Augmenting Quasi-Tree Search Algorithm for Maximum Homogenous Information Flow with Single Source / Multiple Sinks

Koichi FUJITA and Hitoshi WATANABE
Graduate School of Engineering, SOKA University
1-236, Tangi-Cho, Hachioji-Shi, Tokyo, JAPAN 192-8577
Tel: +81-426-91-9419, Fax: +81-426-91-9312
e-mail: khujita@edu.t.soka.ac.jp, hitoshiw@t.soka.ac.jp

Abstract: This paper presents a basic theory of information flow from single sending point to multiple receiving points, where new theories of algebraic system called "*Hybrid Vector Space*" and flow vector space play important roles. Based on the theory, a new algorithm for finding maximum homogenous information flow is proposed, where homogenous information flow means the flow of the same contents of information delivered to multiple clients at a time. Effective multi-routing algorithms for tree-shape delivery rout search are presented.

1. INTRODUCTION

In recent years, Internet technology has emerged as the major driving force in communication network, where large amount of data packets are processed and delivered. One of the main obstacles for rapid delivery of data packet is the delay caused by processing time at each router. In order to improve efficiency of delivery throughout the network, it is useful to process a series of data packets as a single flow of information, where successive packet is not required the routing process and directly transfer to the next destination. This approach can reduce repeater-delay remarkably. Along this direction, there exists a system called MPLS (Multi-Protocol Label Switching) suitable for "one to one" communication network, where a series of packets is transferred along a path from the sending point to the receiving point. In recent information delivery system such as CDN (contents delivery network), "one to many" or "many to many" communication network become important. Multi-cast communication method is one of such "one to many" system, where information is transferred along a tree-shape rout and is copied at the branching node of the tree-shape rout.

For transmission of large amount of information in a network at its maximum efficiency, it is preferable to utilize available network resources as many as possible. From this viewpoint, most of previous systems including multi-cast are insufficient. The purpose of this study is to present a new theory of information flow, where conventional flow conservation law is not always valid due to *merge* and *copy* properties of information. Based on the theory, an effective multi-routing algorithm for "single-source multiple-sinks" (SSMS) information flow network is proposed. In section 2 and 3, general properties of information flow and an algebraic system called "*Hybrid Vector Space*" are introduced, which are especially useful for SSMS information network analysis. Information flow distribution in SSMS network can be well represented by means of a set of flow vectors as described in section 4. The maximum homogenous flow problem of SSMS information network and its solving algorithm are presented on section 5 and 6. Here, homogenous information flow means the flow of the same contents of information delivered to multiple clients at a time, such as a content delivery service is an example of such services. Experimental results of tree-shape delivery rout search algorithm are described in section 7.

Theory and algorithms introduced in this paper may play an important role in future advanced information network.

2. THEORY OF INFORMATION FLOW NETWORK

2.1 Information Flow/Information Flow Rate

[Definition2.1] *Information Flow*: Successive sequence of information units delivering from a sender to receivers.

$\boldsymbol{F}(s,t)$: Set of information units delivering from source s to sink t in unit time interval.

$\boldsymbol{f}(u,v)$: Set of information units delivering through edge (u,v) in unit time interval.

[Definition2.2] *Information Flow Rate*: Number of information units per unit time interval. (bit/sec, packet/sec)

$F(s,t)=|\boldsymbol{F}(s,t)|$: Information flow rate from source s to sink t.

$f(u,v)=|\boldsymbol{f}(u,v)|$: Information flow rate through edge (u,v).

2.2 Information Flow Network

When the terminal and router are modeled with a vertex and a communication line is modeled with an edge. Edge capacity and information flow correspond to communication band-width and information delivered from the sender to the receiver respectively.

[Definition2.3] *Information Flow Network*: N

Let $N=(G,S,T,C,\Phi)$ be an information flow network with l-connected directed graph $G=(V,E)$ ($l\geq 2$), a set of m vertices $V=\{v_i\}$ ($i=1,2,\ldots,m$), a set of n edges $E=\{e_k\}$ ($k=1,2,\ldots,n$), a set of κ information sources $S=\{s_i\}$ ($i=1,2,\ldots,\kappa$), a set of λ information sinks $T=\{t_j\}$ ($j=1,2,\ldots,\lambda$), a set of n edge capacities $C=\{c(u,v)\}$ $(u,v)\in E$, $c(u,v)\geq 0$, and a set of edge flow $\Phi=\{\boldsymbol{f}(u,v)\}$, $|\boldsymbol{f}(u,v)|\leq c(u,v)$.

