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기하학적 비선형 구조물의 설계 민감도해석 및 위상최적설계
Design Sensitivity Analysis and Topology Optimization of Geometrically Nonlinear Structures

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ABSTRACT

A continuum-based design sensitivity analysis (DSA) method for non-shape problems is developed for geometrically nonlinear elastic structures. The non-shape problem is characterized by the design variables that are not associated with the domain of system like sizing, material property, loading, and so on. Total Lagrangian formulation with the Green-Lagrange strain and the second Piola-Kirchhoff stress is employed to describe the geometrically nonlinear structures. The spatial domain is discretized using the 4-node isoparametric plane stress/strain elements. The resulting nonlinear system is solved using the Newton-Raphson iterative method. To take advantage of the derived analytical sensitivity in topology optimization, a fast and efficient design sensitivity analysis method, adjoint variable method, is employed and the material property of each element is selected as non-shape design variable. Combining the design sensitivity analysis method and a gradient-based design optimization algorithm, an automated design optimization method is developed. The comparison of the analytical sensitivity with the finite difference results shows excellent agreement. Also application to the topology design optimization problem suggests a very good insight for the layout design.

1. Introduction

Over the past few years, a lot of research efforts have been devoted to develop DSA methods for structural systems. The design sensitivity is defined as the variation of performance measures with respect to the design variation.(1) In the continuum DSA approach, the design sensitivity expressions are obtained by taking the first order variation of the continuum variational equation that represents the structural system. The continuum DSA methods developed so far can handle four types of design variables: sizing, shape, material property, and configuration design variables. In this paper, a continuum method for the non-shape problems like material property DSA is considered.

Topology optimization is a method that helps designers to find a suitable structural layout for the required structural performances. Ever since Bendsøe and Kikuchi(2) introduced the homogenization method, a lot of

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attention has been devoted to develop topology optimization methods.[(3)~(6)] For the topology optimization, the design variables are the parameters of material distribution for the structural systems. Therefore a lot of design parameters are necessarily involved to find optimal material distribution and their gradients, so called design sensitivities, are required in gradient-based optimization methods. Linearly elastic structures are so far considered in topology optimization problems. However, for the precise layout design, a topology optimization method that uses continuum based DSA method is developed to design geometrically nonlinear structures.

2. Geometrically Nonlinear Analysis Using Total Lagrangian Formulation

The left subscript and superscript denote the reference and designated configuration numbers, respectively. The right subscript and superscript represent the tensor component and the iteration counter, respectively. The equilibrium of a deformable body can be expressed using the principle of virtual work in the total Lagrangian formulation as[(7),(8)]

$$a(\mathbf{z},\bar{\mathbf{z}}) = \ell(\bar{\mathbf{z}}), \quad \text{for all } \bar{\mathbf{z}} \in \mathbf{Z} \tag{1}$$

where

$$a(\mathbf{z},\bar{\mathbf{z}}) \equiv \iiint_{{}^0\Omega} {}^n_0S_{ij}\, {}^n_0\bar{\varepsilon}_{ij}\, \mathrm{d}^0\Omega \tag{2}$$

and

$$\ell(\bar{\mathbf{z}}) \equiv \iiint_{{}^0\Omega} {}^n_0f_i\, \bar{z}_i\, \mathrm{d}^0\Omega + \iint_{{}^0\Gamma_t} {}^n_0T_i\, \bar{z}_i\, \mathrm{d}^0\Gamma_t \equiv {}^nR \tag{3}$$

and $\mathbf{z}$, $\bar{\mathbf{z}}$, $\mathbf{Z}$, $a(\mathbf{z},\bar{\mathbf{z}})$ and $\ell(\bar{\mathbf{z}})$ are displacement, virtual displacement, the space of kinematically admissible virtual displacement, strain energy form, and load form, respectively. ${}^d_0S_{ij}$, ${}^d_0\bar{\varepsilon}_{ij}$, ${}^d_0f_i$, ${}^d_0T_i$ and ${}^dR$ are the second Piola-Kirchhoff stress tensor, the Green-Lagrange strain tensor corresponding to $\bar{\mathbf{z}}$, external body forces, surface tractions, and external virtual work at the current configuration (n), measured with respect to the initial configuration, respectively, and ${}^0\Omega$ and ${}^0\Gamma_t$ are domain and traction boundary at the initial configuration, respectively. The Green-Lagrange strain tensor at the configuration (n+1), measured with respect to the initial configuration is defined as

