# Time Hopping Sequences Based on Pseudo Random Codes for Ultra Wideband Impulse Radio Systems*

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**Abstract:** A new form of spread spectrum technique called the ultra wideband impulse radio (UWB-IR) system has drawn much attention for future high speed wireless communication services. In this paper, a new type of time hopping sequences constructed from multiple distinct $m$-sequences of the same order, is proposed for multiple access in the UWB-IR systems. Simulation results reveal that the proposed time hopping sequences achieve comparable or even better bit error rate performance than the ideal random sequences, and can be effectively applied in various multiple access situations.

## 1. Introduction

Recently, ultra wideband impulse radio (UWB-IR) system [1,2,3] has drawn much attention as a new type of spread spectrum multiple access communication system in such applications as in-building wireless LAN and military communication systems[4,5]. Unlike the conventional communication systems which utilize information-bearing continuous RF carriers, the UWB-IR systems transmit a train of baseband pulses named Gaussian monocycle pulses (GMP's) or simply impulses with very short duration in the order of sub-nano seconds.

Digital modulation in the UWB-IR systems is usually accomplished by pulse position modulation (PPM) scheme at a rate of many pulses per symbol, and time hopping with a different hopping sequence for each user provides multiple access capability. The end result is that time hopped, pulse position modulated GMP's have the power spectrum of ultra wide bandwidth spanned from close to DC to several GHz and very low density well below the thermal noise floor, so that the system causes a negligible interference to the existing radio systems. In dense multipath environment, the UWB-IR systems inherently provide with a very fine resolvability mainly due to very short GMP duration, and the use of Rake-type receivers significantly improves the performance[3]. Moreover, utilization of short duration GMP allows an extremely high processing gain, and thus a large number of active users can be accommodated.

Former theoretical studies on the UWB-IR systems include bit error rate analyses for the UWB-IR systems with perfectly random hopping sequences under additive white Gaussian noise (AWGN) channels [1,2,3]. However, random sequences are far from realistic situation. In this paper we propose a new type of time hopping sequences constructed from multiple distinct $m$-sequences of the same order for multiple access in the UWB-IR systems. The proposed time hopping sequences are suitable for more general asynchronous as well as synchronous multiple access situations.

The paper is organized as follows. Section 2 explains the binary UWB-IR systems. Section 3 describes the proposed time hopping sequence and its characteristic. Simulation results are provided in Section 4 to highlight the BER performance of the proposed hopping sequence in both synchronous and asynchronous multiple access situations. Finally, Section 5 concludes the paper.

## 2. Binary UWB-IR Systems

For digital signalling, the UWB-IR system utilizes GMP $p(t)$ which is expressed as

$$p(t) = 2A\sqrt{\pi e}\frac{t}{\tau}e^{-2\pi(t/\tau)^2} \quad . \tag{1}$$

where $A$ denotes the amplitude and $\tau$ is a parameter related to the width of the GMP. For the GMP $p(t)$ in (1), 3-dB bandwidth is approximately given as $1.16/\sqrt{\pi\tau}$ Hz. Typically, $\tau$ is in the order of sub-nano seconds, resulting in ultra wide bandwidth of few GHz.

Data modulation in the UWB-IR system is usually accomplished by PPM scheme at a rate of many pulses per symbol, and time hopping with a different hopping sequence for each user provides multiple access capability. The end result is that time hopped, pulse position modulated GMP's have the power spectrum of ultra wideband and very low density well below the thermal noise floor. In the receiver of the UWB-IR system, the receive antenna that is modeled as a differentiator[6] yields a signal $w(t)$ for a GMP input $p(t)$ as

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$$w(t) = 2A\sqrt{\pi e}\left(\frac{1}{\tau} - \frac{4\pi t^2}{\tau^3}\right)e^{-2\pi(t/\tau)^2}. \tag{2}$$