2.3 Flow Conservation Law in Information Flow Network

[Definition2.4] *Generalized Information Flow Network*

$N=(G,S,T,C,F)$ is defined as a generalized flow network if $C=\{c_k\}$ and $F=\{f_k\}$ ($k=1,2,\ldots,n$) satisfy the following conditions:

(a) Capacity Constraint Conditions: $f(u,v)\leq c(u,v)$ $(u,v)\in E$

(b) Flow Conditions:

$$\sum_{u\in IN(v)} f(u,v) - \sum_{w\in OUT(v)} f(v,w) = R(v) (v,w)\in E$$

(b-1) Balance state : $R(v)=0$

(b-2) Excess state : $R(v)>0$

(b-3) Scarcity state : $R(v)<0$

where $IN(v)$ (OUT(v)) is a set of incident edges coming into (going out from) v.

According to three states of flow conditions defined above, several types of flow networks are possible. If flow conditions are in the balance state (b-1) at every vertices v, the flow network is a conventional flow conservation network. On the other hand, if any vertex v is in excess state (b-2), v should provide a buffer (or storage) to save excess amount $R(v)$ and the network is called as a flow excess network. And if any vertex v is in scarcity state (b-3), shortage amount $R(v)$ at vertex v should be compensated and the network is called as a flow scarcity network.

In the case of commodity flow network, conditions (b-1) and (b-2) are possible but not (b-3). For information flow network, however, all three states are possible for the sake of storing and copying capabilities of a computer equipped at vertex v. From the practical viewpoint, storing information needs less cost and time. Therefore, it is preferable to utilize information flow network without the excess state condition (b-2).

3. HYBRID VECTOR THEORY

3.1 Binary Vector Space

[Definition3.1] *Binary Vector Space with dimension* λ: $\boldsymbol{R}^{\lambda}=\{\boldsymbol{r}\}$

Binary Vector: $\boldsymbol{r}=(r_1,r_2,\ldots,r_\lambda)$

ν-th component of $\boldsymbol{r}$: $r_\nu=0$ or 1 $(\nu=1,2,\ldots,\lambda)$

Basis Binary Vector: $\boldsymbol{e}_\nu=(\delta_{\nu1},\delta_{\nu2},\ldots,\delta_{\nu\lambda})$, $\delta_{\nu i}=0(i\neq\nu)$, $1(i=\nu)$.

Binary Vector Operation: Vector sum $\vee$, Vector product $\wedge$

logical sum $\vee$: $1\vee1=1$, $1\vee0=1$, $0\vee1=1$, $0\vee0=0$

logical product $\wedge$: $1\wedge1=1$, $1\wedge0=0$, $0\wedge1=0$, $0\wedge0=0$

$\boldsymbol{r}_i=(r^i_1,r^i_2,\ldots,r^i_\lambda)\in\boldsymbol{R}^\lambda$, $\boldsymbol{r}_i\vee\boldsymbol{r}_j=(r^i_1\vee r^j_1,\ldots,r^i_\lambda\vee r^j_\lambda)\in\boldsymbol{R}^\lambda$,

$\boldsymbol{r}_i\wedge\boldsymbol{r}_j=(r^i_1\wedge r^j_1,r^i_2\wedge r^j_2,\ldots,r^i_\lambda\wedge r^j_\lambda)\in\boldsymbol{R}^\lambda$, $\boldsymbol{r}\wedge\boldsymbol{e}_\nu=r_\nu\boldsymbol{e}_\nu$

$\boldsymbol{r}=\bigcup_{\nu=1}^{\lambda} r_\nu\boldsymbol{e}_\nu$: vector sum of $r_\nu\boldsymbol{e}_\nu$'s

3.2 Hybrid Vector Space

[Definition3.2] *Algebraic System*: $\boldsymbol{A}=\{\alpha\}$

Algebraic System Operation (low of composition): " · "

$\alpha_1,\alpha_2\in\boldsymbol{A}\rightarrow\alpha_1\cdot\alpha_2=\alpha\in\boldsymbol{A}$

[Definition3.3] *Vector Space with dimension* λ: $\boldsymbol{A}^\lambda=\{\boldsymbol{\alpha}\}$

Vector: $\boldsymbol{\alpha}=(\alpha_1,\alpha_2,\ldots,\alpha_\lambda)$, ν-th component of $\boldsymbol{\alpha}$: $\alpha_\nu\in\boldsymbol{A}$

Vector Operation: " · " $\boldsymbol{\alpha}_i=(\alpha^i_1,\alpha^i_2,\ldots,\alpha^i_\lambda)\in\boldsymbol{A}^\lambda$,

$\boldsymbol{\alpha}_i\cdot\boldsymbol{\alpha}_j=(\alpha^i_1\cdot\alpha^j_1,\alpha^i_2\cdot\alpha^j_2,\ldots,\alpha^i_\lambda\ \alpha^j_\lambda)\in\boldsymbol{A}^\lambda$

[Definition3.4] *Hybrid Vector Space with dimension* λ: $\Lambda^\lambda=\{\rho\}$

Hybrid Vector: $\rho=(\boldsymbol{\alpha},\boldsymbol{r})\in\Lambda^\lambda$

A pair of a vector $\boldsymbol{\alpha}\in\boldsymbol{A}^\lambda$ and a binary vector $\boldsymbol{r}\in\boldsymbol{R}^\lambda$, such that ν-th component of ρ: $\alpha^*_\nu=r_\nu\alpha_\nu=\alpha_\nu$ $(r_\nu=1)$, 0 $(r_\nu=0)$

