Analysis of LSI Circuits Coupled with RCG Interconnects - Asymptotic Method

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Abstract

High frequency digital LSI circuits are usually composed of many sub-circuits coupled with interconnects. They sometimes causes serious problems of the fault switching by time-delays, crosstalks, reflections of signals and so on. Therefore, it is very important to develop a user-friendly simulator for solving these problems. Although a moment matching method is widely used as the reduction technique of interconnects, it may happen to arise erroneous results for evaluating the poles far from the origin. In this paper, we show an asymptotic method in the complex frequency-domain, where we calculate the exact poles and residues giving large effect to the transient responses. Then, the interconnects are replaced by the asymptotic equivalent circuits using the poles and residues. Thus, we can develop a users-friendly simulator using the equivalent circuits.

1. Introduction

The analysis and design of high speed LSI chips are becoming more and more important, because interconnects in LSI chips sometime cause the fault switching operations due to the signal delays, crosstalks and so on. In the last decade, there have been published many papers concerning to the transient responses of lossy interconnects [1-5]. The recursive convolution method combining moment-matching technique [1-3] can be efficiently applied to the analysis of lossy interconnect terminated by nonlinear elements. However, one of the serious problems is that the poles far from origin calculated by the moment-matching technique become erroneous because of the Maclaurin expansion technique and Padè approximation. To overcome the problem, Nakhla et al. have proposed CFH (complex frequency hopping) [4] for getting the exact poles. Unfortunately, the algorithm based on Taylor expansion becomes complex because we need to solve the algebraic equation with complex coefficients. The transient analysis of single line interconnect combining the inverse Laplace transformation and recursive convolution method are programmed in SPICE 3 [6]. However, it is sometimes time consuming depending on the parameters, and cannot be applied to uniform interconnects. In the reference [7], the authors have proposed a technique of replacing the interconnect by the discrete π-type and/or T-type models which are easily applied to SPICE simulator, and it can be only applied to relatively short interconnects. We have proposed another elegant method for calculating the exact poles and residues [8], where the admittance matrix of interconnect can be described in the form of partial fraction. Then, we can get an asymptotic equivalent circuit in the complex frequency domain, which can efficiently calculate the transient responses of LSI circuits with SPICE.

In this paper, let us consider large scale gate-array circuits connected by interconnects in LSI substrate. Note that the capacitance component in the substrate is dominant compared to the inductance, because the per dielectric constant of Silicon is over 10 times. Therefore, we can neglect the inductance component and model the interconnect with RCG multiconductor transmission lines. In this case, all the poles are located on the negative real axis on a complex plane, which makes easy to calculate the poles. We also propose the asymptotic equivalent circuit using the poles and residues. Thus, this simulator is a user-friendly. In section 2, we show how to calculate the exact poles and residues. In section 3, the asymptotic equivalent circuit is synthesized from the partial fractions. We show the illustrative examples in section 4. We found from many examples that the asymptotic method can get the good results even with the lower order approximation.

2. Calculation of poles and residues

Now, consider a uniform N coupled RCG interconnects in substrate. The telegraph equation is described by

$$\left.\begin{aligned} \frac{d\mathbf{V}(x,s)}{dx} &= -\mathbf{R}\mathbf{I}(x,s) \\ \frac{d\mathbf{I}(x,s)}{dx} &= -(\mathbf{G}+s\mathbf{C})\mathbf{V}(x,s) \end{aligned}\right\} \quad (1)$$

in the complex frequency-domain. Thus, we have

$$\left.\begin{aligned} \frac{d^2\mathbf{V}(x,s)}{dx^2} &= \mathbf{R}(\mathbf{G}+s\mathbf{C})\mathbf{V}(x,s) \\ \frac{d^2\mathbf{I}(x,s)}{dx^2} &= (\mathbf{G}+s\mathbf{C})\mathbf{R}\mathbf{I}(x,s) \end{aligned}\right\} \quad (2)$$

