_1, \cdots, v_5\}$, where $v_1 = (1\,2\,0\,0\,1\,1)^T$, $v_2 = (1\,3\,1\,0\,0\,1)^T$, $v_3 = (1\,1\,0\,1\,2\,0)^T$, $v_4 = (1\,3\,2\,1\,0\,0)^T$ and $v_5 = (1\,2\,1\,1\,1\,0)^T = (1/2)v_3 + (1/2)v_4$. Note also that we can classify V_5 as $V_5 = \{V_4, V_5 \backslash V_4\} = \{\{v_1, \cdots, v_4\}, \{v_5\}\}$ by using the rank conditions in (2) of Remarks.

(3) Finally for $\widetilde{U}_5 = \{\widetilde{u}_i\} \in Z_+^{7\times14}$, if each set of vectors $\widetilde{u}_i \in \widetilde{U}_5$ has the seventh nonzero element more than unity, we do not care it because each $u_i \in Z_+^{6\times1}$ obtained from $\widetilde{u}_k \in Z_+^{7\times1}$, k= 8, 11, 14, by deleting the seventh element does not satisfy $Ax = b$.

On the contrary, if we apply the old FM method to $\widetilde{A}$ of this example, we obtain only $\widetilde{U}_5' = \{\widetilde{u}_1, \widetilde{u}_6, \widetilde{u}_8, \widetilde{u}_{12}, \widetilde{u}_{14}\}$, where we have u_1, u_2, and u_3 from $\widetilde{u}_1$, $\widetilde{u}_6$, and $\widetilde{u}_{12}$, respectively, but $\widetilde{u}_8$ and $\widetilde{u}_{14}$ are not solutions for $Ax = b$ and these two belong to the above item (3). ■

6. Conclusions

We have first proposed the modified FM method which can always generate all minimal T-invariants $U_5 \supseteq U_4$, while the old FM method can not always do, for $Ax = 0^{m\times1}$ in P/T Petri nets. We have secondly given the extended FM method which can always generate all particular solutions $V_5 \supseteq V_4$ as well as $U_5 \supseteq U_4$ for $Ax = b \neq 0^{m\times1}$ in P/T Petri nets, by applying the modified FM method to the augmented system $\widetilde{A}\widetilde{x} = 0^{m\times1}$ of the original one $Ax = b$, where $\widetilde{A} = [A, -b] \in Z^{m\times(n+1)}$. We have also shown some criteria for having V_4 from $V_5 \supseteq V_4$, i.e., how to distinguish $V_5 \backslash V_4$ and V_4 [5], whereas the criterion for U_4 from $U_5 \supseteq U_4$ has been well-known [1].

It is noted that the results of this paper are useful for discussing the general properties through the state equation approach because we have also found generators for particular solutions as well as T-invariants at level 4 and 5.

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