Mapping of Fine Grayscale Data into the sRGB Color Space

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Abstract: Fine grayscale data exceeding 8 bits per pixel is mapped into the sRGB color space so as to be displayed on a 24-bit sRGB CRT monitor. The characteristics of the pseudo gray generated by mapping are investigated in terms of the CIELAB color difference and chromaticities.

1. Introduction

In some sophisticated applications such as chest radiography, mammography, and radiographic computed tomography, fine grayscale images are used because of their rich capability in brightness discrimination. At present, a high-end monochrome CRT monitor for displaying those digital images is expensive. In contrast, a popular 24-bit color CRT monitors is quite cheap and its price amounts to just a few hundredths, while its displaying capability for grayscale data is limited within 8 bits per pixel. This paper describes a simple method to treat the fine grayscale data in the sRGB color system at the expense of small color difference.

The problems are to find a new approximate representation of fine gray in nine bits or more in the 8-bit sRGB color system, and to apply the finer-scale pseudo gray representation to the visualization on the 8-bit sRGB CRT monitors. A given grayscale value are mapped onto a particular point in the sRGB color system so that the point may be located around the lightness axis as close as possible.

The resulting pseudo gray in the sRGB color system is different from the true gray. It can be, however, acceptable for some limited applications, unless the introduced chroma reaches a perceivable level, and if the pseudo gray offers sufficient accuracy and linearity along the achromatic axis.

2. Mapping of Fine Grayscale Data

2.1 Mapping into the sRGB Color Space

A given grayscale image is assumed to have been encoded with high accuracy that exceeds 8 bits per pixel (bpp) in the 8-bit sRGB color system. The 8-bit accuracy is common to popular 24-bit color CRT monitors, and the grayscale images in nine or more bpp cannot be displayed as it is, unless a simple truncation takes place as a preprocessing. This is a mismatch between the grayscale information and a displaying device, and causes significant loss of information.

An $(n+8)$-bit grayscale value V can be expressed by

$$V = 2^n V_0 + \Delta V \qquad (1)$$

$$V_0 = \mathrm{floor}[\, V/2^n \,] \qquad (2)$$

where V_0 is the portion of the most significant 8 bits, and ΔV represents the other less significant bits. The grayscale data is expressed by a 3-D vector

$$(V_0 + \Delta V/2^n,\ V_0 + \Delta V/2^n,\ V_0 + \Delta V/2^n) \qquad (3)$$

to be interpreted as an achromatic color, and is mapped onto a point in the 8-bit sRGB color space as shown in Fig. 1. The corresponding destination vector is expressed by

$$(V_0 + \Delta R,\ V_0 + \Delta G,\ V_0 + \Delta B) \qquad (4)$$

where ΔR, ΔG, and ΔB are unknown integers.

The vector $(\Delta R, \Delta G, \Delta B)$ is a tuning vector to be added to the basement vector (V_0, V_0, V_0) so that the distance between the source and destination vectors are minimized in terms of a proper distance measure. Since the basement vector is common to the source and destination vectors, a possible approach to find a solution is to minimize the distance between the tuning vector and the local source vector, $(1, 1, 1)\ \Delta V/2^n$, where the factor is a positive fraction smaller than unity. This is a kind of local optimization problems.

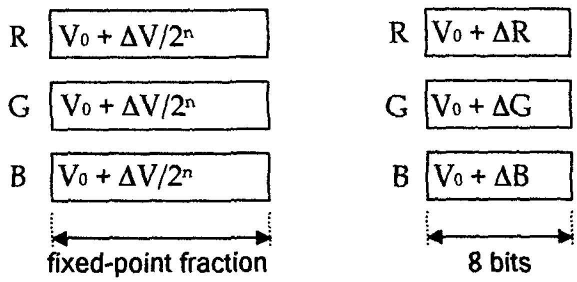

Fig. 1 Mapping between two 3-D vectors. A given grayscale value (*left*) and its pseudo gray (*right*) in the form of an RGB tristimulus vector encoded in 8 bits.