Let us introduce the transfer matrix $\mathbf{P}_v(s)$ and $\mathbf{P}_c(s)$ [9] for describing them in the diagonal forms. Thus, we have

$$\left.\begin{aligned} \mathrm{diag}[\lambda_i(s)^2] &= \mathbf{P}_v(s)^{-1}\mathbf{R}(\mathbf{G}+s\mathbf{C})\mathbf{P}_v(s) \\ \mathrm{diag}[\lambda_i(s)^2] &= \mathbf{P}_c(s)^{-1}(\mathbf{G}+s\mathbf{C})\mathbf{R}\mathbf{P}_v(s) \end{aligned}\right\} \quad (3)$$

where we have the following relations:

$$\left.\begin{aligned} &\mathbf{P}_c(s)^T = \mathbf{P}_v(s)^{-1}, \ \ \mathbf{P}_c(s) = \mathbf{R}^{-1}\mathbf{P}_v(s)\mathbf{\Gamma}(s) \\ &\mathbf{P}_v(s) = (\mathbf{G}+s\mathbf{C})^{-1}\mathbf{P}_c(s)\mathbf{\Gamma}(s), \ \ \mathbf{\Gamma}(s) = \mathrm{diag}[\gamma_i(s)] \end{aligned}\right\} \quad (4)$$

Then, the impedance matrix is described in the following form;

$$\begin{bmatrix} \mathbf{V}(0,s) \\ \mathbf{V}(d,s) \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{11}(s) & \mathbf{Z}_{12}(s) \\ \mathbf{Z}_{21}(s) & \mathbf{Z}_{22}(s) \end{bmatrix} \begin{bmatrix} \mathbf{I}(0,s) \\ \mathbf{I}(d,s) \end{bmatrix} \quad (5)$$

where

$$\begin{aligned}\mathbf{Z}_{11}(s) = \mathbf{Z}_{22}(s) = \mathbf{P}_v(s)\text{diag}[\coth \lambda_i(s)d]\mathbf{P}_c(s)^{-1} \\ \mathbf{Z}_{12}(s) = \mathbf{Z}_{21}(s) = \mathbf{P}_v(s)\text{diag}[\sinh^{-1} \lambda_i(s)d]\mathbf{P}_c(s)^{-1}\end{aligned} \quad (6)$$

Observe that poles of the impedance matrix are found at the locations satisfying $\sinh \lambda_i(s) = 0$. Thus, we have the following theorem for calculating the poles.

Theorem 1: *The locations of poles satisfying relations (6) are found by solving the following equation:*

$$\left|\mathbf{R}(\mathbf{G} + s\mathbf{C}) + \left(\frac{n\pi}{d}\right)^2 \mathbf{I}\right| = 0, \quad n = 1, 2, \ldots \quad (7)$$

Proof: We have from (6) that the poles satisfy the following relation:

$$\det|\mathbf{P}_c(s)\text{diag}[\sinh \lambda_i(s)d]\mathbf{P}_v(s)^{-1}| = 0 \quad (8)$$

or

$$\det|\mathbf{P}_c(s)\text{diag}[\tanh^{-1} \lambda_i(s)d]\mathbf{P}_v(s)^{-1}| = 0 \quad (9)$$

Since $\mathbf{P}_v(s)$ and $\mathbf{P}_c(s)$ are nonsingular for $n \neq 0$, the poles satisfying the above two relations are given by

$$\sinh \gamma_i(s)d = 0, \quad i = 1, 2, \ldots, N \quad (10)$$

where N shows a number of the coupled conductors. Namely, we have

$$\gamma_i(s)d = jn\pi, \quad i = 1, 2, \ldots, N, \quad n = 1, 2, \ldots \quad (11)$$

Therefore, the characteristic equation given by (3) must satisfy the relation (7).

Q.E.D.