Basic Hybrid Vector: $\boldsymbol{h}_\nu=(\alpha_\nu,\boldsymbol{e}_\nu)\in\Lambda^\lambda$

A pair of a vector $\alpha_\nu\in\boldsymbol{A}$ and a basic vector $\boldsymbol{e}_\nu$

Hybrid Vector Operation: "⊗"

$\rho_i=(\boldsymbol{\alpha}_i,\boldsymbol{r}_i)\in\Lambda^\lambda$, $\rho_i\otimes\rho_j=(\boldsymbol{\alpha}^*_i\cdot\boldsymbol{\alpha}^*_j,\boldsymbol{r}_i\vee\boldsymbol{r}_j)=(\boldsymbol{\alpha},\boldsymbol{r})=\rho\in\Lambda^\lambda$,

$\boldsymbol{\alpha}=\boldsymbol{\alpha}^*_i\cdot\boldsymbol{\alpha}^*_j=(\alpha^*_{1i}\cdot\alpha^*_{1j},\alpha^*_{2i}\cdot\alpha^*_{2j},\ldots,\alpha^*_{\lambda i}\cdot\alpha^*_{\lambda j})\in\boldsymbol{A}^\lambda$, $\boldsymbol{r}=\boldsymbol{r}_i\vee\boldsymbol{r}_j\in\boldsymbol{R}^\lambda$

A hybrid vector can be represented as the combination of basic hybrid vectors

$$\rho=(\boldsymbol{\alpha},\boldsymbol{r})=(\boldsymbol{\alpha},\bigcup_{\nu=1}^{\lambda}r_\nu\boldsymbol{e}_\nu)=\sum_{\nu=1}^{\lambda}{}_{\otimes}r_\nu(\alpha_\nu,\boldsymbol{e}_\nu)=\sum_{\nu=1}^{\lambda}{}_{\otimes}r_\nu\boldsymbol{h}_\nu$$

[Definition3.5] *Homogenous Hybrid Vector*: $\hat{\rho}=(\alpha,\boldsymbol{r})\in\Lambda^\lambda$

$\rho=(\boldsymbol{\alpha},\boldsymbol{r})\in\Lambda^\lambda$, if ν-th component of $\boldsymbol{\alpha}$: $\alpha_\nu=\alpha\in\boldsymbol{A}$ $(\nu=1,2,\ldots,\lambda)$ then ρ called homogenous hybrid vector $\hat{\rho}$.

A pair of an element $\alpha\in\boldsymbol{A}$ and a binary vector $\boldsymbol{r}\in\boldsymbol{R}^\lambda$, such that ν-th component of ρ: $\alpha^*_\nu=r_\nu\alpha_\nu=0$ $(r_\nu=0)$, α_ν $(r_\nu=1)$

$$\hat{\rho}=(\alpha,\boldsymbol{r})=(\alpha,\bigcup_{\nu=1}^{\lambda}r_\nu\boldsymbol{e}_\nu)=\sum_{\nu=1}^{\lambda}{}_{\otimes}r_\nu(\alpha_\nu,\boldsymbol{e}_\nu)=\sum_{\nu=1}^{\lambda}{}_{\otimes}r_\nu\boldsymbol{h}_\nu$$

Magnitude of homogenous hybrid vector: $|\hat{\rho}|=|(\alpha,\boldsymbol{r})|=|\alpha|$

Homogenous Hybrid Vector Operation: "⊗"

$\hat{\rho}_i=(\alpha_i,\boldsymbol{r}_i)\in\Lambda^\lambda$, $\hat{\rho}_i\otimes\hat{\rho}_j=(\alpha^*_i\cdot\alpha^*_j,\boldsymbol{r}_i\vee\boldsymbol{r}_j)=(\alpha,\boldsymbol{r})=\hat{\rho}\in\Lambda^\lambda$,

$\alpha=\alpha^*_i\cdot\alpha^*_j=(\alpha^*_{1i}\cdot\alpha^*_{1j},\alpha^*_{2i}\cdot\alpha^*_{2j},\ldots,\alpha^*_{\lambda i}\cdot\alpha^*_{\lambda j})\in\boldsymbol{A}^\lambda$, $\boldsymbol{r}=\boldsymbol{r}_i\vee\boldsymbol{r}_j\in\boldsymbol{R}^\lambda$

If $\boldsymbol{r}_i=\boldsymbol{r}_j=\boldsymbol{r}$, $\hat{\rho}_i\otimes\hat{\rho}_j=(\alpha_i\cdot\alpha_j,\boldsymbol{r})=(\alpha,\boldsymbol{r})=\hat{\rho}\in\Lambda^\lambda$, $\alpha=\alpha_i\cdot\alpha_j\in\boldsymbol{A}$, $\boldsymbol{r}\in\boldsymbol{R}^\lambda$