$${}^{n+1}_0\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial^{n+1}z_i}{\partial^0x_j} + \frac{\partial^{n+1}z_j}{\partial^0x_i} + \frac{\partial^{n+1}z_k}{\partial^0x_i}\frac{\partial^{n+1}z_k}{\partial^0x_j} \right) \tag{4}$$

and the constitutive law is

$${}^{n+1}_0S_{ij} = C_{ijkl}\, {}^{n+1}_0\varepsilon_{kl} \tag{5}$$

where C_{ijkl} are the components of the elasticity tensor.

Since the strain energy form is nonlinear in its arguments, Equation (1) cannot be solved directly. In this paper, an incremental-iterative scheme is adopted to solve nonlinear systems. In the incremental analysis, the external load is gradually incremented and the solution of each load step is sought based on the previous equilibrium solution. With the solution of configuration (n), the displacements, strains and stresses at configuration (n+1) can be decomposed as

$$ {}^{n+1}z_i = {}^{n}z_i + \Delta z_i \tag{6}$$

$$ {}^{n+1}_{0}S_{ij} = {}^{n}_{0}S_{ij} + {}_{0}\Delta S_{ij} \tag{7}$$

$$ {}^{n+1}_{0}\varepsilon_{ij} = {}^{n}_{0}\varepsilon_{ij} + {}_{0}\Delta \varepsilon_{ij} \tag{8}$$

The incremental Green-Lagrange strain tensor ${}_{0}\Delta\varepsilon_{ij}$ can be divided into its linear and nonlinear parts as

$$ {}_{0}\Delta\varepsilon_{ij} = {}_{0}\Delta e_{ij} + {}_{0}\Delta\eta_{ij} \tag{9}$$

where

$$ {}_{0}\Delta e_{ij} = \frac{1}{2}\left({}_{0}\Delta z_{i,j} + {}_{0}\Delta z_{j,i} + {}^{n}_{0}z_{k,i}\; {}_{0}\Delta z_{k,j} + {}_{0}\Delta z_{k,i}\; {}^{n}_{0}z_{k,j}\right) \tag{10}$$

$$ {}_{0}\Delta\eta_{ij} = \frac{1}{2}\, {}_{0}\Delta z_{k,i}\; {}_{0}\Delta z_{k,j} \tag{11}$$

Using the incremental decomposition, the equilibrium equation at the configuration (n+1) becomes

$$ \iiint_{{}^0\Omega} {}_{0}\Delta S_{ij}\left({}_{0}\Delta\bar{e}_{ij} + {}_{0}\Delta\bar{\eta}_{ij}\right) \mathrm{d}^0\Omega + \iiint_{{}^0\Omega} {}^{n}_{0}S_{ij}\; {}_{0}\Delta\bar{\eta}_{ij}\, \mathrm{d}^0\Omega = {}^{n+1}R - \iiint_{{}^0\Omega} {}^{n}_{0}S_{ij}\; {}_{0}\Delta\bar{e}_{ij}\, \mathrm{d}^0\Omega \tag{12}$$

In general, Equation (12) is highly nonlinear in the incremental displacement Δz_i. The linearized incremental equation is obtained by using the approximation ${}_{0}\Delta\bar{\varepsilon}_{ij} = {}_{0}\Delta\bar{e}_{ij}$ and ${}_{0}\Delta S_{ij} = {}_{0}C_{ijkl}\; {}_{0}\Delta e_{kl}$ as

$$ \iiint_{{}^0\Omega} {}_{0}C_{ijkl}\; {}_{0}\Delta e_{kl}\; {}_{0}\Delta\bar{e}_{ij}\, \mathrm{d}^0\Omega + \iiint_{{}^0\Omega} {}^{n}_{0}S_{ij}\; {}_{0}\Delta\bar{\eta}_{ij}\, \mathrm{d}^0\Omega = {}^{n+1}R - \iiint_{{}^0\Omega} {}^{n}_{0}S_{ij}\; {}_{0}\Delta\bar{e}_{ij}\, \mathrm{d}^0\Omega \tag{13}$$