Corollary 1.1: *The poles $p_{0,i}$ and residues $k_{0,i}$ for $n = 0$ are given in the following relations:*

$$\text{diag}[p_{0,i}] = -\mathbf{H}^T\mathbf{S}\mathbf{Q}^T\mathbf{G}\mathbf{Q}\mathbf{S}\mathbf{H}, \ \text{diag}[k_{0,i}] = \frac{1}{\text{diag}[C_{ii}]d} \quad (12)$$

Proof: The pole at $n = 0$ in (7) are found from the following relation:

$$\begin{aligned}&\lim_{\lambda_i \to 0} \mathbf{P}_v(s)\text{diag}[\sinh^{-1} \lambda_i(s)d]\mathbf{P}_c^{-1}(s) \\ &= (\mathbf{G} + s\mathbf{C})^{-1}\mathbf{P}_c(s)\mathbf{\Gamma}(\mathbf{s})(\mathbf{\Gamma}(s)d)^{-1}\mathbf{P}_c^{-1}(s) \\ &= (\mathbf{G} + s\mathbf{C})^{-1}/d\end{aligned} \quad (13)$$

Next, let us transform the above relation into the diagonal form [10]:

$$(\mathbf{G}+s\mathbf{C})^{-1}/d = \left(\mathbf{H}^T\mathbf{S}\mathbf{Q}^T\mathbf{G}\mathbf{Q}\mathbf{S}\mathbf{H} + s\mathbf{I}\right)^{-1} \text{diag}[\frac{1}{C_{ii}d}] \quad (14)$$

where $\mathbf{S} = \text{diag}[C_{ii}]^{-\frac{1}{2}}$, and $\mathbf{Q}$ and $\mathbf{H}$ are the transfer matrices for $\mathbf{C}$ and $\mathbf{S}\mathbf{Q}^T\mathbf{G}\mathbf{Q}\mathbf{S}$, respectively. Thus, the poles and residues are given by (12).

Q.E.D.

Theorem 2: *Assume that* **R**, **C**, **G** *are positive real symmetric matrices. Then, all the poles of RCG interconnect are located on the negative real axis.*

Proof: The poles satisfying relation (7) are given by

$$\left|(\mathbf{G} + s\mathbf{C}) + \left(\frac{n\pi}{d}\right)^2 \mathbf{R}^{-1}\right| = 0, \quad n = 1, 2, \ldots \quad (15)$$

For simplicity, we rewrite the matrix (15) as follows:

$$s\mathbf{C} + \mathbf{H}_n = \mathbf{0}, \quad \text{where } \mathbf{H}_n = \left(\frac{n\pi}{d}\right)^2 \mathbf{R}^{-1} + \mathbf{G} \quad (16)$$

Observe that $\mathbf{H}_n$ is also a positive real symmetric matrix. Thus, it can be transformed into the following diagonal form:

$$\mathbf{S}\mathbf{Q}^T(\mathbf{H}_n + s\mathbf{C})\mathbf{Q}\mathbf{S} = \mathbf{S}\mathbf{Q}^T\mathbf{H}_n\mathbf{Q}\mathbf{S} + s\mathbf{I} = \mathbf{0} \quad (17)$$

Since $\mathbf{S}\mathbf{Q}^T\mathbf{H}_n\mathbf{Q}\mathbf{S}$ is the positive real symmetric matrix, the poles satisfying (17) are negative real numbers.

Q.E.D.

Now, consider the numerical technique for the calculation of poles. Eq.(7) can be rewritten as follows:

$$|s\mathbf{I} + \mathbf{A}| = 0, \quad \text{for } \mathbf{A} = (\mathbf{R}\mathbf{C})^{-1}\left(\left(\frac{n\pi}{d}\right)^2 \mathbf{I} + \mathbf{R}\mathbf{G}\right) \quad (18)$$

This relation is the characteristic equation of $\mathbf{A}$, so that we can apply Bocher formular [11] to get the polynomial as follows:

Bocher formular: For a $n \times n$ matrix $\mathbf{A}$, set

$$|s\mathbf{I} + \mathbf{A}| = \alpha_0 + s\alpha_1 + \cdots + s^{n-1}\alpha_{n-1} + s^n = 0 \quad (19)$$

