가상농도법(Pseudo Concentration Method)을 이용한 센터-게이트 디스크 (Center-gated disk) 에서의 섬유배향에 대한 수치모사

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Numerical Simulation of Fiber Suspensions in Center-gated Disk using a Pseudo Concentration Method

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Introduction

Many numerical simulations for the prediction of fiber orientation in injection-molded composites have been developed based on the Hele-Shaw approximation [1], which neglects the details of the flow field at the melt front and gate regions. While fiber orientation predictions based on the Hele-Shaw model showed reasonable results far from the gate and melt front regions [2,3,4,5,6,7,8], it is necessary that a complete solution of three-dimensional governing equations including accurate treatment of melt front region must be considered for the proper simulations.

Filling of the two-dimensional (plane or axisymmetric) geometry was analyzed [9]. Melt front was tracked using PCM (Pseudo Concentration Method) [10,11]. However, calculated melt front for center-gated disk showed unsatisfactory shape and it was due to the corner region where the appropriate wall boundary conditions cannot be applied. First simulations of fiber orientation in three-dimensional molded features and the first comparisons with experiments were reported [12]. VOF method [13] was used to track the melt front. However for the center-gated disk simulation, skewing melt front shape was acquired and solutions have diverged.

In this work, the aim is to develop numerical scheme to cover the details of the flow field for axisymmetric three-dimensional geometry discarding previous numerical assumptions such as Hele-Shaw model. Melt front is captured by PCM. Finite element method with penalty function formulation is implemented for incompressible non-Newtonian, non-isothermal Stokes flow. SUPG (Streamline Upwind Petrov-Galerkin) technique [14] is adopted for the stability of convection-dominated problem. For the fiber orientation prediction, second-order orientation tensor evolution equation with IBOF closure approximation [15] is used.

Governing Equations

The melt is assumed to be incompressible, non-Newtonian and non-isothermal Stokes flow, hence the governing equations are as follows:

Mass and momentum conservation equations

$$u_{i,i} = 0 \tag{1}$$

$$\sigma_{ij,j} = 0 \tag{2}$$

$$\sigma_{ij} = -P\delta_{ij} + \tau_{ij} \tag{3}$$

where u_i is i-th component of the velocity, σ_{ij} is the Cauchy stress tensor, P is the hydrostatic pressure, δ_{ij} is the identity tensor and τ_{ij} is the deviatoric stress tensor with following Dinh-Armstrong model [16];

$$\tau_{ij} = \eta\dot{\gamma}_{ij} + \eta N u_{k,l} a_{ijkl} \tag{4}$$

$$N = \frac{\pi n L^3}{6\ln(2h/D)} \tag{5}$$

where $\dot{\gamma}_{ij} = u_{j,i} + u_{i,j}$ is the rate of deformation tensor, η is the solvent viscosity, n is the number of fibers per unit volume, h is average inter-fiber distance between neighboring fibers, D is fiber diameter and L is its length .

Energy equation

$$\rho C_p\left(\frac{\partial T}{\partial t} + u_r\frac{\partial T}{\partial r} + u_z\frac{\partial T}{\partial z}\right) = \frac{1}{r^a}\frac{\partial}{\partial r}\left(r^a k_r\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + Q \tag{6}$$

$$a = \begin{cases} 0 : \text{2-dimensional case } (\mathrm{r} \to \mathrm{x}, \mathrm{z} \to \mathrm{y}) \\ 1 : \text{axisymmetric case} \end{cases}$$

where ρ is the density, C_p is the thermal capacity, T is the temperature, k_r, k_z is the thermal conductivity in r-direction and z-direction, respectively and Q is the viscous dissipation.

Pseudo-concentration convection equation

$$F_{,t} + u_i F_{,i} = 0 \tag{7}$$

Pseudo-concentration function is defined as F and $F = 1$ corresponds to polymer, whereas $F = 0$ stands for air.