As for global optimizations, any solution has to consider the mapping between the source vector of Eq. (4) and its destination vector of Eq. (3) rather than that between ΔV and $(\Delta R, \Delta G, \Delta B)$, since any color space available at present is non-uniform. Although a solution to this problem could be solved by an exhaustive search in the form of a one-to-one mapping table, such a solution would not be attractive for practical use because of its extensive storage space. Furthermore, a truly optimal solution has to match with comprehensive conditions including viewing conditions and visual appearance evaluated by the human visual system. This problem is so hard to be solved.

The problem in this subsection is thus to find a set of 2^n integer-valued tuning vectors of (ΔR, ΔG, ΔB) to a fixed sequence between 0 and 2^n-1. The transformation formulae from sRGB to CIEXYZ are defined as follows [4-5].

$$X = 0.4124R_s + 0.3576G_s + 0.1805B_s \quad (5a)$$
$$Y = 0.2126R_s + 0.7152G_s + 0.0722B_s \quad (5b)$$
$$Z = 0.0193R_s + 0.1192G_s + 0.9505B_s \quad (5c)$$

X, Y, and Z are the CIE tristimulus values. The CIE chromaticity coordinates for ITU-R BT.709 reference primaries are as follows.

$$x = 0.6400,\ y = 0.3300 \text{ for } R_s \quad (6a)$$
$$x = 0.3000,\ y = 0.6000 \text{ for } G_s \quad (6b)$$
$$x = 0.1500,\ y = 0.0600 \text{ for } B_s \quad (6c)$$

The standard illuminant white is CIE D_{65} of which chromaticity is given by x=0.3127 and y=0.3290.

CIELAB is widely used to describe color difference in many industries. It is widely accepted as a standard interchange color space [11]. It is transformed from CIEXYZ, as follows [6-10].

$$L^* = 116 f(Y/Y_n) - 16 \quad (7a)$$
$$a^* = 500\,[\, f(X/X_n) - f(Y/Y_n)\,] \quad (7b)$$
$$b^* = 200\,[\, f(Y/Y_n) - f(Z/Z_n)\,] \quad (7c)$$

where

$$f(p) = p^{1/3} \quad \text{for } p > 0.008856 \quad (8a)$$
$$f(p) = 7.787p + 16/116, \text{ otherwise} \quad (8b)$$

L* is referred to as lightness. a* and b* represent chroma components. X_n, Y_n, and Z_n are the tristimulus values of the standard illuminant. The CIELAB color difference between two colors is defined as follows [6-10].

$$\Delta E_{ab}^* = [\,(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2\,]^{1/2} \quad (9)$$

where Δ stands for the difference between two quantities in issue.

In this way, once a given grayscale value V has been transformed onto a vector in CIELAB, then what follows is to find a mapping of the local source vector $\Delta V/2^n$ (1,1,1) to a tuning vector (ΔR_s, ΔG_s, ΔB_s) in sRGB so that both color difference and lightness difference between the true gray and the pseudo gray may be tuned to zero as close as possible. To this end, an exhaustive search will be applied for $n = 4$ and a set of 15 vectors will have been found. The solution set of those vectors has to satisfy additional two conditions. That is, it has to keep the natural order correspondence to the integer sequence of 1 through 15. Secondly, the segmentation of the lightness scale has to be uniform as fine as possible. In other words, the bin sizes along the quantized lightness scale must be as regular as possible.

The solution in 12-bit accuracy is listed in Tables 1, where the lightness difference and the color difference have been computed for a fixed basement, V_0=25. The values of color difference and lightness difference are valid for this basement value. If 11-bit accuracy is desirable, 8 rows that are marked by a *single plus* in the rightmost column are removed away. If 10-bit accuracy is appropriate, 4 *double plus*-marked rows are furthermore suppressed and only three vectors are left in the table.

Finally, a pseudo gray color is computed by adding the common scalar value V_0 to the tuning vector (ΔR_s, ΔG_s, ΔB_s). Since the pseudo gray color belongs to the 8-bit sRGB color space, it looks like as if the number of gray levels had been augmented by a factor of the total numbers of tuning vectors.