4. INFORMATION FLOW VECTOR THEORY

4.1 Information Flow and Information Flow Rate

[Definition4.1] *Information Flow* f: Set of information units per unit time

[Definition4.2] *Information Flow Rate* $f=|f|$: Number of information units per unit time

[Definition4.3] *Algebraic System of Information Flow*: $\Phi=\{f\}$

Information Flow Operation (low of composition):" · "→"∪"(union of sets), " · "→"∩"(intersection of sets), $f_1,f_2\in\Phi$,

$f_1\cup f_2=f_3\in\Phi$, f_3:the union of information flow f_1 and f_2,

$f_1\cap f_2=f_4\in\Phi$, f_4:the intersection of information flow f_1 and f_2

[Definition4.4] *Algebraic System of Information Flow Rate*: $\boldsymbol{F}=\{f\}$

Information Flow Rate Operation (low of composition):

" · " → "+" (addition), " · " → "⊕" (compound addition),

" · " → "*" (compound product), $f_1,f_2\in\boldsymbol{F}$,

$f_1+f_2=|f_1|+|f_2|=f_3\in\boldsymbol{F}$, f_3: Total number of information flow units of f_1 and f_2,

$f_1\oplus f_2=|f_1\cup f_2|=f_4\in\boldsymbol{F}$, f_4: Number of disjoint information units of f_1 and f_2,

$f_1*f_2=|f_1\cap f_2|=f_5\in\boldsymbol{F}$, f_5: Number of duplicated information units of f_1 and f_2,

$f_1\oplus f_2=f_1+f_2-f_1*f_2=f_4\in\boldsymbol{F}$

4.2 Information Flow Vector Space

[Definition4.5] *Information Flow Vector Space with dimension* λ: $\Psi^\lambda=\{\psi\}$

Homogenous Information Flow Vector: $\psi=(f,\boldsymbol{r})=(f^*_1,f^*_2,\ldots,f^*_\lambda)=\boldsymbol{f}^*\in\Psi^\lambda$

A pair of a information flow $f\in\Phi$ and a binary vector $\boldsymbol{r}\in\boldsymbol{R}^\lambda$, such that ν-th component of ψ: $f^*_\nu=r_\nu f=0$ $(r_\nu=0)$, f $(r_\nu=1)$ $(\nu=1,2,\ldots,\lambda)$, Magnitude of homogenous information flow vector: $|\psi|=|(f,\boldsymbol{r})|=|f|$

Homogenous Information Flow Vector Operation: "⊗"

$\psi_i=(f_i,\boldsymbol{r}_i)=(f^*_{1i},f^*_{2i},\ldots,f^*_{\lambda i})=\boldsymbol{f}^*_i\in\Psi^\lambda$ $f^*_{\nu i}=r^i_\nu f_i=f_i$ $(r^i_\nu=1)$, 0 $(r^i_\nu=0)$, $\psi_i\otimes\psi_j=(\boldsymbol{f}^*_i\cup\boldsymbol{f}^*_j,\boldsymbol{r}_i\vee\boldsymbol{r}_j)=(\boldsymbol{f},\boldsymbol{r})=\psi\in\Psi^\lambda$, $\boldsymbol{f}=\boldsymbol{f}^*_i\cup\boldsymbol{f}^*_j=(f^*_{1i}\cup f^*_{1j},f^*_{2i}\cup f^*_{2j},\ldots,f^*_{\lambda i}\cup f^*_{\lambda j})\in\Psi^\lambda$, $\boldsymbol{r}=\boldsymbol{r}_i\vee\boldsymbol{r}_j\in\boldsymbol{R}^\lambda$

If $\boldsymbol{r}_i=\boldsymbol{r}_j=\boldsymbol{r}$, $\psi_i\otimes\psi_j=(f_i\cup f_j,\boldsymbol{r})=(f,\boldsymbol{r})=\psi\in\Psi^\lambda$, $f=f_i\cup f_j\in\Phi$, $\boldsymbol{r}\in\boldsymbol{R}^\lambda$

$$\sum_{i=1}^{\mu}{}_{\otimes}\psi_i=\sum_{i=1}^{\mu}{}_{\otimes}(f_i,\boldsymbol{r}_i)=(\bigcup_{i=1}^{\mu}\boldsymbol{f}^*_i,\bigcup_{i=1}^{\mu}\boldsymbol{r}_i)=(\boldsymbol{f}^*,\boldsymbol{r})\in\Psi^\lambda,\ \boldsymbol{f}^*=\bigcup_{i=1}^{\mu}\boldsymbol{f}^*_i\in\Phi^\lambda$$

Magnitude of information flow vector:

$$|\sum_{i=1}^{\mu}{}_{\otimes}\psi_i|=|(\bigcup_{i=1}^{\mu}\boldsymbol{f}^*_i,\bigcup_{i=1}^{\mu}\boldsymbol{r}_i)|=|\bigcup_{i=1}^{\mu}\boldsymbol{f}^*_i|$$