Then, $\alpha_0, \alpha_1, \ldots, \alpha_{n-1}$ are gien by

$$\left.\begin{aligned}\alpha_{n-1} &= -\text{trace}(\mathbf{A}) \\ \alpha_{n-2} &= -\tfrac{1}{2}\left[\alpha_{n-1}\text{trace}(\mathbf{A}) + \text{trace}(\mathbf{A}^2)\right] \\ &\ldots\ldots \\ \alpha_0 &= -\tfrac{1}{n}\left[\alpha_1\text{trace}(\mathbf{A}) + \alpha_2\text{trace}(\mathbf{A}^2) + \right. \\ &\qquad \left.\cdots + \alpha_{n-1}\text{trace}(\mathbf{A}^{n-1}) + \text{trace}(\mathbf{A}^n)\right]\end{aligned}\right\} \quad (20)$$

Hence, we can numerically solve (19) with Bairstow method and so on.

Next, let us calculate the residues of (5) [4].

Theorem 3: *The residues of* $\mathbf{Z}_{12}(s)$ *and* $\mathbf{Z}_{21}(s)$ *in (5) for the pole* s_i *is given by*

$$\mathbf{k}_{12,i} = -\mathbf{P}_v(s)\text{diag}\left[\frac{1}{\cosh(\lambda_i(s)d)\frac{\partial \lambda_i(s)}{\partial s}d}\right]\mathbf{P}_c(s)^{-1}\Bigg|_{s=s_i} \quad (21)$$

where $\frac{\partial \lambda_i(s)}{\partial s}$ *is obtained in the following:*

$$\begin{bmatrix}\frac{\partial \mathbf{U}_i}{\partial s} \\ \frac{\partial \lambda_i(s)}{\partial s}\end{bmatrix} = \begin{bmatrix}\mathbf{R}(\mathbf{G} + s\mathbf{C}) - \lambda_i(s)^2\mathbf{I} & -2\lambda_i(s)\mathbf{U}_i \\ \mathbf{U}_i^T & 0\end{bmatrix}^{-1} \times \begin{bmatrix}-\mathbf{R}\mathbf{C}\mathbf{U}_i \\ 0\end{bmatrix} \quad (22)$$

where $\mathbf{U}_i$ is the eigenvector for $\lambda_i(s_i)$.

Corollary 3.1: *The residues of* $\mathbf{Z}_{11}(s)$ *and* $\mathbf{Z}_{22}(s)$ *are given by*

$$\mathbf{k}_{11,i} \doteq \mathbf{P}_v(s)\text{diag}\left[\frac{1}{\frac{\partial \lambda_i(s)}{\partial s}d}\right]\mathbf{P}_c^{-1}(s)\Bigg|_{s=s_i} \quad (23)$$

Observe that the residues of (21) and (23) are the same except for the signs. Then, the impedance matrix is described by the partial fractions using these poles and the residues in the following form:

$$Z_{ij}(s) = \frac{k_{1,ij}}{s - p_1} + \frac{k_{2,ij}}{s - p_2} + \frac{k_{3,ij}}{s - p_3} + \cdots \quad (24)$$

It is very important that how many poles should be chosen to approximate the matrix. We now consider the special case of a single interconnect whose impedance matrix is given by

$$\begin{bmatrix} V(0,s) \\ V(d,s) \end{bmatrix} = Z_0(s) \begin{bmatrix} \coth\lambda(s)d & \sinh^{-1}\lambda(s)d \\ \sinh^{-1}\lambda(s)d & \coth\lambda(s)d \end{bmatrix} \times \begin{bmatrix} I(0,s) \\ I(d,s) \end{bmatrix} \qquad (25)$$

$$Z_0(s) = \sqrt{R/(G+sC)}, \quad \lambda(s) = \sqrt{R(G+sC)}$$

For example, we set the parameters as follows:
$R = 50[\Omega/\mu m]$, $C = 62.8[pF/\mu m]$, $G = 0.1[S/\mu m]$ $d = 1[\mu]$. The frequency responses of $Z_{11}(s) = Z_{22}(s)$ and $Z_{12}(s) = Z_{21}(s)$ for the order 20 approximation are shown in Fig.1 (a) and (b), respectively. We found from the results that we have a good result in all the frequency-domain even for the lower approximation.