Utilizing the receive antenna output signals, the transmitted binary PPM signals $s^{(k)}(t)$ for the $k$-th user with time hopping can be equivalently modeled as

$$s^{(k)}(t) = \sum_j w\left(t - jT_f - c_j^{(k)}T_c - \delta d_{\lfloor j/N_s \rfloor}^{(k)}\right), \tag{3}$$

where $T_f$ is the pulse repetition interval which is several hundred or thousand times larger than the GMP duration. In (3), $c_j^{(k)}$ corresponds to the decimally represented $j$-th time hopping code for the $k$-th user, and $T_c$ is the controllable unit time. Denoting $N_h$ as the maximum value of the hopping code, $T_c$ should satisfy $N_h T_c \le T_f$ to avoid any collision with the next pulse. Also in (3), $\delta$ represents the time interval between the data bits "0" and "1'" in the PPM, and $d_{\lfloor j/N_s \rfloor}^{(k)}$ is the $\lfloor j/N_s \rfloor$-th data bit of the $k$-th user where $N_s$ denotes the number of pulse repetitions per data bit, yielding the actural data transmission rate

$$R_s = 1/N_s T_f \text{ (bps)}. \tag{4}$$

Under an additive white Gaussian (AWGN) channel, the input signal $r(t)$ to a correlator receiver with synchronous multiple access of $N_u$ users in a downlink, can be represented as [2,3]

$$r(t) = \sum_{k=1}^{N_u} A_k s^{(k)}(t-\tau) + n(t), \tag{5}$$

where $A_k$ denotes attenuation of the $k$-th user's signal $s^{(k)}(t)$, $\tau$ is the delay which is supposed to be the same for all the users in a synchronous system, and $n(t)$ is the AWGN with zero-mean and two-sided power spectral density $N_0/2$ Watt/Hz. For the $i$-th bit $d_i^{(1)} \in \{0,1\}$ of the first user in the synchronous multiple access situation, (5) can be rewritten as

$$\begin{aligned} r(t) &= A_1 s^{(1)}(t-\tau) + n_{tot}(t) \\ &= A_1 w_{bit}(t - \delta d_i^{(1)}) \\ &\quad + \left(\sum_{k=2}^{N_u} A_k s^{(k)}(t-\tau) + n(t)\right), \end{aligned} \tag{6}$$

with

$$w_{bit}(t) \equiv \sum_{j=iN_s}^{(i+1)N_s-1} v\left(t - jT_f - c_j^{(1)}T_c - \tau\right). \tag{7}$$

In this case, the reference signal $v_{bit}(t)$ used in the first user's correlator receiver is given by [2,3]

$$\begin{aligned} v_{bit}(t) &\equiv w_{bit}(t) - w_{bit}(t-\delta) \\ &= \sum_{j=iN_s}^{(i+1)N_s-1} v\left(t - jT_f - c_j^{(1)}T_c - \tau\right), \end{aligned} \tag{8}$$

where $v(t) \equiv w(t) - w(t-\delta)$ , and the decision statistic becomes

$$\int_{\in T_i} r(t) v_{bit}(t) dt. \tag{9}$$

Here, $T_i$ represents the non-overlapping time intervals of $N_s$ GMPs for the bit $d_i^{(1)}$. For large $N_u$, $n_{tot}(t)$ becomes approximately Gaussian-distributed, and the BER $P_b$ can be obtained as follows under an assumption of a perfect delay acquisition [1].

$$\begin{aligned} P_b &= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{S_{out}(N_u)}}^{\infty} e^{-x^2/2} dx \\ &= Q\left(\sqrt{S_{out}(N_u)}\right) = Q\left(\sqrt{\frac{(A_1 N_s m_p)^2}{\sigma_{tot}^2(N_u)}}\right) \end{aligned} \tag{10}$$

where

$$m_p \equiv \int_{-\infty}^{\infty} w(t)[w(t) - w(t-\delta)]dt = \int_{-\infty}^{\infty} w(t)v(t)dt, \tag{11}$$

$$\sigma_{tot}^2(N_u) \equiv E\left(\left[\int_{\in \mathrm{T_i}} n_{tot}(t) v_{bit}(t)\right]^2\right), \tag{12}$$

with statistical expectation operator $E(\cdot)$.

