An Effective Tangential Cutting Algorithm for Rapid Prototyping

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Abstract: For laser-cutting rapid prototyping systems that fabricate objects with layers having a certain degree of thickness, it is necessary both to minimize surface distortion and to generate dynamically feasible laser cutting trajectories. An effective tangential layer cutting algorithm is developed for this requirements. An energy function is defined in terms of tangential line lengths and their distances. And the energy is minimized for the tangential lines to closely describe a layer suffice. Our method is applied to 3D model samples and the generated tangential lines effectively approximate layer surface and make laser trajectory smooth.

1. Introduction

Rapid Prototyping(RP) systems transform 3D models to standard STL files, extract layer infromation from the files, fabricate layers, and stack those layers to form 3D objects[1, 2, 3]. Layer-based building process that stacks vertically-cut layers typically result in solid objects bearing artifacts such as staircases. To reduce the staircase effect, layer thickness can be made thinner. However, this method requires much more time in manufacturing.

In this paper, an effective tangential layer cutting algorithm is developed to solve surface distorsion problem inherent to layer-based building process and to produce dynamically feasible laser-cutting trajectories for laser-cutting systems. Tangential layers permit a layer having a certain degree of thickness and the overall building time of 3D objects can be reduced significantly. While layer-based building process that cuts layer vertically requires only cross-sectional slice information, tangential cutting method requires surface information of adjoined two cross-sections as well as the layer surface information between the two cross-sections.

To approximate a 3D layer surface with laser traces, a set of line segments connecting top and bottom slice contours are considered. The problem of finding effective tangential line segments is formulated as an energy minimization problem where an energy is defined and minimized. This paper is organized as follows. In section 2, layers are formed by STL slicing algorithm. In section 3, an energy is defined and a tangential layer cutting algorithm is described. Section 4 shows the application result of the proposed algorithm and a concluding remark is given in section 5.

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2. Layer Formation by Slicing

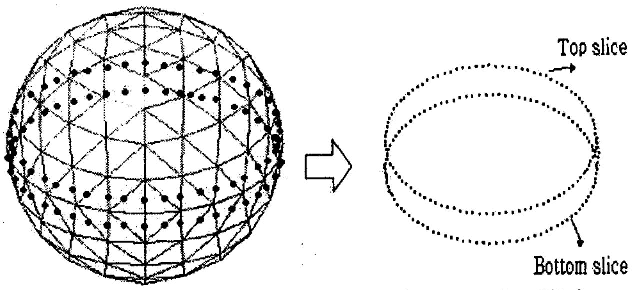
A slice is a cross-section of an 3D object at a height and a layer consists of two slices of a height range. That is, a layer connects a top and bottom slices and has a surface between the two. Since most 3D objects can be represented in STL format, we implemented an STL slicer to generate layers.

Each slice is generated by computing the intersection points between a slicing plane and STL triangular surfaces[4]. To test whether a slicing plane intersects any line segment of an STL facet or not, the relation between a plane equation f(x, y, z) and a point P(x, y, z) is considered as follows:

1) if point P is on plane f, $f(P) = 0$,
2) if point P is above plane f, $f(P) > 0$,
3) if point P is below plane f, $f(P) < 0$.

Each of the two points of a facet line is applied to the plane equation and their values are multiplied. If the multiplied result is negative, the line of the facet intersects the slicing plane.

After selecting a crossing line segment, the intersecting point can be found by binary search. Figure 1 shows that a layer is created from adjoinning two cross-sections.



(a) 3D Model (b) Layer by Slicing

Figure 1. Layer Production by Slicing

3. Forming Tangential Line Segments

Since most surface reconstruction algorithms describe a layer by many triangular facets, these methods are not suitable in generating smooth laser trajectory[4]. Laser beam of a layer-based RP system must move smoothly along the edge surface of a layer. Therefore, a layer must be approximated by a set of tangential line segments rather than triangular facets. This set consists of line segments that are laid along the two curved upper and lower slice contours as illustrated in Figure 2.

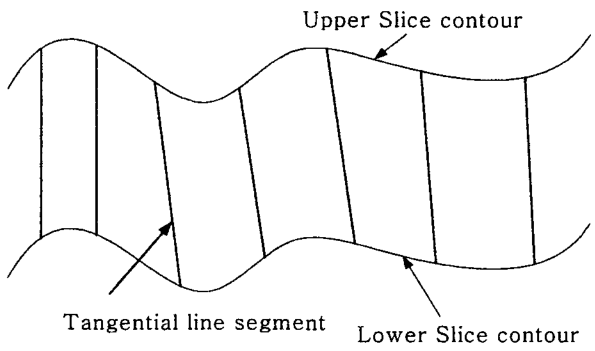


Figure 2. Tangential Line Segments

Tangential line segments are optimized by energy relaxation method. For this, one-to-one correspondence between upper and lower slice contour points is made. A line segment that connects the upper and lower contour points defines a tangential line segment. A middle slice contour is additionally extracted from the 3D model by an extra slicing operation for closer approximatation of the layer surface. As shown in figure 3, the length of a tangential line segment influences the degree of a layer surface smoothness and the distance between a tangential line segment and the point of middle slice contour influences the degree of layer surface approximation.

