An Adaptive Photon Mapping in the Use of Automatic Differentiation

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Abstract: Photon mapping is an efficient global illumination technique for realistic image synthesis that has been developed in computer graphics. In this paper, an adaptive photon mapping in the use of automatic differentiation is proposed. Since the automatic differentiation is used when photons emit from the light sources through the scene, we can check the variation of surrounding shape. Therefore, we can decrease the number of photons and generate an image in relatively low computational cost.

1. Introduction

Ray tracing[1] is well known as one of the most popular rendering techniques in computer graphics. In the method, light rays are traced backwards from the observer to the light sources. This approach can only handle mirror reflections, refractions and direct illumination. Important effects such as depth of field, motion blur, caustics, indirect illumination, and glossy reflection cannot be visualized by the method.

One of the efficient techniques to simulate these effects is photon mapping[2]. It is an efficient global illumination technique for realistic image synthesis that has been developed in computer graphics. The idea is to break ray tracing into two passes: the first casts photons into the scene from the light sources, and the second collects the photons to produce an image. The photon map is constructed by photons emitted from the light sources and traced through a scene. It contains information on all photons hitting surfaces, and this information can be used to render the scene efficiently.

However, it is not necessary to store an excessive number of photons when the lighting at the position is simple. We must get the information of photons on the part where sharp shape exists (e.g., the edges of objects). In this paper, an adaptive photon mapping algorithm will be proposed in the use of the automatic differentiation[3], which can detect edges of shapes and emit photons efficiently. In the proposed method, each calculation of the photon mapping has its differentiation with respect to coordinates. This approach makes the photon mapping adaptive so that the number of photons decreases and the computational cost becomes low.

2. Photon Mapping

In order to generate, store and use illumination as a set of points, photon map is defined as a data structure to process these points. Photon tracing is a technique used to generate the points representing the illumination in a scene. Since this algorithm is ray-tracing-based and since the photon map uses points as primitives, we get the advantages of the ray-tracing algorithm: complex geometry, no meshing, specular reflections, and more.

The photon mapping is a two-pass method which consists of photon tracing and rendering:

Photon tracing:

It is the technique that build the photon map by tracing photons from the light sources and storing them as they interact with the diffuse surfaces in the scene. When a photon hits a specular or refractive surface, it is not stored in the photon map (see Figure 1).

Rendering:

The photon map can be visualized directly by the simple ray tracing. For all diffuse surfaces the radiance is estimated by the obtained photon map, and standard recursive ray tracing is used for specular and refractive surfaces (see Figure 2).

The full global illumination consists of the four components, i.e., direct, specular, caustics, and indirect illumination. Direct illumination and specular reflection (or transmissions) are evaluated by using ray tracing. Caustics are rendered by using the radiance estimate from the caustic photon map which is built by tracing photons only towards the specular surface in the model and storing them. Indirect diffuse illumination is approximately evaluated by using the radiance estimate from the global photon map which is built by tracing photons toward all objects in the model.

The main advantage of photon mapping is both very versatile and fast. However, the photon map may store an excessive number of photons on the simple lighting. Therefore, it takes excessive computational cost in the method.

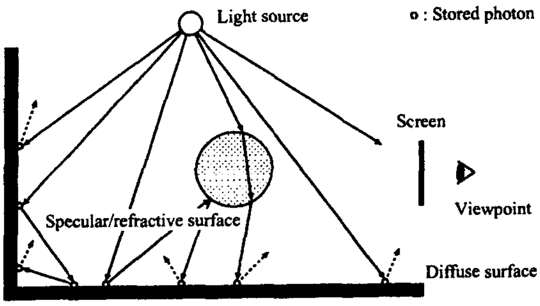

Figure 1. Photon tracing

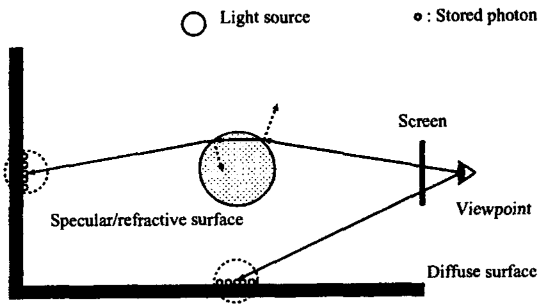

Figure 2. Rendering using photon map

3. Automatic Differentiation

Suppose that f is a function of $x_i, i = 1, \ldots, n$, and that f consists of a set of procedures $w = \phi(u, v)$ or $w = \psi(u)$, where ϕ is a binary operator (e.g., $+, -, \times, \cdots$) and ψ is a unary operator (e.g., sin, cos, exp, $\cdots$). If $\frac{\partial u}{\partial x_i}$ and $\frac{\partial v}{\partial x_i}$ are given, then $\frac{\partial w}{\partial x_i}$ can be obtained with the chain-rule of differentiation. By calculating such $\frac{\partial w}{\partial x_i}$ simultaneously with calculation of the w, $\frac{\partial f}{\partial x_i}$ can be automatically calculated when the value of f is calculated. Primary approximation of f at a certain perturbation h can thus be obtained as follows:

$$f + \frac{\partial f}{\partial x_i} h. \quad (1)$$

Table 1 shows some examples of the rule of $\frac{\partial w}{\partial x_i}$.

