A Study on the Closed Linear Movement of the Center of Mass in the Rotatory Movement of a Rigid Body

Byung -Tae Chung
Dept.Computer Science, Incheon City College
#235 Dohwa-Dong ,Nam-Gu , Incheon 402-750,KOREA
e-mail:btchung@icc.ac.kr, Fax:-82-32-760-8955
Phone:+82-32-760-8721

Abstract: It is understood so far that the center of mass does not make any linear movement from the rotatory movement of a rigid body in the closed system. However, it has been found that the center of mass of the system could make a closed linear movement due to production of an instantaneous center of mass by the Coriolis force in the rotatory movement of a rigid body in the closed system. The nature of the closed linear movement in the non-inertial system and that of the open movement in the inertial system are different from each other. That is, the closed movement is described like the time integration of frictional forces, which is different from the open movement usually considered and described like the time integration of external forces. It is shown in this paper that the Coriolis force, called a fictitious force in the classical mechanics, is similar to the frictional force so that it causes to move the center of mass of a closed system. In this paper, following an explanation of the closed linear movement of a non-inertial system and the open movement of an inertial system, the source of the closed linear movement phenomenon of a rotatory rigid body is presented.

1. Introduction

In the classical mechanics, the Coriolis force (Fc) is known as a fictitious force [1], [2] and is regarded as not actually existing. However, in the modern mechanics, it is regarded as a real force affecting the system under consideration. It will be presented in this paper that the Coriolis force actually takes a real non-inertial movement effect from the relation between the Coriolis force and the instantaneous center of mass (ICM) in the closed system under the constraining condition of ω. In this paper, the effect of the non-inertial movement is the cause of the positional movement of the center of mass (CM) [3], [4], that is, a closed movement of mass in the system. So far, however, this has been regarded as impossible, because it is against the Newton's laws if a closed moving system intends to make a linear positional movement by internal propulsion without any external expulsion.

For closed systems developed up-to now, there are spherical mobile robots (SMR) [5] and spherical vehicle control systems (SVCS) [6]-[8], etc., which utilize the movement of the center of gravitational forces. However, these corresponds to eccentric movements made by rolling or by using gravitational forces and frictional forces on the rough surface. Two kinds can be considered for the movement of an object. That is, the open movement and the closed movement [9]. The movement produced by external forces and kept moving by the inertia can be called the open movement, while the movement produced as a result of applying external forces to an object in both the forward and backward directions with a difference in time for the same time period can be called the closed movement.

In this paper, if a Coriolis force is produced by the rotatory movement of mass in the closed system, then it is shown to be related to the ICM and thus becomes the source of a non-inertial system. We will present the concept by a vector model and explain the Coriolis force becomes the source of the closed system. Even though it is produced momentarily, it will be explained to be a real force actually existing.

2. Non-inertial closed linear movement by the Coriolis force produced in the rotatory movement of a rigid body

First, a linear closed movement is defined here. If the same amount of external forces is applied to a rigid body in the forward and backward directions with a difference in time for a constant time period as in Fig. 1, then a closed movement p [9] is produced as Eq. (1).

$$\int^{e}[F\delta(t)-F\delta(t-e)]dt = P\,[u(t)-u(t-e)]=\overline{P} \quad (1)$$

$$\int^{e}\overline{P}\,dt = ml = C \neq 0 \quad (2)$$

The integration of a closed movement equation

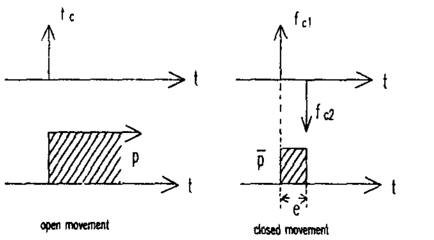

Fig. 1. Closed movement and open movement.

to infinity by time leads to a non-zero value of C, as in Eq. (2). The integration for the open movement, however, gives a value of infinity for C, as in Eq. (3).

$$\int^{\infty} P dt = \infty \quad (3)$$

For reference, if there is a non-zero value taken by C in the system, the CM of the whole system becomes to move. If the outside mass is denoted by M and the inside mass by m in an inertial system, as shown in Fig. 2, then when M moves a distance ℓ at an initial stand-still state, the total amount of movement of the CM is given as Eq. (4).

$$CM = M\cdot\frac{l}{M+m} \quad (4)$$

From the equation, a closed movement can be produced if there exists Fc instead of F in the system. Figure 3 shows that a Coriolis force Fc is produced on the centroid which is a trajectory of the ICM.