Figure 2 shows the local mapping characteristics between grayscale values and the lightness values of the pseudo gray. As seen in the plots, the *monotonic increasing* property between the grayscale value and the lightness of the pseudo gray is satisfied as intended. Approximate linearity between them is also observed, while the bin size of quantization accuracy is non-uniform. It is worth to note that errors in both lightness and color difference vanish at two endpoints. The lightness of the pseudo gray extends its value over the entire range between 0 and 255, and this mapping gamut is implemented by the basement integer V_0 added to the tuning vector. As a consequence, the global linearity and quantization accuracy are equally maintained over all unit intervals on the entire grayscale, where 'unit' implies one digital count in 8-bit sRGB.

Table 1 Tuning Vectors for 12-bit Grayscale Data. ΔL^* and ΔE_{ab}^* are valid to the basement value of V_0=25.

ΔV	ΔR_s, ΔG_s, ΔB_s	ΔL*	ΔE_{ab}^*	Note
0	0, 0, 0	0.000	0.000	
1	0, 0, 1	-0.007	1.037	++
2	1, 0, -1	-0.011	1.276	+
3	1, 0, 0	-0.018	0.716	+++
4	1, 0, 1	-0.015	1.264	++
5	2, 0, -1	-0.029	1.773	+
6	1, 0, 2	0.013	2.177	+
7	2, 0, 0	0.009	1.420	+++
8	2, 0, 1	0.002	1.759	+
9	-1, 1, 1	-0.009	1.427	++
10	3, 0, 0	-0.009	2.113	+
11	-1, 1, 2	0.028	1.727	+
12	0, 1, 0	0.024	1.264	+++
13	0, 1, 1	0.018	0.708	+
14	0, 1, 2	0.011	1.222	++
15	1, 1, 0	0.007	1.033	+

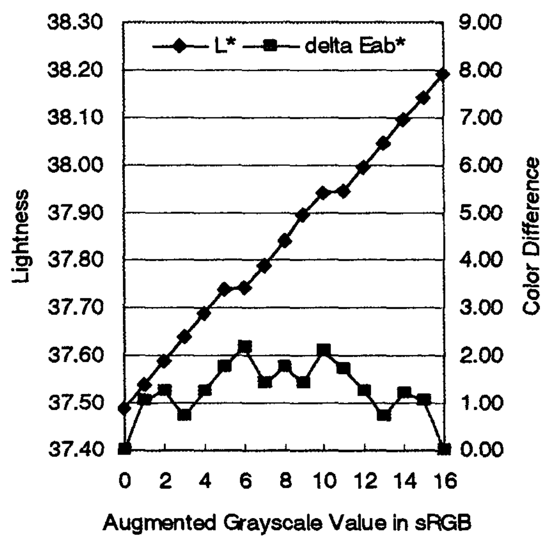

Fig. 2 Local mapping characteristics.

2.2 Quantization and Gamma Correction

If a given grayscale data Q has been encoded in (n+8) bits, the number of gray tones is 2^{n+8}. In contrast, the number of tones in the pseudo gray is given by 255 x 2^n + 1. In order to solve this mismatch, a given grayscale data Q is quantized before its mapping to the pseudo gray, as follows.

$$V = \text{floor}\left[(2^8 - 1)2^n q + 0.5 \right] \quad (10)$$

where q is given by the scaling

$$q = Q/(2^{n+8} - 1) \quad (11)$$

and is a real number between 0 and 1.

Unfortunately, some tuning vectors cannot be applied at black end or white end, because any value outside [0, 255] is impossible in 8-bit sRGB. Those inhibited pseudo grays are replaced by another admissible pseudo gray of which lightness is closest to that of the inhibited pseudo gray.

In order for the pseudo gray to be evaluated, the reference gray has to be identified. It is hence appropriate to refer to so-called gamma. A CRT monitor has a strong non-linear transfer function between input and output. A typical CRT gamma is 2.2 for sRGB [4-5] and NTSC [6, 15]. The CRT gamma has to be corrected prior to displaying an image on CRT monitors. Most of cameras are thus designed to correct the CRT gamma, and hence most digital images have been gamma corrected and then encoded in digital data. This is a legacy since the advent of television [3-6, 15]. In the sRGB color system, significant effort has been paid to the gamma [4-5, 11-14], and the viewing gamma is described by

$$\gamma_V = \gamma_C \gamma_D \quad (12)$$

where γ_C and γ_D are camera gamma and display gamma. The viewing gamma is 1.125 in sRGB.