Fiber evolution equation

The evolution equation for the second order orientation tensor a_{ij} [17] can be expressed as

$$\frac{Da_{ij}}{Dt} = -\frac{1}{2}\left(\omega_{ik}a_{kj} - a_{ik}\omega_{kj}\right) + \frac{1}{2}\lambda\left(\dot{\gamma}_{ik}a_{kj} + a_{ik}\dot{\gamma}_{kj} - 2\dot{\gamma}_{kl}a_{ijkl}\right) + 2D_r\left(\delta_{ij} - 3a_{ij}\right) \tag{8}$$

$$\lambda = \left(\frac{(L/D)^2 - 1}{(L/D)^2 + 1}\right)$$

where $\omega_{ij} = u_{j,i} - u_{i,j}$ is the rotation rate tensor. $\dot{\gamma}$ is the generalized shear rate, defined as

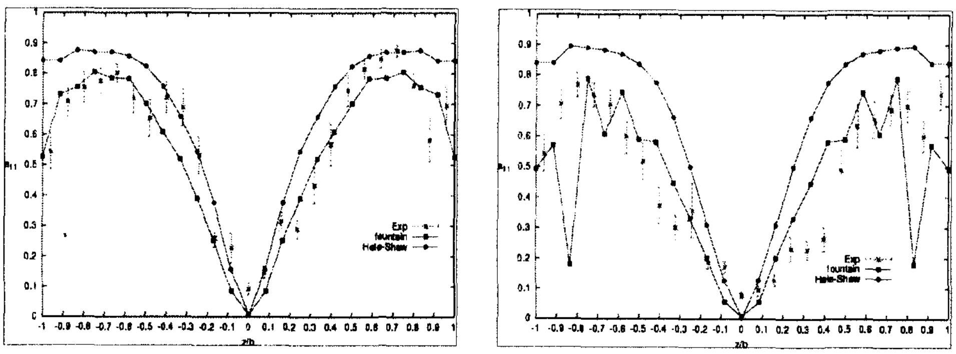
$$\dot{\gamma} = \sqrt{\frac{1}{2}\dot{\gamma}_{ij}\dot{\gamma}_{ji}} \tag{9}$$

In the present study, $D_r = C_I\dot{\gamma}$ [18] is adopted for the numerical calculations of the fiber orientation.

Numerical results

Suspension of nylon 6/6 reinforced with 43wt% (v_f =0.23) of glass fibers with L=210 μm and D=11 μm [3] is used. Processing conditions includes inlet temperature of T_{inlet} =550K, mold wall temperature of T_{wall} =347K, and the filling time of t_{fill} =2.5s. Outer radius of center-gated disk is R_o =76.2mm and thickness is $2b$ =3.18mm. Random fiber orientation and parabolic velocity profile is assumed at the gate.

Fig.1 shows the effect of fountain flow vs. Hele-Shaw (lubrication approximation) on a_{11} component (fiber-fluid decoupled case, $C_I = 0.001$) including experimental data [3]. Predictions with fountain flow nearly match the experimental data. On the other hands, previous results by Hele-Shaw model [6,7] over-predict a_{11}. Some differences between the numerical results and the experimental data might arise from neglecting packing stage of injection molding cycle. Therefore, fountain flow effect is very crucial for the exact prediction of orientation state and Hele-Shaw model must be discarded.



(a) (b)

Fig.1 Effect of fountain flow vs. Hele-Shaw (lubrication approximation) on a_{11} component for non-Newtonian fiber suspension flow (fiber-fluid decoupled case, $C_I = 0.001$) as a function of thickness positions (z/b) for center-gated disk; (a) at r/b=22.8 (b) at r/b=40.4

Conclusion

Finite element numerical analysis of axisymmetric three-dimensional geometry (including two-dimensional plane case) has been performed for fiber suspensions using PCM as a melt-front capturing technique. Comparison between the predictions and the experimental data for center-gated disk shows almost exact qualitative agreements. Importantly, including fountain flow effect induces wide core layers of orientation distributions and consequently makes the predictions closer to experimental data. It has been achieved by discarding Hele-Shaw approximation and special technique around melt front region.

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