## 3. Time Hopping Sequence Based on Pseudo Random Codes

Most previous studies [1,2,3] have assumed ideal random sequences for analysis of system performance, however, random sequences are far from realistic situation. On the other hand, we have considered in [7,8] a time hopping sequence generator composed of a pseudo random or maximal length sequence ($m$-sequence) generator and a decimalizing module. In the scheme, a sequence is obtained by decimalization of $n$ memory elements in an $n$-stage $m$-sequence generator, and other sequences are simply obtained with different offsets or initial conditions. The time hopping sequences in [7,8] are suitable for synchronous multiple access situations where perfect frame synchronization and identical delays among different users are to be guaranteed.

We propose in this paper the time hopping sequences which are constructed from *multiple* distinct $m$-sequences of the same order. Unlike the sequences in [7,8] the proposed sequences are suitable for more general asynchronous as well as synchronous multiple access situations. Consider an $n$-stage $m$-sequence generator with the following primitive generator polynomial

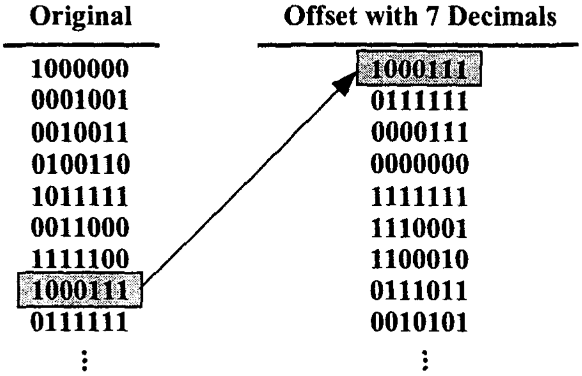
$$\begin{gathered} g(x) = x^n + g_{n-1}x^{n-1} + \cdots + g_1 x + 1 \\ (g_i \in \{0,1\},\ i = 1, \cdots, n-1). \end{gathered} \tag{13}$$

Note that an $m$-sequence obtained by (13) achieves the maximum period of $L \equiv 2^n - 1$ bits. Moreover, the number of different primitive polynomials (or different $m$-sequences) of degree $n$ is $M_n = \frac{1}{n}\phi(L)$ where the "Euler-totient function" $\phi(L)$ denotes the number of positive integers less than and relative primes with $L$. For instance, $L = 1023$ and $M_n = 60$ for $n = 10$.

Suppose that for a positive integer $l$ a frame consists of $N_h \equiv 2^l$ slots with $T_f = N_h T_c$ and total of $N_u (\ge N_h)$ users are to be accessed. In this case, a time hopping sequence for a user is generated by decimalizing an $m$-sequence of length $L$ bits by successive non-overlapping $l$ bit intervals. Thus, the period of a time hopping sequence becomes $N \equiv \left\lfloor \frac{L}{l} \right\rfloor$, and the last $L - \left\lfloor \frac{L}{l} \right\rfloor$ bits of the $m$-sequence are intentionally discarded. Moreover, by run-length property [9] of the $m$-sequences, a time hopping code can take decimal values ranging from 0 to $N_h - 1$ provided that $1 \le l \le n-2$. An $m$-sequence of length $L = 1023$ ($n = 10$) generated by $g(x) = x^{10} + x^3 + 1$ with an initial state of 0000000001, and a decimally represented time hopping sequence with $l \equiv 7$, are illustrated in Figure 1. Based on the shifting property [9] of the $m$-sequences, other time hopping sequences are constructed with integer multiples of an offset $f$ (in decimals), as shown in Figure 2 where $f = 7$. The offset should be determined by considering various factors such as frame duration and maximum difference of propagation delays among the users.

| **Binary** | **Decimal** |
|---|---|
| **1000000** | **64** |
| **0001001** | **9** |
| **0010011** | **19** |
| **0100110** | **38** |
| **1011111** | **95** |
| **0011000** | **24** |
| **1111100** | **124** |
| **⋮** | **⋮** |

Figure 1 : Example of an $m$-sequence of length $L = 1023$ ($n = 10$) generated by $g(x) = x^{10} + x^3 + 1$ with an initial state of 0000000001, and a decimally represented time hopping sequence with $l \equiv 7$.



| **Original** | **Offset with 7 Decimals** |
|---|---|
| **1000000** | **1000111** |
| **0001001** | **0111111** |
| **0010011** | **0000111** |
| **0100110** | **0000000** |
| **1011111** | **1111111** |
| **0011000** | **1110001** |
| **1111100** | **1100010** |
| **1000111** | **0111011** |
| **0111111** | **0010101** |
| **⋮** | **⋮** |

Figure 2 : Construction of a new time hopping sequence by an offset of $f = 7$ decimals.