The pseudo gray can be directly applied to the legacy grayscale data. In the following description, the grayscale data encoded after gamma correction of 2.2 is referred to as the legacy data, and a linearly encoded data is referred to as linear data, which is a consequence of the camera gamma of unity. To display those fine grayscale linear data on an 8-bit sRGB CRT monitor, the display gamma is corrected before the pseudo gray mapping.

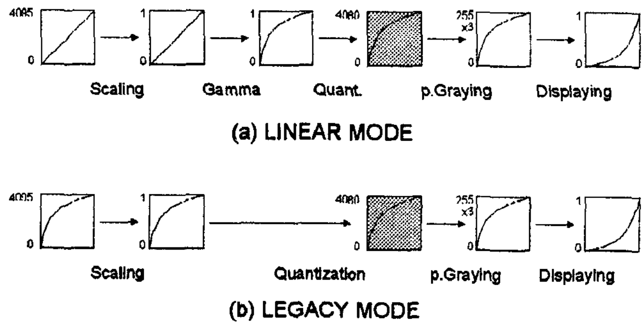

Fig. 3 (a) Linear mode and (b) legacy mode in 12-bit accuracy. The pseudo gray is encoded into 8-bit sRGB and is decoded/displayed on an 8-bit sRGB CRT monitor so as to be evaluated in CIELAB. The reference gray for evaluations is marked by shaded squares.

3. Numerical Evaluation of the Pseudo Gray

The grayscale data quantized in 255 x 2^n + 1 tones is used as the reference gray. It is defined by the 8-bit sRGB encoded values after scaling, gamma adjustment, and quantization. All evaluations are performed on a virtual screen after decoding onto an 8-bit sRGB CRT monitor.

The color difference that is identical to the lightness difference between a given (n+8)-bit grayscale data and its reference gray has been examined for every tone for every gamma among 1.0, 1.125, and 2.2. The maximum difference in lightness has been found to be 0.035 and 0.109 for 12 and 10-bit accuracy data, respectively.

Figure 4 shows the color difference and lightness difference between reference gray and pseudo gray over the whole range of the quantized grayscale in 10-bit accuracy for legacy data. The viewing gamma is assumed to be unity. The grayscale value extends its value 0 through 255, and the number of pseudo gray colors plotted in the figure is 1021 in 10-bit accuracy. Hence the values in the horizontal axis can be a fraction such as 1/4, and the axis is referred to as the fractional grayscale in 8-bit sRGB. At a glance, one would see there were three plots for a single color difference, and also it is so as well as the lightness difference. However, it is not real. Upper three plots belong to the color difference, and lower three plots belong to the lightness difference. Those split cluster plots are produced by the different error levels in tuning vectors of which behavior has been shown in Fig. 2. There is seen a single point of ΔL^* apart from its clusters at the white end in the figure. This is a consequence of the negotiated replacement of an inhibited tuning vector by an admissible one described in the previous subsection.

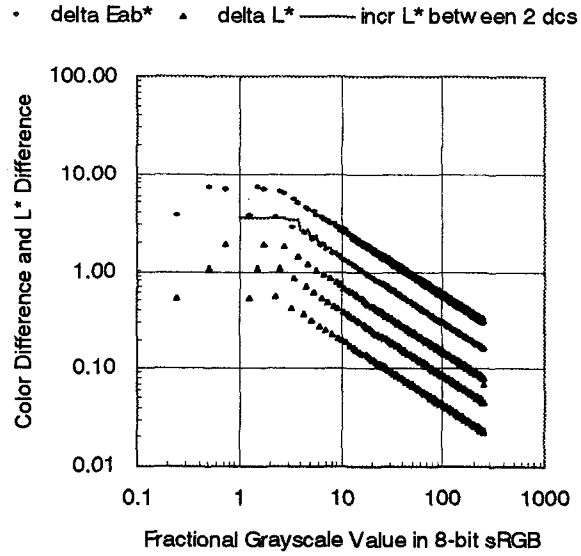

Fig. 4 Color difference (*upper*) and lightness difference (*lower*) in the case of 10-bit accuracy for legacy data. A unity viewing gamma is assumed. The thin line shows the incremental lightness between two successive digital counts in 8-bit sRGB.

