

# A Tabu Search Algorithm to Optimal Weight Selection in Design of Robust $H_\infty$ Power System Stabilizer

S. Dechanupaprittha and I. Ngamroo

Electrical Power Engineering Program

Sirindhorn International Institute of Technology,

Thammasat University, Pathumthani 12121, Thailand

Phone: (+66) 2986-9009 ext. 3315, E-mail: sanchai@siit.tu.ac.th

**Abstract:** This paper proposes a tabu search (TS) algorithm to optimal weight selection in design of robust  $H_\infty$  power system stabilizer (PSS). In  $H_\infty$  control design, the weight selection and the representation of system uncertainties are the major difficulties. To cope with these problems, TS is employed to automatically search for the optimal weights. On the other hand, the normalized coprime factorization (NCF) is used. The  $H_\infty$  controller can be directly developed without  $\gamma$ -iteration. Also, the pole-zero cancellation phenomena are prevented. The performance and robustness of the proposed PSS under different loading conditions are investigated in comparison with a robust tuned PSS by examining the case of a single machine infinite bus (SMIB) system. The simulation results illustrate the effectiveness and robustness of the proposed PSS.

## 1. Introduction

Due to low damping in interconnected power systems, disturbances cause the problem of low frequency oscillations (0.2-2.5 Hz) in power systems. To extend the power system stability limit, a power system stabilizer (PSS) is used as an auxiliary device that provides a supplementary feedback signal to the generator for improving the dynamic stability of a system [1], [2].

Recently,  $H_\infty$  control techniques have been applied to PSS design problems [3], [4], [5]. However, the importance and difficulties in the selection of weights of  $H_\infty$  optimization problem arise due to the trade-off relationship between sensitivity ( $S(s)$ ) and complementary sensitivity ( $T(s)$ ) functions. The  $\gamma$ -iteration is also required in these works. Later, the robust  $H_\infty$  PSS via normalized coprime factorization (NCF) approach was presented by Ngamroo [6]. With an advantage of NCF, the robust  $H_\infty$  controller can be directly synthesized without  $\gamma$ -iteration. The technique for weight selection was systematically developed. Nevertheless, engineering skills are necessary to properly select the weights.

In this paper, the new procedure of optimal weight selection is concerned in design of robust  $H_\infty$  PSS to compromise the performance and robustness of the system. The optimal weights are achieved by employing TS, which is regarded as a promising technique for combinatorial optimizations [7]. Also, NCF with loop-shaping design is used in the design procedure. The proposed PSS is examined in comparison with a robust tuned PSS [8] on a single machine infinite bus (SMIB) system.

The organization of this paper is as follows. Firstly, the linearized model of SMIB is introduced in Section 2. Next, Section 3 describes  $H_\infty$  control design via NCF with loop-shaping design, weight formulation, and objective function. Section 4 describes TS to optimal weights selection. Subsequently, the experimental results are discussed in Section 5. Lastly, conclusion is given.

## 2. System Model

In this paper, a SMIB system equipped with a simplified exciter, and the proposed PSS, as shown in Fig.1, is considered. A synchronous generator with a terminal voltage  $V_t$  is connected to an infinite bus with a voltage  $E_b$  through a lossless transmission line having external reactance  $X_e$ .  $V_{ref}$  denotes a reference voltage. The dynamic behavior of this system is represented by a linearized model of Heffron-Phillips [9], as delineated in Fig.2.

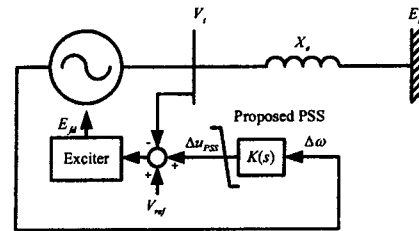


Figure 1. The proposed PSS with SMIB system.

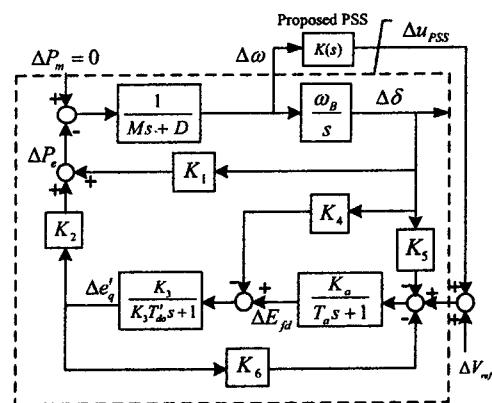


Figure 2. Linearized model of SMIB with proposed PSS.