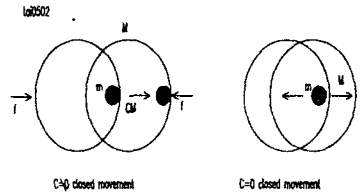

Fig. 2. Closed linear movement between the outside mass M and inside mass m in the inertial system.

If there is an axis of rotation rotating constantly (ω =constant) at an arbitrary point of the outside mass M of the system, and M and m move away constantly from each other, then an ICM is produced at the point of m on the line connecting the central axis of rotation (RCM) and that of the ICM (IA). At that instant, a force Fc is produced at the point of the ICM in the vertical direction to the IA connecting the RCM and ICM, as shown in Fig. (3).

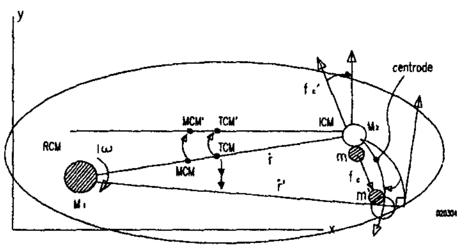

Fig. 3. Vector model of the relation of the ICM and Fc in the rotatory movement of the closed system.

The produced ICM moves making a centroid. The RCM makes a trajectory of a momentary arc centering around the central axis of the ICM since there is an Fc in the ICM. As a result, the MCM also rotates centering around the axis of the ICM and thus the TCM makes a positional movement also. This result is the case that the Fc produced by the action of the RCM and ICM reacts first in the inertial coordinate system and then reacts later in the rotatory coordinate system. Accordingly, since there occurs a time phase delay in the action and reaction between the two coordinate systems, Eqs. (8) and (10) can be obtained. In the case that r is variable and ω is constant, if a momentary angular acceleration is forced to be made to the RCM, then Eq. (5) is obtained. From this equation, it can be noted that the Fc just results in temporarily by the inertial mass I.

$$-\omega\frac{dI}{dt} = \tau_c = \gamma Fc \qquad (5)$$

For simplicity, a simplified model is developed by introducing a constraining condition of $\dot{r}$ and ω in Fig. 3. In this model, if it is assumed that $\dot{r}$ and ω are constant and the centrifugal force and centripetal force cancel each other, then the equations of rotational mechanics, Eqs. (6) and (7), are simplified as follows:

The Inertial System

$$2mr'\omega\delta(t) = 0 \qquad (6)$$

The rotatory System

$$-2mr'\omega\delta(t-e) = 0 \qquad (7)$$

When Eqs. (6) and (7) are combined together, Eq. (8) is obtained for a certain instant time.

$$F_c'\delta(t) - F_c\delta(t-e) = 0 \qquad (8)$$

Integration of both sides with the time period e gives Eq. (9).

$$\int^e [F_c'\delta(t) - F_c(t-e)]dt = \overline{P} \neq 0 \qquad (9)$$

An intermediate value of C in this vector model can be calculated as in Eqs. (10) and (11).

$$\overline{\delta} = \delta(t) - \delta(t-e) \qquad (10)$$

$$C = \int^{\infty}\int^{\infty} F_c\,\overline{\delta}\,dt \qquad (11)$$

where C=m · ℓ [kg/m] is defined as a quantity with dimensions of mass multiplied by distance.

3. Conclusion

If an equation for the Fc is defined and integrated by introducing a time phase difference in the inertial and non-inertial systems, the Fc makes a closed movement.

The open momentum and closed momentum are conservative except for the time duration (collision time: ⊿t) of existence of the Fc for the vector model moving in the closed system. However, during the time duration of ⊿t, the CM makes movement because there is a non-zero value taken by C and there exists a closed movement independently.

However, the internal force is zero even though there is some time interval. In the L-frame [10], an Fc produced during the rotatory movement in the closed system makes movement of the CM internally, but only the centrifugal force and centripetal force can be seen externally.

The phenomenon of positional movement of the CM in the present paper has a new non-inertial closed linear movement form. This means that when a closed system moves by inertia in space with zero gravity and no air, the trajectory of movement of the CM could be changed by the action and reaction on rotations in the system.

Also, there is a possibility of obtaining an open movement by applying some internal inertial force offsetting the velocity of the CM with another mass while producing an Fc which is the cause of the positional movement of the CM. As a conclusion, it can be found that in an arbitrary system, the Fc makes the CM of the system to be a closed linear movement [9] by an inertial force temporarily produced by the inertial mass. This kind of system movements can be applied to the simulation program of dynamics, and also internal propelling engines for position control of space vehicles, space stations, and satellites, etc.

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