As observed in Fig.4, both differences decrease, as the grayscale value increases. This is a natural consequence, because the color difference behavior produced by the tuning vectors is identical in every unit interval along the grayscale and, at the same time, the lightness difference will decrease in proportion to the relative magnitude to the basement value. It is observed that the decreasing rate obeys the first-order derivative of the one third-power law between lightness and Y-tristimulus value.

Although the color difference in Fig. 4 is large at the black end, it takes values smaller than unity over quite a broad range of the grayscale. In fact, the color difference is smaller than unity, when the 8-bit grayscale digital count exceeds 44, which is approximately equivalent to 49 in lightness. For the legacy data, 90% among the full dynamic range of the linear grayscale exceed the gamma corrected value of $0.351 = 0.1^{1/2.2}$ that is equivalent to the grayscale count of 89 among 255. This fact is restated as follows. For legacy 10-bit grayscale images, 90% among all gray values on the linear grayscale can be displayed on an 8-bit sRGB CRT monitor within the color difference of 0.62. This would be quite satisfactory, because it is hard to perceive such color difference smaller than 0.3, 1.2, 3.0 or 5.0. The critical limit differs by literature [3, 6-10], because it depends on viewing conditions and the perceptual capability of individuals.

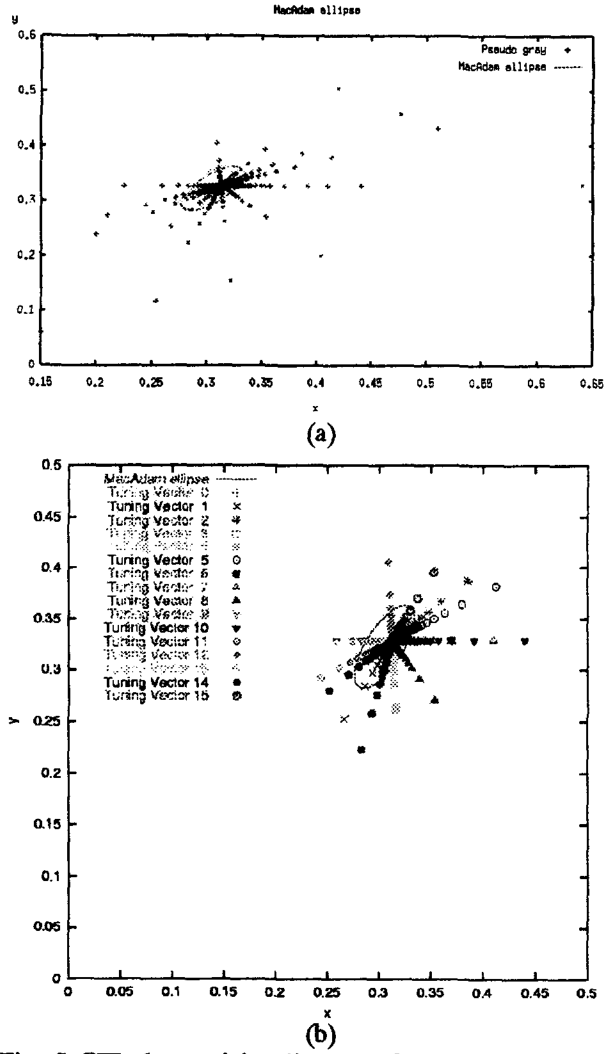

Fig. 5 CIE chromaticity diagrams for pseudo gray colors and the MacAdam ellipse. (a) a whole plot for all pseudo gray colors in 12-bit accuracy, (b) a plot for pseudo gray colors brighter than 50 in lightness.

Another discussion can be developed in terms of the CIE chromaticity diagram. Figure 5 (a) plots the locations of the pseudo gray colors. As seen in the figure, the plots are classified in fifteen clusters on respective radiating semi-linear segments that correspond to the 15 tuning vectors in 12-bit accuracy. A limited number of pseudo gray colors of which digital count exceeds 45 are plotted in part (b). The chromaticities of those pseudo grays are found to concentrate within considerably small area.

4. Concluding Remarks

To explore the possibility in displaying fine grayscale images on 8-bit sRGB CRT monitors, pseudo gray colors have been computed and their objective data have been presented in terms of color difference and chromaticities. The described pseudo gray can be sufficiently useful in some limited but real applications.

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