The state equations of system in Fig.2 can be expressed as

$$\Delta \dot{X} = A\Delta X + B\Delta u_{PSS}, \quad (1)$$

$$\Delta Y = C\Delta X + D\Delta u_{PSS}, \quad (2)$$

$$\Delta u_{PSS} = K(s)\Delta\omega, \quad (3)$$

where  $\Delta X = [\Delta\delta \ \Delta\omega \ \Delta e'_q \ \Delta E_{fd}]^T$  and  $\Delta Y = [\Delta\omega]$ .  $\Delta\delta$  denotes the deviation of rotor angle,  $\Delta\omega$  is the deviation of rotor speed,  $\Delta e'_q$  is the deviation of internal voltage of the generator, and  $\Delta E_{fd}$  is the deviation of field voltage.  $\Delta u_{PSS}$  is the control output signal of the proposed PSS ( $K(s)$ ), which uses only  $\Delta\omega$  as a feedback input signal. The details of modelling procedures are discussed more completely in [9]. Note that the system (1) is a single input single output (SISO) system, which is referred to as the nominal plant  $G$ .

### 3. Design Methodology

In this section the  $H_\infty$  control design via NCF with loop-shaping design is introduced. The weight formulation and the objective function are described.

#### 3.1 $H_\infty$ Control Design via NCF with Loop-shaping Design

Here, the robust stabilization problem is considered by using NCF [10] to represent a nominal plant model  $G = M^{-1}N$ , where  $M$  and  $N$  are coprime and normalized stable transfer functions. Then a perturbed plant  $G_\Delta$  can be described by  $G_\Delta = (M + \Delta M)^{-1}(N + \Delta N)$ , where  $\Delta M$  and  $\Delta N$  are stable unknown transfer functions which represent unstructured uncertainties. For  $\|\Delta N \ \Delta M\|_\infty \leq 1/\gamma$ ,  $1/\gamma$  indicates the limitation on a size of perturbations that can exist without destabilizing the closed-loop system of Fig. 3. With NCF, the occurrence of hidden plant modes due to pole-zero cancellations is prevented. Moreover, the NCF model can represent uncertainties when a nominal stable plant  $G$  becomes unstable after being perturbed.

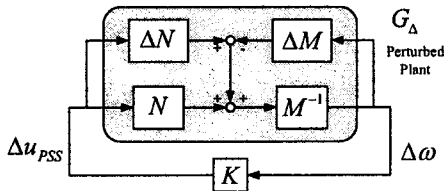


Figure 3.  $H_\infty$  robust stabilization problem.

In practice, a loop-shaping design is required to achieve desired closed-loop performances [10]. As shown in Fig. 4, a pre-compensator ( $W_1$ ) and a post-compensator ( $W_2$ ) are employed to construct the shaped plant  $G_S = W_2GW_1$ , enclosed by a solid line, prior to synthesis of the  $H_\infty$  controller. Subsequently, the robust controller  $K = W_1K_\infty W_2$ , enclosed by a dotted line, is developed where  $K_\infty$  is the  $H_\infty$  synthesized controller.

By applying NCF with loop-shaping design, it was shown in [10] that the robust controller  $K$  would stabilize the perturbed plant if and only if the shaped plant

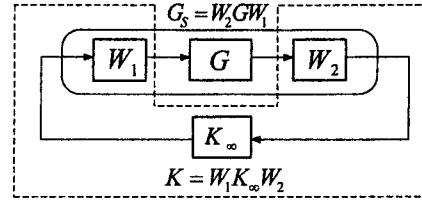


Figure 4. A Shaped plant  $G_S$  and robust controller  $K$ .

$G_S$  is stable and the following four-block problem (4) is satisfied.

$$\gamma_{\min} \triangleq \left\| \begin{array}{cc} S_S & S_S G_S \\ K_\infty S_S & K_\infty S_S G_S \end{array} \right\|_\infty < \gamma \quad (4)$$

Notice that  $S_S = (I - GK)^{-1}$  is the sensitivity function for a positive feedback arrangement in Fig. 4. To guarantee the stability of a shaped plant  $G_S$ , the selected  $\gamma$  must be greater than  $\gamma_{\min}$ . In this paper,  $\gamma$  is set to 1.1 times of  $\gamma_{\min}$ .

By virtue of NCF, the value of  $\gamma_{\min}$  can be explicitly calculated by  $\gamma_{\min} = \sqrt{1 + \lambda_{\max}(XZ)}$ , where  $\lambda_{\max}$  denotes the maximum eigenvalue of  $XZ$ . For a minimal state-space realization  $(A, B, C, D)$  of  $G_S$ ,  $X$  and  $Z$  are the unique positive solutions to the generalized control algebraic Riccati equation (GCARE) and the generalized filtering algebraic Riccati equation (GFARE), respectively.