4.3 Information Flow Rate Vector Space

[Definition4.6] *Information Flow Rate Vector Space with dimension* λ: $\Lambda^{\lambda}=\{\varphi\}$

Homogenous Information Flow Rate Vector:

$\varphi=(f,\boldsymbol{r})=(f^{*}_{1},f^{*}_{2},\ldots,f^{*}_{\lambda})=\boldsymbol{f}^{*}\in\Lambda^{\lambda}$

A pair of a information flow rate $f\in\boldsymbol{F}$ and a binary vector $\boldsymbol{r}\in\boldsymbol{R}^{\lambda}$, such that ν-th component of $\varphi: f^{*}_{\nu}=r_{\nu}f=f\,(r_{\nu}=1)$, $0\,(r_{\nu}=0)$ $(\nu=1,2,\ldots,\lambda)$, Magnitude of homogenous information flow rate vector: $|\varphi|=|(f,\boldsymbol{r})|=f=|f|$

Homogenous Information Flow Rate Vector Operation:

"$\otimes$" $\varphi_i=(f_i,\boldsymbol{r}_i)=(f^{*}_{1i},f^{*}_{2i},\ldots,f^{*}_{\lambda i})=\boldsymbol{f}^{*}_{i}\in\Lambda^{\lambda}, f^{*}_{\nu i}=r^{i}_{\nu}f_i=f_i$ $(r^{i}_{\nu}=1)$, $0(r^{i}_{\nu}=0)$, $\varphi_i\otimes\varphi_j=(\boldsymbol{f}^{*}_{i}\oplus\boldsymbol{f}^{*}_{j}, \boldsymbol{r}_i\vee\boldsymbol{r}_j)=(\boldsymbol{f},\boldsymbol{r})=\varphi\in\Lambda^{\lambda}, \boldsymbol{f}=\boldsymbol{f}^{*}_{i}\oplus\boldsymbol{f}^{*}_{j}=(f^{*}_{1i}\oplus f^{*}_{1j},f^{*}_{2i}\oplus f^{*}_{2j},\ldots,f^{*}_{\lambda i}\oplus f^{*}_{\lambda j})\in\Lambda^{\lambda}, \boldsymbol{r}=\boldsymbol{r}_i\vee\boldsymbol{r}_j\in\boldsymbol{R}^{\lambda}$

If $\boldsymbol{r}_i=\boldsymbol{r}_j=\boldsymbol{r}$, $\varphi_i\otimes\varphi_j=(f_i\oplus f_j,\boldsymbol{r})=(f,\boldsymbol{r})=\varphi\in\Lambda^{\lambda}$, $f=f_i\oplus f_j\in\boldsymbol{F}$, $\boldsymbol{r}\in\boldsymbol{R}^{\lambda}$,

$\varphi_i\underline{\otimes}\varphi_j=(f_i+f_j,\boldsymbol{r})=(f^{*},\boldsymbol{r})=\varphi^{*}\in\Lambda^{\lambda}, f^{*}=f_i+f_j\in\boldsymbol{F}, \boldsymbol{r}\in\boldsymbol{R}^{\lambda}$, $\sum_{i=1}^{\mu}{}_{\otimes}\varphi_i=$

$\sum_{i=1}^{\mu}{}_{\otimes}(f_i,\boldsymbol{r}_i)=(\sum_{i=1}^{\mu}{}_{\oplus}\boldsymbol{f}^{*}_{i},\bigcup_{i=1}^{\mu}\boldsymbol{r}_i)=(\boldsymbol{f}^{*},\boldsymbol{r})\in\Lambda^{\lambda},\boldsymbol{f}^{*}=\sum_{i=1}^{\mu}{}_{\oplus}\boldsymbol{f}^{*}_{i}\in\boldsymbol{F}^{\lambda}$

Magnitude of information flow rate vector:

$|\sum_{i=1}^{\mu}{}_{\otimes}\varphi_i|=|(\sum_{i=1}^{\mu}{}_{\oplus}\boldsymbol{f}^{*}_{i},\bigcup_{i=1}^{\mu}\boldsymbol{r}_i)|=|\sum_{i=1}^{\mu}{}_{\oplus}\boldsymbol{f}^{*}_{i}|$

5. APPLICATION OF THE INFORMATION FLOW VECTOR THEORY IN SSMS INFORMATION FLOW NETWORK

5.1 Definitions in SSMS Information Flow Network

[Definition5.1]

Edge Flow $f(u,v)$: information flow on edge (u,v)

Edge Flow Rate $f(u,v)=|f(u,v)|$: information flow rate on edge (u,v)

Destination Vector $\boldsymbol{r}_i=(r^{i}_{1},r^{i}_{2},\ldots,r^{i}_{\lambda})$: binary vector which indicates destination of flow. If $r^{i}_{\nu}=1$, then the destination of $f_i(u,v)$ or $f_i(u,v)$ is ν-th sink t_{ν}, and if $r^{i}_{\nu}=0$, then ν-th sink t_{ν} is not the destination.