The right-hand side of Equation (13) represents the "out-of-balance virtual work" after the solution, produced by the previous linearization. Using the Newton-Raphson iterative method the steps are repeated until the difference between the external and the internal virtual work is negligible within a certain convergence measure.
Define another energy bilinear form as[8]

$$ \begin{aligned} a^{*}({}^{n}\mathbf{z};\Delta\mathbf{z},\Delta\bar{\mathbf{z}}) &\equiv \iiint_{{}^0\Omega} {}_{0}C_{ijkl}\; {}_{0}\Delta e_{kl}\; {}_{0}\Delta\bar{e}_{ij}\, \mathrm{d}^0\Omega + \iiint_{{}^0\Omega} {}^{n}_{0}S_{ij}\; {}_{0}\Delta\bar{\eta}_{ij}\, \mathrm{d}^0\Omega \\ &= \iiint_{{}^0\Omega} {}_{0}C_{ijkl}\left({}_{0}\Delta z_{k,l}\, {}_{0}\Delta\bar{z}_{i,j} + {}^{n}_{0}z_{m,i}\, {}_{0}\Delta z_{k,l}\, {}_{0}\Delta\bar{z}_{m,j} + {}^{n}_{0}z_{m,k}\, {}_{0}\Delta z_{m,l}\, {}_{0}\Delta\bar{z}_{i,j}\right. \\ &\quad \left. + {}^{n}_{0}z_{m,k}\, {}^{n}_{0}z_{n,i}\, {}_{0}\Delta z_{m,l}\, {}_{0}\Delta\bar{z}_{n,j}\right) \mathrm{d}^0\Omega + \iiint_{{}^0\Omega} {}^{n}_{0}S_{ij}\; {}_{0}\Delta z_{k,i}\, {}_{0}\Delta\bar{z}_{k,j}\, \mathrm{d}^0\Omega \end{aligned} \tag{14}$$

The fact that the incremental material property tensor and the second Piola-Kirchhoff stress tensor are symmetric with respect to their indices has been used for the second equality in Equation (14). Thus, the energy bilinear form $a^{*}({}^{n}\mathbf{z};\bullet,\bullet)$ in Equation (14) is symmetric in its arguments.

3. Design Sensitivity Analysis

3.1 Energy and Load Forms and Static Response

Consider a structural system in its final equilibrium configuration corresponding to a given design $\mathbf{u}$. The equilibrium equation for the structural system can be written, using Equation (1),

$$a_{\mathbf{u}}(\mathbf{z},\bar{\mathbf{z}}) = \ell_{\mathbf{u}}(\bar{\mathbf{z}}), \quad \text{for all } \bar{\mathbf{z}} \in \mathbf{Z} \tag{15}$$

where the subscript $\mathbf{u}$ is used to indicate that the equilibrium equation corresponds to the design $\mathbf{u}$. The variational equation corresponding to the perturbed design $\mathbf{u} + \tau\delta\mathbf{u}$ are written as

$$a_{\mathbf{u}+\tau\delta\mathbf{u}}(\mathbf{z}_\tau,\bar{\mathbf{z}}) = \ell_{\mathbf{u}+\tau\delta\mathbf{u}}(\bar{\mathbf{z}}), \quad \text{for all } \bar{\mathbf{z}} \in \mathbf{Z} \tag{16}$$

The first order variations of each term in Equation (15) with respect to its explicit dependence on the design variable $\mathbf{u}$ are defined as

$$a'_{\delta\mathbf{u}}(\mathbf{z},\bar{\mathbf{z}}) \equiv \frac{d}{d\tau} a_{\mathbf{u}+\tau\delta\mathbf{u}}(\tilde{\mathbf{z}}_\tau,\bar{\mathbf{z}})\bigg|_{\tau=0} \tag{17}$$

$$\ell'_{\delta\mathbf{u}}(\bar{\mathbf{z}}) \equiv \frac{d}{d\tau} \ell_{\mathbf{u}+\tau\delta\mathbf{u}}(\bar{\mathbf{z}})\bigg|_{\tau=0} \tag{18}$$