The  $H_\infty$  controller shown in Fig. 4, which is the suboptimal controller for the selected  $\gamma > \gamma_{\min}$  can be determined by

$$K_\infty = \begin{bmatrix} A + BF + \gamma^2(L^T)^{-1}ZC^T(C + DF) & \gamma^2(L^T)^{-1}ZC^T \\ B^T X & -D^T \end{bmatrix} \quad (5)$$

where  $F = -S^{-1}(D^T C + B^T X)$  and  $L = (1 - \gamma^2)I + XZ$ . Consequently, the robust stabilization controller  $K = W_1K_\infty W_2$  is readily developed [11].

#### 3.2 Weight Formulation

As shown in Fig. 4, two weights are used for constructing  $G_S$ . The weights  $W_1$  and  $W_2$  are assumed as lead-lag transfer functions as follow.

$$W_i = K_{W_i}(T_{1W_i}s + 1)/(T_{2W_i}s + 1) \quad (6)$$

where  $K_{W_i}$ ,  $T_{1W_i}$ , and  $T_{2W_i}$  are searched parameters of the  $i^{\text{th}}$  weight, which are verified as feasible solutions if selected weights construct a stable  $G_S$ .

#### 3.3 Objective Function

The objective function used in this paper is based on the value of  $\gamma$ ,  $M_S = \|S(s)\|_\infty$ , and  $M_T = \|T(s)\|_\infty$ . The characteristics of  $S(s)$  and  $T(s)$  are considered to achieve the system performance. Here,  $\gamma_0$  is specified as a required stability margin, therefore,  $\Delta_\gamma = |\gamma - \gamma_0|$  is minimized to guarantee the required specification. The small  $S(s)$  leads to good disturbance attenuations.

While the small  $T(s)$  indicates good robust stability. The identity  $S(s) - T(s) = I$  states a trade-off between performance and robust stability for a positive feedback arrangement. Based on the maximum peak criteria,  $M_S$  and  $M_T$  differ at most by one [11]. To compromise performance and robustness,  $\Delta_{ST} = |M_S - M_T|$  should to be small.

Therefore, the control problem can be formulated as the following optimization problem.

$$\text{Min} \quad F = \Delta_\gamma + \Delta_{ST} \quad (7)$$

$$\text{Subj} \quad K_{Wi}^{\min} \leq K_{Wi} \leq K_{Wi}^{\max}, \quad (8)$$

$$T_{1Wi}^{\min} \leq T_{1Wi} \leq T_{1Wi}^{\max}, \quad (9)$$

$$T_{2Wi}^{\min} \leq T_{2Wi} \leq T_{2Wi}^{\max}, \quad (10)$$

The minimum and maximum values of the gain  $K_{Wi}$  are set to 1 and 500, respectively. While the minimum and maximum values of  $T_{1Wi}$  and  $T_{2Wi}$  are set to 0.001 and 10, respectively. In this paper, TS is employed to solve this control optimization problem and search for optimal weight parameters.

#### 4. TS to Optimal Weight Selection

TS is an iterative improvement procedure that can start from any initial feasible solution and attempt to determine a better solution. As a meta-heuristics, TS is based on a local search technique with the ability to escape from being trapped in local optima [7]. Here, components of the proposed TS are briefly discussed. Also, the Proposed TS procedure is given.

##### 4.1 TS Components

**Encoding and Decoding:** The concatenated encoding method is used to encode each parameter into a binary string normalized over its range and also stack each normalized string in series with each other to construct the string individual. The same number of  $n$  bits is used for each parameter. And decoding is a reverse process to obtain the actual value of each parameter prior to evaluation of objective function [12].

**Trial Solution Generation:** To generate a trial solution, one bit of a binary string is flipped at a time. The maximum number of trial solutions in each iteration is referred to a neighborhood solution space (NS). In this paper, NS is set to 90% of total number of bits in a string individual ( $[0.90 \times n \times NP]$ ) where  $NP$  is a number of parameter searched [12].

**Tabu List Restriction:** Tabu List (TL) is utilized to keep attributes (bit positions) that created the best solution in the past iterations for iterations so that they cannot be used to create new solution candidates. As the iteration proceeds, a new attribute enters into a TL and the oldest one is released. Particularly, the size of TL is the control parameter of TS. The size of TL that provided good solutions usually grows with the size of the problem. In this paper,  $\lfloor \sqrt{n \times NP} \rfloor$  is used to determine the best size of TL [7].