Basic Flow Vector: $\boldsymbol{h}_{\nu}(f(u,v))=(f(u,v),\boldsymbol{e}_{\nu})$

Basic Flow Rate Vector: $\boldsymbol{h}_{\nu}(f(u,v))=(f(u,v),\boldsymbol{e}_{\nu})$

Edge Flow Vector $\psi_i(u,v)=(f_i(u,v),\boldsymbol{r}_i)$: A pair of a edge flow $f_i(u,v)$ and a destination vector $\boldsymbol{r}_i$, Magnitude of edge flow vector: $|\psi_i(u,v)|=|(f_i(u,v),\boldsymbol{r}_i)|=|f_i(u,v)|$, Decomposition of edge flow vector: $\psi_i(u,v)=\sum_{\nu=1}^{\lambda}{}_{\otimes}r_{\nu}(f(u,v),\boldsymbol{e}_{\nu})=\sum_{\nu=1}^{\lambda}{}_{\otimes}r_{\nu}\boldsymbol{h}_{\nu}(f(u,v))$

Edge Flow Rate Vector $\varphi_i(u,v)=(f_i(u,v),\boldsymbol{r}_i)$: A pair of a edge flow rate $f_i(u,v)$ and a destination vector $\boldsymbol{r}_i$, Magnitude of edge flow rate vector: $|\varphi_i(u,v)|=|(f_i(u,v),\boldsymbol{r}_i)|=|f_i(u,v)|=f_i(u,v)$

Decomposition of edge flow rate vector:

$\varphi(u,v)=(f(u,v),\bigcup_{\nu=1}^{\lambda}r_{\nu}\boldsymbol{e}_{\nu})=\sum_{\nu=1}^{\lambda}{}_{\otimes}r_{\nu}(f(u,v),\boldsymbol{e}_{\nu})=\sum_{\nu=1}^{\lambda}{}_{\otimes}r_{\nu}\boldsymbol{h}_{\nu}(f(u,v))$

$\psi_i(u,v)$ and $\varphi_i(u,v)$ with destination binary vector $\boldsymbol{r}_i$ are homogenous hybrid vectors on edge (u,v), whose destinations are determined by $\boldsymbol{r}_i$.

Edge Flow Vector $\rho(u,v)=\sum_{i=1}^{\mu}{}_{\otimes}\psi_i(u,v)=\sum_{i=1}^{\mu}{}_{\otimes}(f_i(u,v),\boldsymbol{r}_i)$: μ edge flow vector on edge (u,v) Magnitude of edge flow vector:

$|\rho(u,v)|=|\sum_{i=1}^{\mu}{}_{\otimes}\psi_i(u,v)|=|\sum_{i=1}^{\mu}{}_{\otimes}(f_i(u,v),\boldsymbol{r}_i)|=|\bigcup_{i=1}^{\mu}f_i(u,v)|$

Basic vector representation of edge flow vector:

$\rho(u,v)=\sum_{\nu=1}^{\lambda}{}_{\otimes}(\phi_{\nu}(u,v),\boldsymbol{e}_{\nu})=\sum_{\nu=1}^{\lambda}{}_{\otimes}\boldsymbol{h}_{\nu}(\phi_{\nu}(u,v))$

ν-th component of $\rho(u,v)$: $\phi_{\nu}(u,v)=\bigcup_{i=1}^{\mu}f_i(u,v)r^{i}_{\nu}$

Edge Flow Rate Vector $\gamma(u,v)=\sum_{i=1}^{\mu}{}_{\otimes}\varphi_i(u,v)=\sum_{i=1}^{\mu}{}_{\otimes}(f_i(u,v),\boldsymbol{r}_i)$: μ edge flow rate vector on edge (u,v)

Magnitude of edge flow rate vector:

$|\gamma(u,v)|=|\sum_{i=1}^{\mu}{}_{\otimes}\varphi_i(u,v)|=|\sum_{i=1}^{\mu}{}_{\oplus}f_i(u,v)|=|\bigcup_{i=1}^{\mu}f_i(u,v)|$

Basic vector representation of edge flow rate vector:

$\gamma(u,v)=\sum_{\nu=1}^{\lambda}{}_{\otimes}(\phi_{\nu}(u,v),\boldsymbol{e}_{\nu})=\sum_{\nu=1}^{\lambda}{}_{\otimes}\boldsymbol{h}_{\nu}(\phi_{\nu}(u,v))$

ν-th component of $\gamma(u,v)$: $\phi_{\nu}(u,v)=\sum_{i=1}^{\mu}{}_{\oplus}f_i(u,v)r^{i}_{\nu}$

Information Flow Distribution on a SSMS Information Flow Network: Γ

Set of edge flow rate vectors $\Gamma=\{\gamma(u,v)\}$ $(u,v)\in\boldsymbol{E}$ on a SSMS information flow network N is defined as the *Information Flow Distribution* of N provided that the following conditions are satisfied;