Hence, we can construct $\left\lfloor \frac{L}{fl} \right\rfloor$ different time hopping sequences from an $m$-sequence (or a primitive polynomial) of length $L$, and the total of $M_n \left\lfloor \frac{L}{fl} \right\rfloor$ time hopping sequences can be constructed from all the primitive polynomials of degree $n$. As an example, for $n = 10$, $l = 7$ and $f = 7$, simultaneous multiple access of $M_n \left\lfloor \frac{L}{fl} \right\rfloor$ users are possible. Note that the degree $n$ of primitive polynomials should be selected to satisfy the following condition

$$N_u \le \frac{1}{n}\phi(2^n - 1)\left[\frac{2^n - 1}{f\ell}\right]. \tag{14}$$

Let us define the normalized occurrence of coincidences (NOC) $C(k,m)$ of the $k$-th and $m$-th time hopping sequences as

$$C(k,m) \equiv \frac{1}{N}\sum_{j=1}^{N}\delta\left(c_j^{(k)} - c_j^{(m)}\right), \tag{15}$$

where $c_j^{(k)}$ denotes the slot number (in decimals) of the $k$-th sequence in the $j$-th frame $[(j-1)T_f, jT_f)$, and $\delta(\cdot)$ is Kronecker delta function. Figure 3 shows the worst case $C(k',m)$ for $N_h = 128$, $N_u = 1200$ and $N = 146$ where $k' = \arg\max_k \{\max_{\mu \ne k} C(k,u)\}$ and $m = 1, \cdots, N_u$. In the figure, the leftmost largest value corresponds to $C(k',k') = 1$ and the other values correspond to $C(k',m)$ for $m \ne k'$. We observe from the figure that the proposed time hopping sequences achieve quite low NOC values, and this highlights the good performance of the sequences.

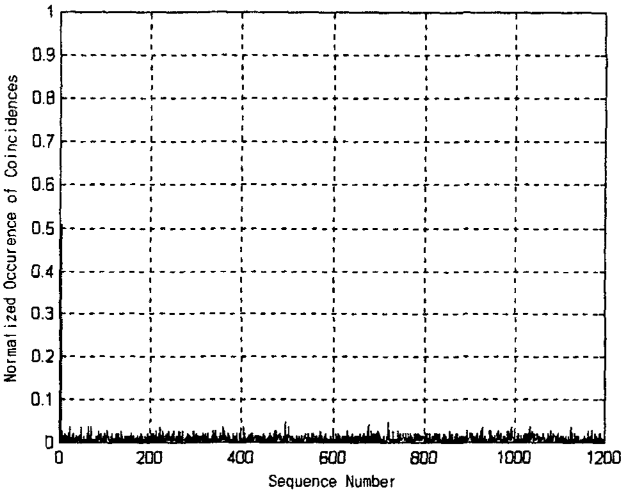


Figure 3 : The worst case NOC $C(k',m)$ of the proposed time hopping sequences for $N_h = 128$, $N_u = 1200$ and $N = 146$.

## 4. Simulation Results

To evaluate the performance of the proposed time hopping sequences, we consider the following multiple access situations. In the first case called "synchronous system", we assume propagation delays $\tau_k$ $(k = 1, \cdots, N_u)$ and the initial sequence offsets of $N_u$ users are identical. In the second case called "asynchronous system #1", the delays $\tau_k$ are all set to 0, however, initial sequence offsets are randomly chosen within the period. Finally, in a more general "asynchronous system #2", delay and initial offset of each user are randomly determined. The time delay of the desired user ($k \equiv 1$) is $\tau_1 = 0$ and other users' time delays are selected randomly in 0, 25, 50, 75, 100 samples from one GMP $w(t)$ which is shown in Figure 4. In the simulation, we consider $A = 10^{-10}$ , $\tau = 0.3$ nsec and $\delta = 0.1626$ nsec[7]. The signal-to-noise ratio (SNR) of each user is set to 9.8 dB, $N_s = 20$ and $N_h = 128$. For the time hopping sequences, $n = 10$ , $l = 7$ and $f = 7$ are utilized. Figure 5 compares bit error rate (BER) performances of the ideal random sequences and the proposed sequences according to the number of users in the "synchronous system". We observe that the proposed sequences achieve comparable or even better performance than the ideal random sequences. Figure 6 shows the BER performances of the proposed sequences for various multiple access situations. It is observed from the figure that the proposed sequences can be quite effectively applied to different multiple access situations.