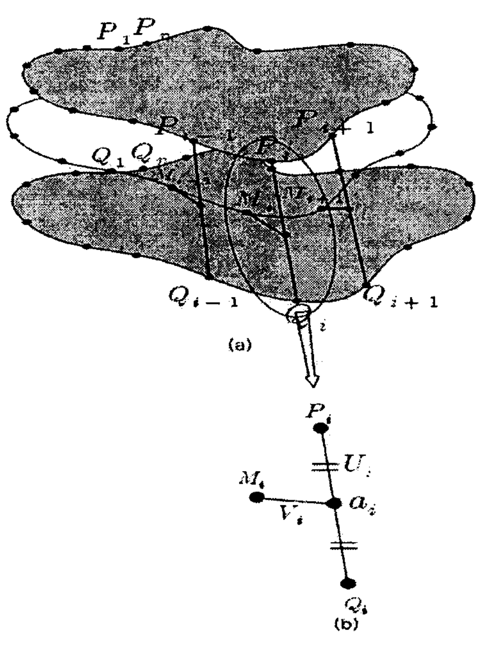


Figure 3. Layer Formulation

So, an energy function is defined by the length term and the distance term. The minimization of the length term achieves the smoothness of laser beam movements due to vertical preference, and the minimization of the distance guarantees the closer layer surface approximation. The symbols used in figure 3 are

P_i : *i*-th point coordinate of upper slice contour,

Q_i : *i*-th point coordinate of lower slice contour,

M_i : *i*-th point coordinate of middle slice contour,

U_i : Tangential line segment connecting P_i and Q_i,

a_i : Middle point of U_i,

V_i : Distance from a_i to M_i.

And the line connecting P_i and Q_i forms the *i*-th tangential line segment. The energy function of *i*-th line segment is

$$E_i(P_i,Q_i) = \frac{1}{2}\left(K_V V_i^2 + K_U U_i^2\right) \quad (1)$$

where

$$V_i = \sqrt{\left(x_{mi} - \left(\frac{x_{Pi}+x_{Qi}}{2}\right)\right)^2 + \left(y_{mi} - \left(\frac{y_{Pi}+y_{Qi}}{2}\right)\right)^2} \quad (2)$$

is the distance between the middle contour point to the middle point of U_i, and

$$U_i = \sqrt{(x_{Pi}-x_{Qi})^2 + (y_{Pi}-y_{Qi})^2} \quad (3)$$

is the distance from the upper and lower contour points. The constants K_U and K_V denote the convergence rate of energy function. By substituting equations (2) and (3) into equation (1), the energy function becomes

$$E_i(P_i,Q_i) = \frac{1}{2}K_V\left(\sqrt{\left(x_{mi} - \left(\frac{x_{Pi}+x_{Qi}}{2}\right)\right)^2 + \left(y_{mi} - \left(\frac{y_{Pi}+y_{Qi}}{2}\right)\right)^2}\right)^2 + \frac{1}{2}K_U\left(\sqrt{(x_{Pi}-x_{Qi})^2 + (y_{Pi}-y_{Qi})^2}\right)^2 \quad (4)$$

The total energy sum is

$$E = \sum_{i=1}^{n} E_i(P_i,Q_i) \quad \text{for i=0,...,n} \quad (5)$$

If this energy sum is minimized, then all the tangential line segments are shortened as much as possible and placed near the middle contour as close as possible. This minimization is done by gradient descent method and the derivative of each energy function can be approximated by

$$\frac{\partial E_i}{\partial Q_i} \approx \frac{\Delta E_i}{\Delta Q_i} = \left[\frac{E_i(P_i, Q_i + \Delta Q_i) - E_i(P_i,Q_i)}{\Delta Q_i}\right], \quad (6)$$

where the small variation along lower contour is denoted by ΔQ.

4. Implementation and Experiment

Initially, n points of lower, upper, and middle contours are chosen equally spaced along each contour. For simplicity, the chosen middle and upper points are fixed, and only the lower points are allowed to move along the lower contour for the optimization by gradient descent method.