where the '~' denotes that the dependence on the design variation is suppressed and $\bar{\mathbf{z}}$ is independent of τ. Define the first variation of the solution of Equation (15) with respect to the design $\mathbf{u}$ as

$$\mathbf{z}' \equiv \frac{d}{d\tau}\mathbf{z}(\mathbf{u}+\tau\delta\mathbf{u})\bigg|_{\tau=0} = \lim_{\tau\to 0}\frac{\mathbf{z}(\mathbf{u}+\tau\delta\mathbf{u}) - \mathbf{z}(\mathbf{u})}{\tau} \tag{19}$$

Using the above definition, it can be noted that the order of taking the first order variation and the partial derivative of the state can be interchanged as $(z_{i,j})' = (z'_i)_{,j}$. Using the chain rule of differentiation and Equations (17) and (19),

$$\frac{d}{d\tau}\left[a_{\mathbf{u}+\tau\delta\mathbf{u}}(\mathbf{z}(\mathbf{u}+\tau\delta\mathbf{u}),\bar{\mathbf{z}})\right]\bigg|_{\tau=0} = a'_{\delta\mathbf{u}}(\mathbf{z},\bar{\mathbf{z}}) + a^*_{\mathbf{u}}(\mathbf{z};\mathbf{z}',\bar{\mathbf{z}}) \tag{20}$$

Taking the first order variation of both sides of Equation (15) yields[8]

$$a^*_{\mathbf{u}}(\mathbf{z};\mathbf{z}',\bar{\mathbf{z}}) = \ell'_{\delta\mathbf{u}}(\bar{\mathbf{z}}) - a'_{\delta\mathbf{u}}(\mathbf{z},\bar{\mathbf{z}}), \text{for all } \bar{\mathbf{z}} \in \mathbf{Z} \tag{21}$$

3.2 Adjoint Variable Method

Consider a measure of structural performance that can be written in integral form, at the final equilibrium configuration with reference configuration at time 0,

$$\psi = \iiint_{{}^0\Omega} g(\mathbf{z}, {}_0\nabla\mathbf{z}, \mathbf{u}+\tau\delta\mathbf{u})\, \mathrm{d}^0\Omega \tag{22}$$

Taking the first order variation of Equation (22) with respect to design gives

$$\begin{aligned}\psi' &\equiv \frac{\mathrm{d}}{\mathrm{d}\tau}\left[\iiint_{{}^0\Omega} g(\mathbf{z}(\mathbf{u}+\tau\delta\mathbf{u}), {}_0\nabla\mathbf{z}(\mathbf{u}+\tau\delta\mathbf{u}), \mathbf{u}+\tau\delta\mathbf{u})\, \mathrm{d}^0\Omega\right]\Bigg|_{\tau=0} \\ &= \iiint_{{}^0\Omega} (g_{\mathbf{z}}\,\mathbf{z}' + g_{{}_0\nabla\mathbf{z}}\, {}_0\nabla\mathbf{z}' + g_{\mathbf{u}}\,\delta\mathbf{u})\, \mathrm{d}^0\Omega\end{aligned} \tag{23}$$

An adjoint equation is introduced by replacing $\mathbf{z}'$ in Equation (23) by a virtual displacement $\bar{\lambda}$ and by equating terms involving $\bar{\lambda}$ in Equation (23) to the bilinear energy form $a^*_{\mathbf{u}}(\mathbf{z};\lambda,\bar{\lambda})$ defined in Equation (21), which yields the adjoint equation for the adjoint variable λ

$$a^*_{\mathbf{u}}(\mathbf{z};\lambda,\bar{\lambda}) = \iiint_{{}^0\Omega} (g_{\mathbf{z}}\,\bar{\lambda} + g_{{}_0\nabla\mathbf{z}}\, {}_0\nabla\bar{\lambda})\, \mathrm{d}^0\Omega, \quad \text{for all } \bar{\lambda} \in \mathbf{Z} \tag{24}$$

where a solution is λ desired. Since $\mathbf{z}' \in \mathbf{Z}$, evaluate Equation (23) at $\bar{\lambda} = \mathbf{z}'$ to obtain

$$a^*_{\mathbf{u}}(\mathbf{z};\lambda,\mathbf{z}') = \iiint_{{}^0\Omega} (g_{\mathbf{z}}\,\mathbf{z}' + g_{{}_0\nabla\mathbf{z}}\, {}_0\nabla\mathbf{z}')\, \mathrm{d}^0\Omega \tag{25}$$