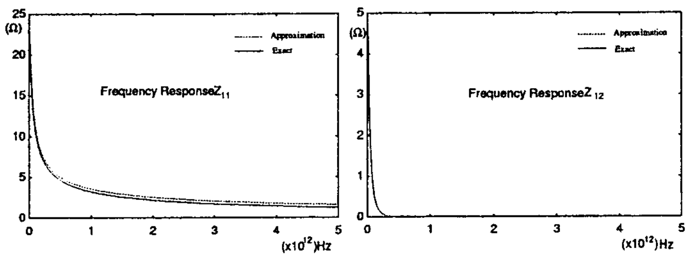

Fig.1: Approximation by partial fraction (Oder=20)

Therefore, we can also expect the good results of the transient responses using the asymptotic equivalent circuit.

3. Asymptotic equivalent circuit

We found in section 2 that all the solutions are located on the negative real axis. In this case, only the poles located near the origin will give large effect to the transient responses, so that we will choose few poles around the origin. Now, let us expand (6) into the partial expansion as follows:

$$Z_{11}(s) = Z_{22}(s)$$
$$= \frac{k_0}{s+p_0} + \sum_{i=1}^{M} \left[\frac{k_{2i-1}}{s+p_{2i-1}} + \frac{k_{2i}}{s+p_{2i}} \right] \qquad (26.1)$$

$$Z_{12}(s) = Z_{21}(s)$$
$$= \frac{k_0}{s+p_0} + \sum_{i=1}^{M} \left[-\frac{k_{2i-1}}{s+p_{2i-1}} + \frac{k_{2i}}{s+p_{2i}} \right] \qquad (26.2)$$

where M is defined as the **order of approximation** of our asymptotic equivalent method. Now, set

$$Z_1(s) = \sum_{i=0}^{M} \frac{k_{2i}}{s+p_{2i}}, \quad Z_2(s) = \sum_{i=1}^{M} \frac{k_{2i-1}}{s+p_{2i-1}} \qquad (27)$$

where

$$p_0 = \frac{G}{C}, \quad p_i = \frac{RG + \left(\frac{i\pi}{d}\right)^2}{RC}, i = 1, 2, \ldots, 2M$$

$$k_0 = \frac{1}{Cd}, \quad k_i = \frac{2}{Cd}, i = 1, 2, \ldots, 2M$$

Then, eq.(5) can be written as follows:

$$\begin{bmatrix} V(0,s) \\ V(d,s) \end{bmatrix} = \begin{bmatrix} Z_1(s)+Z_2(s) & Z_1(s)-Z_2(s) \\ Z_1(s)-Z_2(s) & Z_1(s)+Z_2(s) \end{bmatrix} \times \begin{bmatrix} I(0,s) \\ I(d,s) \end{bmatrix} \qquad (28)$$

Thus, the equivalent circuit is synthesized by the use of current-controlled voltage sources (V_d, V_s) as shown in Fig.2.