Table 1. Examples of the rule $\frac{\partial f}{\partial x_i}$

w	$\frac{\partial w}{\partial x_i}$
$u \pm v$	$\frac{\partial w}{\partial x_i} = \frac{\partial u}{\partial x_i} \pm \frac{\partial v}{\partial x_i}$
$u \times v$	$\frac{\partial w}{\partial x_i} = \frac{\partial u}{\partial x_i} v + u \frac{\partial v}{\partial x_i}$
$\sin(u)$	$\cos(u) \frac{\partial u}{\partial x_i}$

This technique is called automatic differentiation. Unlike numerical differentiation, automatic differentiation does not need any information on neighborhood of the point. In addition, the technique can be used for an algorithm including iteration or branch as well as some formulas. In this paper, the features are applied to the photon mapping.

4. Usage of Automatic Differentiation in Photon Tracing

In the photon tracing, a position where emitted photon intersect a surface is defined as follows:

$$\begin{aligned} x &= V_{D_x} t + P_{L_x}, \\ y &= V_{D_y} t + P_{L_y}, \\ z &= V_{D_z} t + P_{L_z}, \end{aligned} \quad (2)$$

where V_D is direction vector of emitted photon and P_L is coordinates where the photon emit on the light source. V_D is defined as follows:

$$\begin{aligned} V_{D_x} &= P_{N_x} - P_{L_x}, \\ V_{D_y} &= P_{N_y} - P_{L_y}, \\ V_{D_z} &= P_{N_z} - P_{L_z}, \end{aligned} \quad (3)$$

where P_N is coordinates where V_D is normalized. In this paper, P_{N_x}, P_{N_y} and P_{N_z} are assumed to consist of their own value and partial differentiation with respect to x, y and z, such as the following formulas:

$$\begin{aligned} P_{N_x} &< \frac{\partial P_{N_x}}{\partial x}, \frac{\partial P_{N_x}}{\partial y}, \frac{\partial P_{N_x}}{\partial z} >, \\ P_{N_y} &< \frac{\partial P_{N_y}}{\partial x}, \frac{\partial P_{N_y}}{\partial y}, \frac{\partial P_{N_y}}{\partial z} >, \\ P_{N_z} &< \frac{\partial P_{N_z}}{\partial x}, \frac{\partial P_{N_z}}{\partial y}, \frac{\partial P_{N_z}}{\partial z} >. \end{aligned} \quad (4)$$

In the use of the automatic differentiation, the set of differentials is calculated simultaneously by the calculation of its own value.

For example, we consider calculation of intersection between a ball and a ray. Suppose that the ball is defined as the following:

$$x^2 + y^2 + z^2 - r^2 = 0. \quad (5)$$

Substitute (2) for (5) and simplify the equation:

$$At^2 + Bt + C = 0, \quad (6)$$

where A, B and C are real numbers. If the discriminant D of (6) is greater than 0:

$$D = B^2 - 4AC > 0, \quad (7)$$

then the intersection exists. Here $(\frac{\partial D}{\partial x}, \frac{\partial D}{\partial y}, \frac{\partial D}{\partial z})$ has been calculated by the automatic differentiation. Then, we can check the variation of D in the neighborhood of the coordinate of P_N. Suppose that $D > 0$ at the center of the pixel, and that the following inequality holds true:

$$D + \frac{\partial D}{\partial x}h_x + \frac{\partial D}{\partial y}h_y + \frac{\partial D}{\partial z}h_z < 0. \quad (8)$$

Then it is predicted that the ray will not intersect the ball in the neighborhood. Here h_x, h_y and h_z are variations between the coordinate of P_N and the edge of them along with the axis, respectively.

In the other case, suppose that $D < 0$, and that the following inequality holds true:

$$D + \frac{\partial D}{\partial x}h_x + \frac{\partial D}{\partial y}h_y + \frac{\partial D}{\partial z}h_z > 0. \quad (9)$$

Then it is predicted that the ray intersects the ball in the neighborhood. Condition (8) and (9) are checked at the points of triangles in Figure 3, where h_x, h_y and h_z are variations between P_N and the each triangle, respectively.

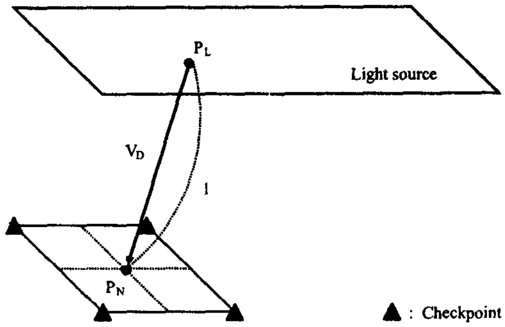

Figure 3. Checkpoints at the neighbor of P_N

5. Adaptive Photon Mapping in the Use of Automatic Differentiation

In this paper, an adaptive photon mapping is proposed in the use of the automatic differentiation. Each calculation of the photon mapping has its differentiation with respect to coordinates. An algorithm of the proposed method is as follows:

Step 1: Start the photon tracing using the automatic differentiation and construct the photon map.