Since both $\bar{\mathbf{z}}$ and λ are in $\mathbf{Z}$, evaluate Equation (21) at $\bar{\mathbf{z}} = \lambda$

$$a^*_{\mathbf{u}}(\mathbf{z};\mathbf{z}',\lambda) = \ell'_{\delta\mathbf{u}}(\lambda) - a'_{\delta\mathbf{u}}(\mathbf{z},\lambda) \tag{26}$$

Using symmetry of the energy bilinear form $a^*_{\mathbf{u}}(\mathbf{z};\bullet,\bullet)$ in its arguments,

$$\iiint_{{}^0\Omega} (g_{\mathbf{z}}\,\mathbf{z}' + g_{{}_0\nabla\mathbf{z}}\, {}_0\nabla\mathbf{z}')\, \mathrm{d}^0\Omega = \ell'_{\delta\mathbf{u}}(\lambda) - a'_{\delta\mathbf{u}}(\mathbf{z},\lambda) \tag{27}$$

where the right side is linear in $\delta\mathbf{u}$ and can be evaluated once the state $\mathbf{z}$ and adjoint variable λ are determined as solutions of original nonlinear equation and linear adjoint equation, respectively. Substituting this result into Equation (23), we have

$$\psi' = \iiint_{{}^0\Omega} g_{\mathbf{u}}\,\delta\mathbf{u}\, \mathrm{d}^0\Omega + \ell'_{\delta\mathbf{u}}(\lambda) - a'_{\delta\mathbf{u}}(\mathbf{z},\lambda) \tag{28}$$

which expresses the dependence of design sensitivity on the design variation and the form of the last two terms on the right depends on the problem under consideration.

4. Formulation of Topology Optimization

The objective of topology optimization is to find a material distribution that minimizes compliance or strain energy of the structural systems. The material distribution can be represented using the relative density function

$\rho(\mathbf{x})$ that has continuous variation from zero to one, taking the value of 1.0 for solid material and 0.0 for void material. Consider the topology optimization problem.

$$\text{minimize} \quad C = \iiint_{^0\Omega} f_i z_i \, \mathrm{d}^0\Omega + \iint_{^0\Gamma_t} T_i z_i \, \mathrm{d}^0\Gamma_t \tag{29}$$

$$\text{subject to} \quad \iiint_{^0\Omega} \rho \, \mathrm{d}^0\Omega \le V \tag{30}$$

where C and ρ are mean compliance and relative density, respectively. For the topology optimization, the structural domain is discretized into NE finite elements and the relative densities are assumed constant in each element. The design variable, relative density for each element, is associated with material property as,

$$E_i = \rho_i^p E_0, \quad i = 1, 2, \cdots NE \tag{31}$$

$$0 < \rho_{\min} \le \rho_i \le 1 \tag{32}$$

where E_0 is the Young's modulus of the original material and the lower bound of material, ρ_{min}, is introduced to avoid numerical singularity. Since the topology optimization necessarily uses large amount of design variables, gradient-based optimization methods are generally considered. Therefore, the sensitivity of the performance measures, for instance displacement and compliance, with respect to the design variables should be determined in a very efficient way. Among various DSA methods, the continuum-based adjoint variable method is known to be most efficient and accurate and thus widely used in the topology optimization problems. In topology optimization problems, the design variables are material properties. The first order variation of the bilinear energy form with respect to its explicit dependence on the design variable is

$$a'_{\delta\mathbf{u}}(\mathbf{z}, \bar{\mathbf{z}}) = \iiint_{^0\Omega} C'_{ijkl} \, \varepsilon_{ij}(\mathbf{z}) \varepsilon_{kl}(\bar{\mathbf{z}}) \, \mathrm{d}^0\Omega \tag{33}$$