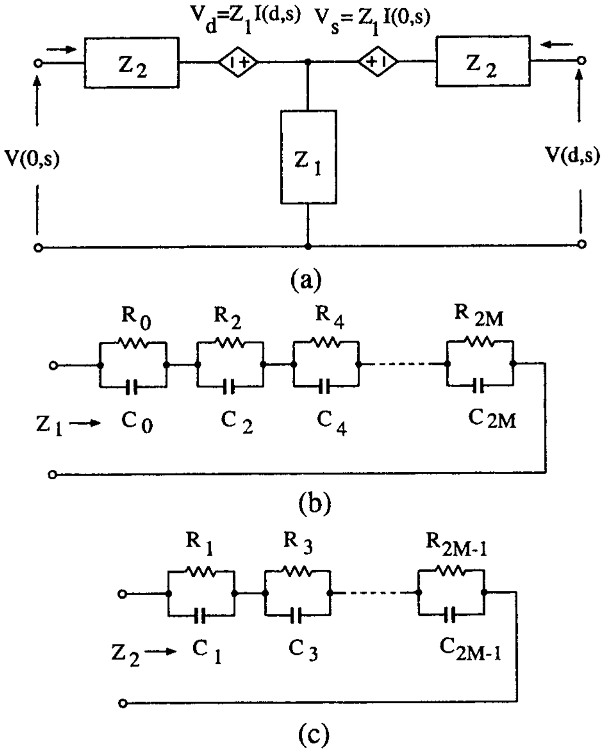

Fig.2 : Asymptotic equivalent circuit

where

$$C_i = \frac{1}{k_i}, \quad R_i = \frac{k_i}{p_i}, \; i = 0, 1, 2, \ldots, 2M \qquad (29)$$

Note that since the asymptotic equivalent circuit is familiar with SPICE, we can easily develop the user-friendly simulator.

4. Illustrative examples

4.1 Interconnect terminated with linear resistors: To show the accuracy of our asymptotic method, we compare our result with the numerical Laplace transformation method [12] which is considered as the exact solution.

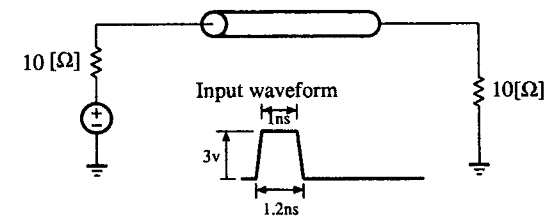

Fig.3(a): Interconnect with linear resistor, $R = 50[\Omega/\mu m]$ $C = 62.8[pF/\mu m]$, $G = 0.1[S/\mu m]$, $d = 0.3\mu m$

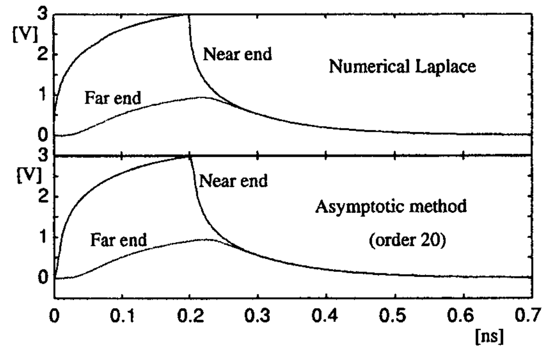

Fig.3(b): Numerical Laplace transformation, (c): Asymptotic method (Order=20)

Observe that both results have exactly same waveforms. We also found from many examples that we can get the good results even for the order 5.

4.2 A fulladder circuit coupled with interconnects: Our asymptotic equivalent circuit can be easily applied to any circuit. Consider a fulladder circuit coupled with interconnects as shown in Fig.4(a).

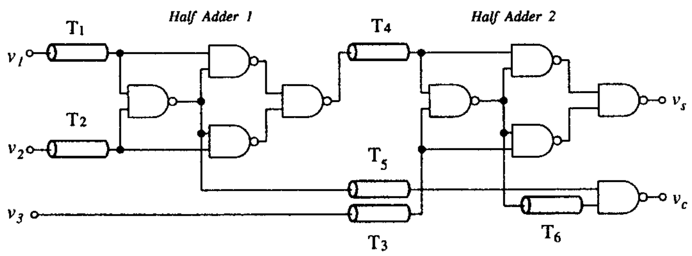

Fig.4(a): Fulladder coupled with interconnects, $R = 50[\Omega/\mu m]$, $C = 0.628[fF/\mu m]$, $G = 0.6[\mu S/\mu m]$, $d = 50[\mu]$