(a) Each component $\phi_{\nu}(u,v)$ of edge flow rate vector $\gamma(u,v)=\sum_{\nu=1}^{\lambda}{}_{\otimes}(\phi_{\nu}(u,v),\boldsymbol{e}_{\nu})$ on edge (u,v) connected to a vertex v satisfies the flow conservation low: $\sum_{u\in IN(v)}\phi_{\nu}(u,v)-\sum_{w\in OUT(v)}\phi_{\nu}(v,w)=0$ $(v\neq s,t_j)$

(b) $\sum_{w\in OUT(v)}|\gamma(v,w)|=F$ $(v=s)$

$\sum_{u\in IN(v)}|\gamma(u,v)|=-F_j\leq F$ $(v=t_j)$ $(j=1,2,\ldots,\lambda)$

(c) $|\gamma(v,w)|=f(v,w)\leq c(v,w)$

Homogenous Information Flow Distribution on a SSMS Information Flow Network: $\hat{\Gamma}$

The information flow distribution Γ on a SSMS information flow network N is defined as the *Homogenous Information Flow Distribution* $\hat{\Gamma}$ provided that the following conditions are satisfied. And N is called a *SSMS Homogenous Information Flow Network*; $F=F_j$ $(j=1,2,\ldots,\lambda)$

F : information generation rate at source s

F_j : information absorption rate at sink t_j

[Definition5.2] *Merging Quantity* and *Copying Quantity*

For the given information flow distribution Γ, merging and copying quantities at a vertex v are defined as follows:

Merging Quantity: $MI(v)=\sum_{u\in IN(v)}|\gamma(u,v)|-|\sum_{u\in IN(v)}{}_{\otimes}\gamma(u,v)|$

Copying Quantity :$CI(v)=\sum_{w\in OUT(v)}|\gamma(v,w)|-|\sum_{w\in OUT(v)}{}_{\otimes}\gamma(v,w)|$

$$|\sum{}_{\otimes}\gamma(u,v)|=|\sum{}_{\otimes}\{\sum_{i=1}{}_{\oplus}\varphi_i(u,v)\}|=|\bigcup\{\bigcup_{i=1}f_i(u,v)\}|$$

Vertex v is called a *Merging Vertex* if $MI(v)\neq0$, vertex v is called a *Copying Vertex* if $CI(v)\neq0$.

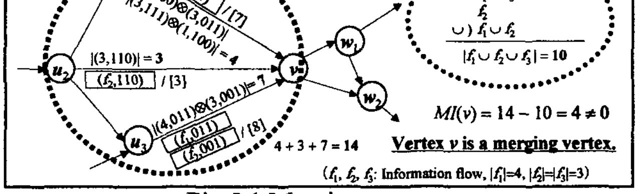

Fig 5.1 Merging vertex v

Compact Information Flow Distribution: An information flow distribution Γ on a SSMS information flow network N is defined as the *Compact Information Flow Distribution*, if it contains no merging vertex in N.

Compact Homogenous Information Flow Distribution:

A compact information flow distribution Γ on a SSMS information flow network N is defined as the *Compact Homogenous Information Flow Distribution*, if it is homogenous information network.

[Definition5.3] *Augmenting Circuit* in SSMS Homogenous Information Flow Network:

Suppose an information flow distribution Γ on a SSMS information flow network N be a homogenous information flow distribution, and L^+ (L^-) be a set of forward (backward) edges in a circuit L of the network. For an information flow rate vector φ_m with destination vector $\boldsymbol{r}_m$,

$\varphi_m=(f_m(u,v),\boldsymbol{r}_m)=\sum_{\nu=1}^{\lambda}{}_{\otimes} r_{m\nu}(f_m(u,v),e_\nu)$ on edge $(u,v)\in L$,

$\Delta(L^+)$ and $\Delta(L^-)$ are defined as follows;

$$\Delta(L^+)=\min_{(u,v)\in L^+}[\max_{\nu}\{\phi_\nu(u,v)\}-f_m(u,v)r_{m\nu}],$$

$$\Delta(L^-)=\min_{(u,v)\in L^-}[f_m(u,v)r_{m\nu}].$$ If $\Delta(L)=\min[\Delta(L^+),\Delta(L^-)]>0$

Then, circuit L is defined as *Augmenting Circuit* with respect to information flow rate vector φ_m.

If an augmenting circuit L with respect to information flow rate vector φ_m exists and has the positive value $\Delta(L)$, then the following augmenting process along the direction of L

$$\gamma(u,v)\otimes(\Delta(L),\boldsymbol{r}_m)\ (u,v)\in L^+,\ \gamma(u,v)\otimes(-\Delta(L),\boldsymbol{r}_m)\ (u,v)\in L^-$$

decreases the amount of merging quantities at merging vertices on L by at least $\Delta(L)$ and increases the amount of copying quantities at copying vertex on L by at least $\Delta(L)$.