The Equation (27) becomes

$$\psi' = \iiint_{^0\Omega} g_{\mathbf{u}} \, \delta\mathbf{u} \, \mathrm{d}^0\Omega - \iiint_{^0\Omega} C'_{ijkl} \, \varepsilon_{ij}(\mathbf{z}) \varepsilon_{kl}(\lambda) \, \mathrm{d}^0\Omega \tag{34}$$

5. Numerical Examples

5.1 Cantilever Plate

The purpose of this example is to verify accuracy and efficiency of the developed analytical sensitivity analysis method. The design sensitivity results are compared with those of the finite differences. Table I shows very good agreement in various loading conditions that produces large deformation. $\Delta C/\Delta E_i$ stands for finite difference sensitivity and $\mathrm{d}C/\mathrm{d}E_i$ represents the analytical sensitivity. For the finite difference method, the compliance of the structure is measured as varying the material property of an element by various amounts. The adjoint variable method proves to be very efficient since only 4 seconds of CPU time are required for the computation whereas 1454 seconds are necessary for the computation of the finite difference sensitivity.

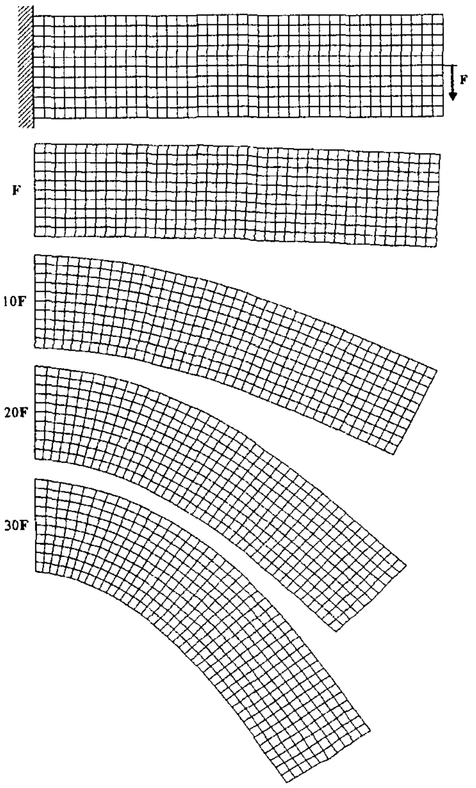
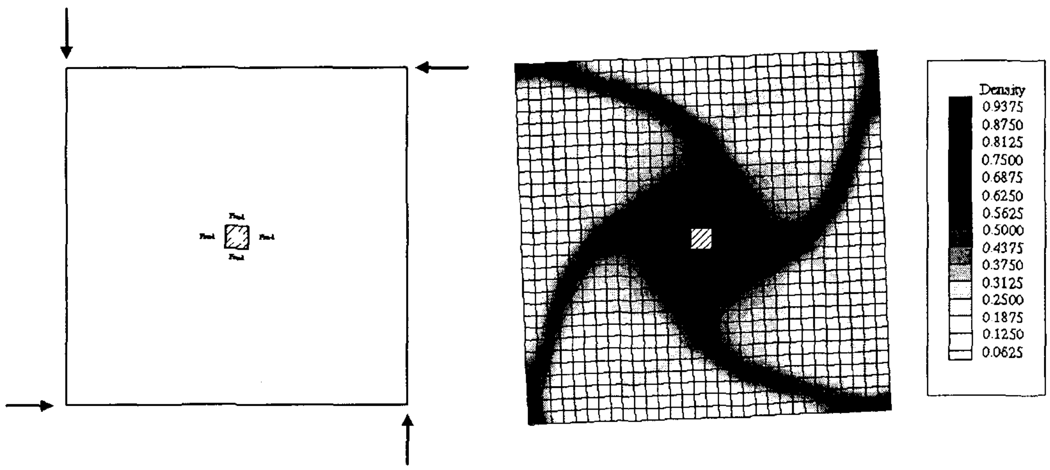