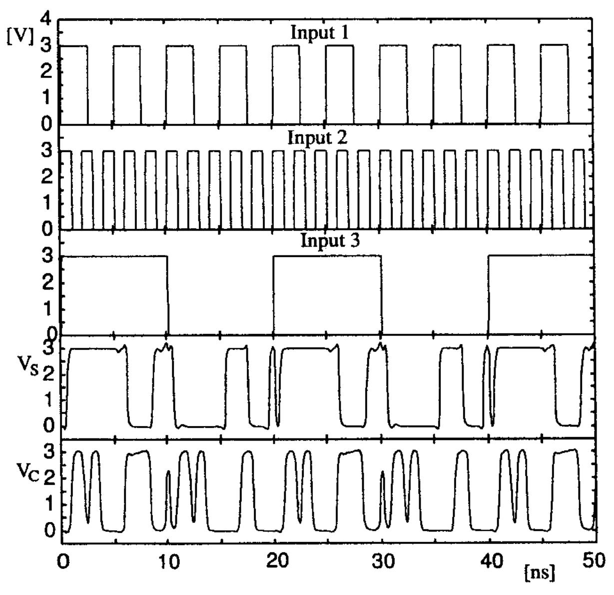

Fig.4(b): Transient response of the fulladder circuit coupled with interconnects

We found from the transient responses Fig.4(b) that the waveforms are largely distorted, and they have large time-delays. They may cause the fault switching.

5. Conclusions and remarks

In this paper, we have proposed an asymptotic equivalent circuit technique for the reduction of interconnects, where the interconnects are replaced by the simple RC circuits. Thus, we can easily get the transient response with SPICE. At first step, we calculate the exact poles and the residues of interconnect, and the impedance matrix is describe the partial fractions. Secondly, we realize the partial fractions by the equivalent circuit called asymptotic circuit. We found from examples that we can get good result even with the low order approximation.

As the future problem, we need to extend our algorithm to large scale multiconductor interconnects.

REFERENCES

[1] L.T.Pillage and R.A.Rohrer, "Asymptotic waveform evaluation for timing analysis," *IEEE Trans. on Computer Aided Design*, vol.CAD-9, pp.352-355, 1990.

[2] E.Chiprout and M.S.Nakhla, *Asymptotic Waveform Evaluation and Moment Matching for Interconnect Analysis*, Kluwer Academic Pub., 1994.

[3] S.Lin and E.S.Kuh, "Transient simulation of lossy interconnects based on the recursive convolution formulation," *IEEE Trans. Circuits and Systems-I*, vol.CAS-39,pp.879-892, 1992.

[4] E.Chiprout and M.S.Nakhla, " Analysis of interconnect networks using complex frequency hopping(CHF)," *IEEE Trans. on Computer Aided Design*, vol.CAD-14, pp.186-200,1995.

[5] R.Achar, P.K.Gunupudi, M.S.Nakhla and E.Chiprout, "Passive interconnect reduction algorithm for distributed/measured networks," *IEEE Trans. Circuits and Systems-II*, vol.CAS-47, pp.287-301, 2000.

[6] J.S.Roychowdhury, A.R.Newton and D.O.Pederson, "Algorithms for the transient simulation of lossy interconnects, *IEEE Trans. on Computer Aided Design*, vol.CAD-13, pp.96-104, 1994

[7] T.Dhaene and D.D.Zutter, "Selection of lumped element models for coupled lossy transmission lines," *IEEE Trans. on Computer Aided Design*, vol.CAD-11, pp.805-815, 1992.

[8] A.Ushida, K.Urabe, Y.Yamagami and Y.Nishio, "Asymptotic equivalent circuits of interconnects based on complex frequency method," *ECCTD'01*, vol.3, pp.29-32, 2001.

[9] Y.Tanji, L.Jiang and A.Ushida, "Analysis of pulse response of multi-conductor transmission lines by a partitioning technique," *IEICE Trans. on Fundamentals*, vol.E77-A,no.12, pp.2017-2027, 1994.

[10] P.M.Chirlian, *Integrated and Active Network Analysis and Synthesis*, Prentic-Hall, 1967.

[11] M.Bocher, *Introduction to Higher Algebra*, Dover, New York, 1964.

[12] T.Hosono, *High Speed Laplace Transformation with BASIC*, Kyouritsu, Japan, 1984.