For the adjustment of *i*-th tangential line segment as shown in figure 4, two points $Q_i + \Delta Q$, $Q_i - \Delta Q$ are considered near point ΔQ. Then the energy difference between point Q_i and point $Q_i + \Delta Q$ is computed using equation (6). For point $Q_i - \Delta Q$, the other energy difference is computed as well. The two energy differences are compared and point Q_i moves by ΔQ to the direction where the more energy decreases.

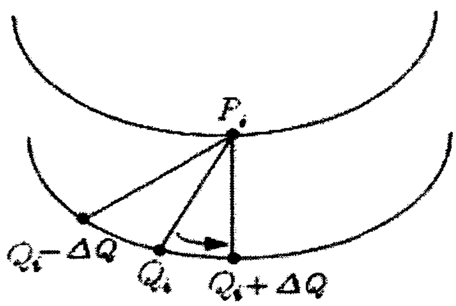
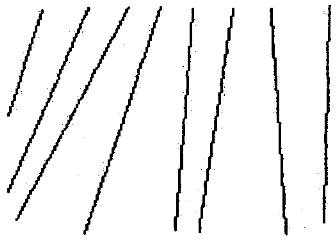


Figure 4. Lower Point Adjustment

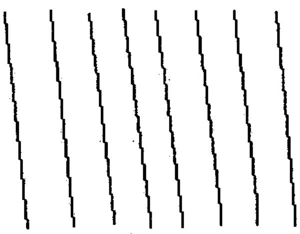
Table 1 shows the gradient descent method that rearranges tangential line segments. Figure 5(a) shows a portion of initial tangential line segments connecting points of upper and lower contours. In figure 5(b), their final configuration after the algorithm is applied.

Table 1. Tangential Line Formation Algorithm

1. Equally spaced n points are selected along upper, lower, and middle slice contours respectively, and fix the upper and middle points.
2. Energy differences between Q_i and $Q_i + \Delta Q$, $Q_i - \Delta Q$ are computed repectively by equation (6) for i-th tangential line segment adjustment.
3. Q_i is moved by ΔQ to direction where more energy decreases.
4. Step2 and 3 are repeated for each n point.
5. Repeat steps 2-4 until the change of energy is below a certain threshold.



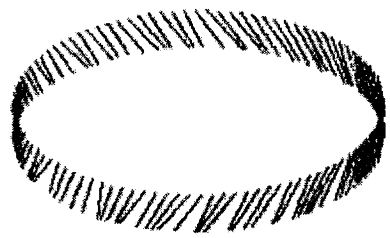
(a) Initial Line Segments



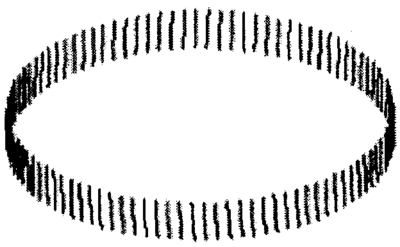
(b) Final Line Segments

Figure 5. Tangential Line Segment Adjustment

Figure 6(a) shows initial line segments of a layer. Figure 6(b) shows the final line segments after optimization. By visual inspection of the two figures, figure 6(b) turns out to have more regular line patterns than figure 6(a). The final line configuration has 34% less energy after optimization. Figure 7 shows that the total energy asymptotically decreases during the optimization process for the 3D object in figure 6. Each sweep consists of 30 point adjustments.



(a) Initial Line Segments (E=409.3987)



(b) Final Line Segments (E=268.9485)

Figure 6. Tangential Line Segment Adjustment(Layer)

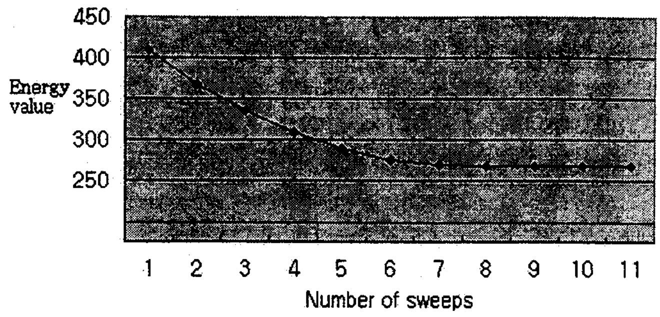


Figure7. Energy Decrease by the Proposed Algorithm

5. Conclusions

In this paper, a tangential layer cutting algorithm is developed in order to reduce surface distortion as well as to generate smooth laser-cutting trajectory. An energy is defined by the length of tangential line segment and the distance to middle plane. Then, the energy is minimized for closer layer surface approximatation and smoother laser cutting motion. The proposed algorithm is tested and verified on several 3D object samples and the experiment shows that the generated tangential line segments are effective. Currently, we are pursuing a further study on improving our algorithm applicable to more complex 3D objects.

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