## 5. Conclusion

In this paper, we proposed a time hopping sequences constructed from multiple distinct $m$ -sequences of the same order for multiple access in the UWB-IR systems. Computer simulation results indicate the proposed time hopping sequences are suitable for more general asynchronous as well as synchronous multiple access situations.

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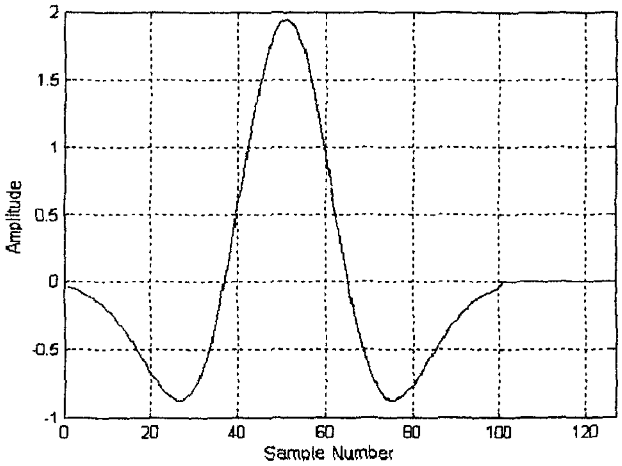

Figure 4 : Recieved gaussian monocycle pulse (GMP) $w(t)$ during one slot duration $T_c$.

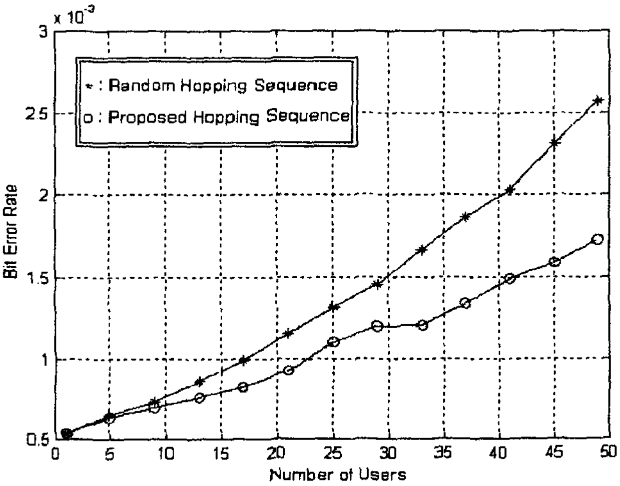

Figure 5 : BER performances of the ideal random sequences and the proposed sequences according to the number of users in synchronous multiple access system.

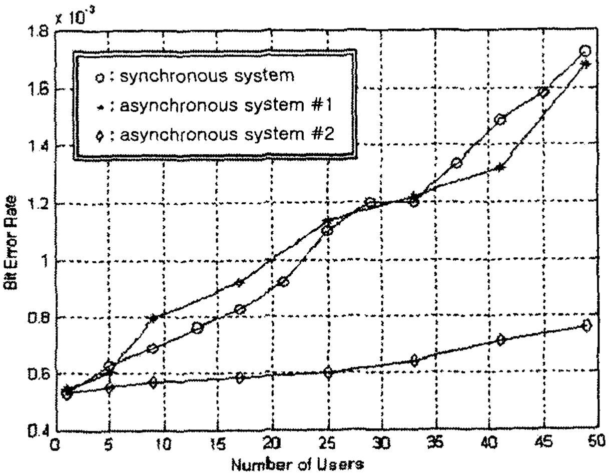

Figure 6 : BER performances of the proposed sequences for various multiple access situations.