**Aspiration Level Criterion:** The aspiration level (AL) criterion allows the attributes included in TL to override its tabu status if it leads to a more attractive solution [7]. The AL used in this paper is satisfied if the tabued attribute yields a solution that is better than the best solution reached at that iteration. After the AL is satisfied, updating TL is carried out by moving the tabued attribute back to the first position of the TL.

**Termination Criteria:** The search will terminate if one of the following criteria is satisfied: (a) the best solution reached does not change for more than 20 iterations or (b) the maximum allowable number of iterations reaches 100.

##### 4.2 Proposed TS Procedure

Firstly, the initial feasible solution is generated arbitrarily. A trial solution is searched if either it is not tabued or, in case of being tabued it passes the AL test. The best solution is always updated during the search process until the termination criteria are satisfied. The following notations is used for the proposed TS procedure:

- $F(X)$  : the objective function of solution  $X$ ,
- $F_b^k$  : the best objective function at iteration  $k$ ,
- $X_0^k$  : the initial feasible solution at iteration  $k$ ,
- $X_m^k$  : a trial  $m$  solution at iteration  $k$ ,
- $X_{cb}^k$  : the current best trial solution at iteration  $k$ ,
- $X_b^k$  : the best solution reached at iteration  $k$ ,
- $k_{\max}$  : the maximum allowable number of iterations.

The TS procedure can be described as follows:

1. Read the constraints of searched parameters, the initial feasible solution  $X_0^k$ , and specification of controller.
2. Specify the size of TL,  $k_{\max}$ , and size of NS.
3. Initialize iteration counter  $k$  and termination counter  $tc$  to zero, and empty TL.
4. Initialize AL by setting  $X_b^k = X_0^k$ .
5. Execute TS procedure:
  - 5.1. Initialize the trial counter  $m$  to zero.
  - 5.2. Generate a trial solution  $X_m^k$  from  $X_0^k$ .
  - 5.3. If  $X_m^k$  is not feasible, go to 5.9.
  - 5.4. If  $X_m^k$  is the first feasible solution, set  $X_{cb}^k = X_m^k$ .
  - 5.5. Perform the Tabu test. If  $X_m^k$  is tabued, then go to 5.8.
  - 5.6. If  $F(X_m^k) < F(X_{cb}^k)$ , set  $X_{cb}^k = X_m^k$ . Otherwise, go to 5.9.
  - 5.7. If  $F(X_m^k) < F(X_b^k)$ , then update the AL by setting  $X_b^k = X_m^k$ . Go to 5.9.
  - 5.8. Perform the AL test. If  $F(X_m^k) < F(X_b^k)$ , set  $X_{cb}^k = X_m^k$ , and update the AL by setting  $X_b^k = X_m^k$ .
  - 5.9. If  $m$  is less than NS,  $m = m + 1$  and go to 5.2.
- 5.10. If there is no feasible solution, set  $X_0^{k+1} = X_b^k$ . Otherwise, set  $X_0^{k+1} = X_{cb}^k$ , and update TL.
6. If  $k = 0$ , go to 8.
7. If  $X_b^k = X_b^{k-1}$ ,  $tc = tc + 1$ . Otherwise,  $tc = 0$ .

8. If  $k < k_{\max}$  and  $tc < 20$ , then  $k = k + 1$ , and go to 5.
9. The  $X_b^k$  is the best solution found.

## 5. Experimental Results

The proposed TS is developed by using MATLAB programming language. In this paper, the tabu length of 9 is used and TS stop after 54 iterations since the best solution reached is unchanged for 20 iterations. The proposed PSS is designed based on the system data obtained from [8]. The initial operating conditions are  $P = 0.8$  pu,  $Q = 0.4$  pu,  $X_e = 0.2$  pu. In the design specification,  $\gamma_0$  is set to 2.5 which is corresponding to 40% allowed coprime uncertainty [11]. From the experiment, the weights searched by TS are  $W_1 = 375(0.0932s + 1)/(0.001s + 1)$  and  $W_2 = (s + 1)/(1.2506s + 1)$ .  $\gamma_{\min}$  and  $\gamma$  are obtained as 2.2727 and 2.5, respectively. As a result, the 8<sup>th</sup> order of the robust controller  $K$  is obtained. Since, the comparisons are made with a 3<sup>rd</sup> order PSS. Hence, the robust controller  $K$  is reduced to 2<sup>nd</sup> order by using a balanced truncation model reduction method and includes a washout circuit with a washout time constant of 10 s [9] to obtain the 3<sup>rd</sup> order proposed PSS as follows.

$$110.28 \frac{(0.05s^3 + 1.5163s^2 + 10s)}{(0.0007s^3 + 0.14s^2 + 10s + 1)} \quad (11)$$

For comparison, simulations are performed and power flow deviations are observed. The proposed PSS is compared with a robust PSS [8]. Note that both PSS have the same control capacity. A small step disturbance of 10% (0.1 pu) in  $\Delta V_{ref}$  is applied at time  $t = 1$  s. Figures 5a-c illustrate the performance and robustness of the proposed PSS under different operating conditions given in Table 1.