5.2 Maximum Homogenous Information Flow Problem in SSMS Information Flow Network

For given SSMS information flow network $N(G,s,T,C,\Gamma)$, it is important to maximize the amount of homogenous information flow $F(s,t_\nu)$ from s to $t_\nu(\nu=1,2,\ldots,\lambda)$, or to maximize the amount of homogenous information flow Rate $F(s,t_\nu)=\hat{F}$ from s to $t_\nu(\nu=1,2,\ldots,\lambda)$. Where magnitude of an homogenous information flow is $|F(s,t_\nu)|=F(s,t_\nu)=\hat{F}$.

[Definition5.4] *Maximum Homogenous Information Flow* is defined as; $\hat{F}_{max}=\max\{F(s,t_\nu)\}=\max\{\hat{F}\}$

[Definition5.5] *Quasi-Tree*: Let P_ν be paths from s to $t_\nu(\nu=1,2,\ldots,\lambda)$ in N, the combination of λ paths P_ν is called quasi-tree and denoted as Ψ.

[Definition5.6] *Augmenting Quasi-Tree*: For a path $P_\nu(\nu=1,2,\ldots,\lambda)$ in quasi-tree Ψ, let P^+_ν be a set of forward edges and P^-_ν be a set of backward edges in path P_ν. For an edge flow rate vector $\gamma(u,v)$ on edge $(u,v)\in P_\nu$, $\Delta(P^+_\nu)$ and $\Delta(P^-_\nu)$ are defined as follows; $\Delta(P_\nu^+)=\min_{(u,v)\in P^+}[c(u,v)-\phi_\nu(u,v)]$,

$\Delta(P_\nu^-)=\min_{(u,v)\in P^-}[\phi_\nu(u,v)]$, $\Delta(P_\nu)=\min[\Delta(P_\nu^+),\Delta(P_\nu^-)]$

If $\Delta(\Psi)=\min_{\nu}[\Delta(P_\nu)]>0$ then, Ψ is defined as *augmenting quasi-tree* with respect to set of edge flow rate vector Γ.

If an augmenting quasi-tree Ψ with respect to Γ exists and has the positive value $\Delta(\Psi)$, then the following augmenting process along the direction of P_ν;

$$\gamma(u,v)\otimes(\Delta(\Psi),e_\nu)(u,v)\in P_\nu^+,\ \gamma(u,v)\otimes(-\Delta(\Psi),e_\nu)\ (u,v)\in P_\nu^-$$

edge flow rate vector $\gamma(u,v)$ on Ψ is updated, and the value of information flow $F(s,t_\nu)$ from s to t can be increased as $\hat{F}+\Delta(\Psi)$.

[Theorem5.1] The necessary and sufficient condition for the set of edge flow rate vector Γ having the maximum homogenous information flow is that there exist no augmenting quasi-tree with respect to Γ.♦

N is called *maximum homogenous information flow network*.

6. ALGORITHM FOR DETERMINATION OF TREE SHAPE DELIVERING ROUTS

1) *Augmenting Quasi-Tree Search Algorithm AQTS*: Find all possible quasi-trees by the labeling scheme and determine the maximum homogenous information network.

2) *Tree Shape Delivering Routs Search Algorithm TDRS*: Find tree shape delivering routs by BSF tree search scheme without augmenting loop in the maximum homogenous information network.

By these algorithms, tree shape delivering routs in the maximum homogenous information flow SSMS network can be determined.

7. A SIMULATION AND CONSIDERATION

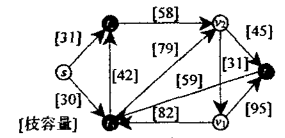

Fig.7.1 SSMS information flow network N

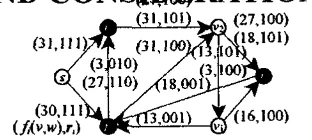

Fig 7.3 maximum homogenous information flow distribution Γ

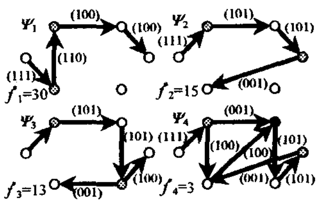

Fig 7.2 augmenting quasi-tree Ψ_1, Ψ_2, Ψ_3, Ψ_4

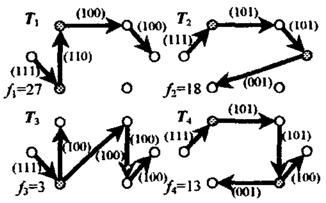

Fig 7.4 tree shape derivering roots T_1,T_2,T_3,T_4

●: merging vertex, ⊗: copying vertex

8. CONCLUSION

A basic theory of information flow is presented. The maximum homogenous information flow problem is soleved and tree shape delivering routs determination algorithm in the SSMS network is presented. Based on the proposed method, the most efficient transfer of large amount of information can be realized by utilizing all available resources of information network. It is expected that the proposed method will be important in the future multimedia information network.

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