Figure 1. Deformed shapes for various loadings

Table . Comparison of DSA accuracy of mean compliance

Load	ΔE_i	$\frac{\Delta C}{\Delta E_i}$	$\frac{dC}{dE_i}$	$\frac{dC}{dE_i} \Big/ \frac{\Delta C}{\Delta E_i} \times 100\%$
F	50	-1.65147e-11	-1.65551e-11	100.24
	150	-8.24196e-13	-8.26367e-13	100.26
	250	-3.56812e-12	-3.57636e-12	100.23
	350	-2.13703e-12	-2.14227e-12	100.25
10F	50	-1.29516e-09	-1.29867e-09	100.27
	150	-4.30471e-11	-4.31664e-11	100.28
	250	-2.66208e-10	-2.66794e-10	100.22
	350	-1.81708e-10	-1.82140e-10	100.24
20F	50	-2.93942e-09	-2.94839e-09	100.31
	150	-4.83939e-11	-4.85374e-11	100.30
	250	-7.08494e-10	-7.10057e-10	100.22
	350	-4.11826e-10	-4.12781e-10	100.23
30F	50	-3.33293e-09	-3.34424e-09	100.34
	150	-2.20261e-11	-2.20976e-11	100.32
	250	-1.12278e-09	-1.12533e-09	100.23
	350	-5.06575e-10	-5.07732e-10	100.23



(a) Design domain and boundary conditions (b) Optimal topology

Figure 2. Topology design example

5.2 Topology Design

The designable domain, boundary, and loading conditions are shown in Figure 2 (a). The material properties and the magnitude of loading are $E = 1.0 \times 10^5$, $\nu = 0.3$, $F = 1000$, respectively. The objective is to minimize the compliance of the structure within the allowable material of 30% to obtain the optimal distribution of the material. The structure consists of 900 plane stress elements and the final material distribution is shown in Figure 2 (b), where the dark region indicates the optimal layout of the structure.

6. CONCLUSIONS

A continuum-based design sensitivity analysis method for the non-shape problems is developed for geometrically nonlinear structural systems. To evaluate the finite element equation of the structure, an incremental analysis scheme in total Lagrangian formulation is employed. The corresponding design sensitivity equation is computed using the tangent stiffness, the original and adjoint responses at the final equilibrium. Therefore no iteration is necessary as shown in Equation (34). A topology optimization method that uses the derived continuum based material property design sensitivity is also developed. The adjoint variable method is more efficient especially in topology optimization problems that have large amount of design variables. For the numerical implementation, the finite element analysis method, the developed design sensitivity analysis method, and a gradient-based optimization method are integrated into a unified framework. The numerical results of topology optimization show very good agreement with intuitive designs.

REFERENCES

1. E.J. Haug, K.K. Choi, V. Komkov, *Design Sensitivity Analysis of Structural System.* Academic Press, New York, 1986.
2. Bendsøe, M.P., Kikuchi, N., "Generating Optimal Topologies in Structural Design Using a Homogenization Method," *Computer Methods in Applied Mechanics and Engineering*, Vol. 71, 1988, pp.197-224.
3. Cho, S., Jung, H., Yang, Y., "Design Sensitivity Analysis for Non-Shape Problems and Its Applications to Topology Optimization," *Proceedings of the 15th Asian Technical Exchange and Advisory Meeting on Marine Structures*, TEAM 2001 Jochiwon, October 2001, pp. 174-182.
4. Sigmund, O., "Systematic Design of Mechanical Systems Using Topology Optimization," *Proceedings of the 2nd ASMO UK / ISSMO conference on engineering design optimization*, Swansea, Wales, UK, July 2000, pp. 5-12.
5. Pedersen, C.B.W., Buhl, T., Sigmund, O., "Topology Synthesis of Large-Displacement Compliant Mechanisms," *International Journal for Numerical Methods in Engineering*, Vol. 50, 2001, pp.2683-2705.
6. Gea, H.C., Luo, J., "Topology Optimization of Structures with Geometrical Nonlinearities. *Computers & Structures*, Vol. 79, 2001, pp.1977-1985.
7. K.J. Bathe, *Finite Element Procedures*, Prentice-Hall, 1996.
8. Choi, K.K., Santos J.L.T., "Design Sensitivity Analysis of Non-linear Structural Systems. Part I : Theory," *International Journal for Numerical Methods in Engineering*, Vol. 24, 1987, pp.2039-2055.