System Parameter	(a) Normal condition	(b) Heavy load & weakline	(c) Plant in unstable state
$P$ (pu)	0.8	0.95	0.95
$Q$ (pu)	0.4	0.5	0.5
$X_e$ (pu)	0.2	0.8	0.8
$D$ (pu)	0	0	-20

Table 1. Operating conditions.

## 6. Conclusion

In this paper, a TS algorithm to optimal weight selection in design of the robust  $H_\infty$  PSS is proposed. By virtue of TS and NCF with loop-shaping design, the proposed method can find an optimal controller for any particular control problem even with the minimum of background necessary to properly define the weights. As a result, the proposed robust controller meets the required specification. In the design, the proposed PSS uses only a rotor speed deviation ( $\Delta\omega$ ) of generator as the feedback input signal. Moreover, the practical realization in power systems can be easily implemented.

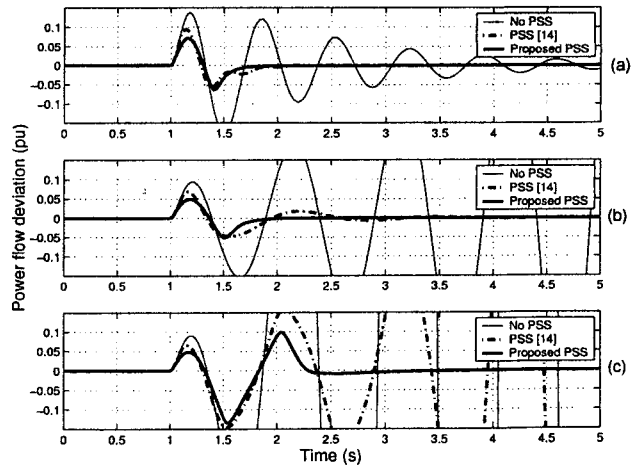


Figure 5. Simulation results.

Consequently, simulation results clearly ensure both the performance and robustness of the proposed PSS.

## References

- [1] F. P. deMello and C. Concordia, "Concepts of synchronous machine stability as affected by excitation control," *IEEE Trans. PAS*, vol. 88, no. 5, pp. 316-329, 1969.
- [2] Y. N. Yu, *Electric Power System Dynamics*, Academic Press, 1983.
- [3] S. Chen and O. P. Malik, " $H_\infty$  Optimization-based power system stabilizer design," in *Proc. 1995 IEE Part C*, vol. 142, no. 2, pp. 179-184.
- [4] R. Asgharian, "A robust  $H_\infty$  power system stabilizer with no adverse effect on shaft torsional modes", *IEEE Trans. EC*, vol. 9, no. 3, pp. 475-481, 1994.
- [5] Q. Zhao and J. Jiang, "Robust controller design for generation excitation systems," *IEEE Trans. Energy Conv.*, vol. 10, no. 2, pp. 201-209, 1995.
- [6] I. Ngamroo, "Design of robust  $H_\infty$  PSS via normalized coprime factorization approach," in *Proc. 2001 IEEE ISCAS*, vol. 2, pp. 129-132.
- [7] F. Glover and M. Laguna, *Tabu Search*, Kluwer Academic Publishers, 2001.
- [8] P. S. Rao and I. Sen, "Robust tuning of power system stabilizers using QFT," *IEEE Trans. Contr. Syst. Tech.*, vol. 7, no. 4, pp. 478-486, 1999.
- [9] K. R. Padiyar, *Power System Dynamics, Stability and Control*, John & Wiley, 1996.
- [10] D. C. McFarlane and K. Glover, "Robust controller design using normalized coprime factor plant descriptions," *Lecture Notes in Control and Information Sciences*, Springer-Verlag, 1990.
- [11] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control, Analysis and Design*, John & Wiley, 1996.
- [12] W. Ongsakul, S. Dechanupaprittha, and I. Ngamroo, "Tabu Search Algorithm for Constrained Economic Dispatch," in *Proc. 2001 CIGRE ICPS*, Sept. 2001, Wuhan, China, pp.428-433.