- **Step 1.1**: Obtain the information on the part where the original ray from the light source hits. Also obtain the information by using the automatic differentiation on the part where the adjacent ray from the light source hits (see Figure 4).
- **Step 1.2**: If both of them are different, emit a new photon to the neighborhood and store the new information in the photon map.

Step 2: After construction of the photon map, render the scene by adaptive distributed ray tracing[4] using the information in the photon map (see Figure 5).

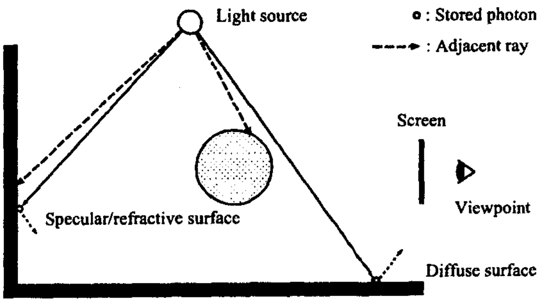

Figure 4. Adaptive photon tracing with automatic differentiation

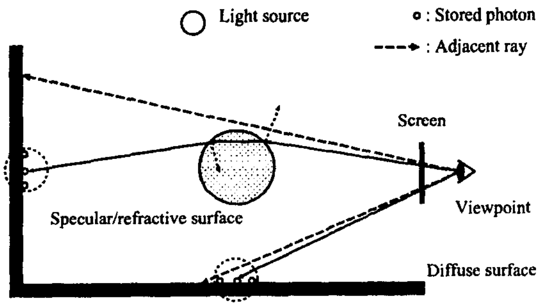

Figure 5. Adaptive distributed ray tracing with automatic differentiation

Although it takes computational cost for the automatic differentiation, we can decrease the number of photons that stored in the photon map and most of the scene can use an interpolated irradiance value in rendering. Therefore, the proposed method makes the photon mapping adaptive so that the computational cost becomes low.

6. Simulation

In this section, we show some results for the box scene (with a non-specular ball to the left, a mirror ball in the middle, and a glass ball to the right) by the conventional method and the proposed method.

Table 2 shows the comparison between the conventional method and the proposed method about the number of emitted photons, stored photons, and the computational cost to generate Figure 8 and 9. It is seen that the number of stored photons in the proposed method is fewer than the one in the conventional method. Figure 6 and 7 shows the position of stored photons in the box scene. Compared with these figures, we can confirm that Figure 7 is stored photons more efficiently than Figure 6. Figure 8 and 9 shows the full global illumination images in the box scene. It is seen that we can obtain the image efficiently by using the proposed method at low computational cost (see Table 2).

Figure 6. The position of stored photons in the box scene by the conventional photon mapping

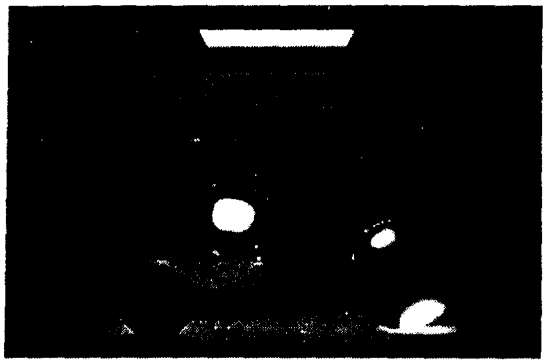

Figure 8. The box scene with full global illumination by the conventional photon mapping

Figure 7. The position of stored photons in the box scene by the proposed adaptive photon mapping

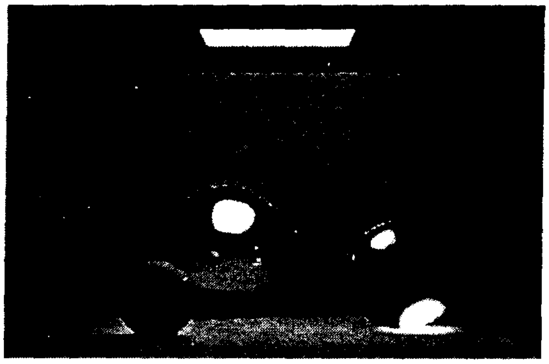

Figure 9. The box scene with full global illumination by the proposed adaptive photon mapping

Table 2. Comparison of number of photons and computational cost

	Figure 6	Figure 7
number of emitted photons for global photon map	100,000	50,000
number of emitted photons for caustic photon map	5,000	2,500
number of stored photons in global photon map	368,558	196,367
number of stored photons in caustic photon map	7,177	3,852
	Figure 8	Figure 9
computational cost [s]	2485.040	630.690

7. Conclusion

In this paper, an adaptive photon mapping in the use of automatic differentiation is proposed. Since the automatic differentiation is used when photons emit from the light sources through the scene, we can check the variation of surrounding shape. Therefore, we can decrease the number of photons and make an image in relatively low computational cost.

Acknowledgement

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