Study on the rainwater recharge model using the groundwater variation and numerical solution of quasi-three dimensional two-phase groundwater flow

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Abstract: A rainwater recharge model, which is combined with the quasi-three dimensional unconfined groundwater flow, is proposed in the present paper. The water budget in the catchments of the planned new campus of Kyushu University is evaluated by the present method that calculates both the surface runoff and groundwater flow simultaneously. The results obtained in the present study reveal that the calculated monthly and annual runoff discharges agree reasonably well with the observed discharge. Combining the rainwater recharge model, the two-phase groundwater flow equation is numerically solved for the entire area including the low land where the salt water intrusion is observed. The calculated depth of the salt-fresh interface agrees reasonably well with the observed ones at several cross sections. On the other hand, however, it is found that the calculated water budget remains uncertain because of lack of information on the accurate potential evapotranspiration including rainfall interception.

In conclusion, however, it is found that the proposed method is applicable for the areas where the horizontal flow is dominant and the interface is assumed to be sharp.

1 INTRODUCTION

The new campus Kyushu University is under construction at present on the hill whose elevation ranges from several 10 meters to 100 meters above sea level, as shown in Fig. 1. In the neighboring area, the shallow groundwater is used for domestic purposes, greenhouses and wineries. It is therefore important to assess the potential drawdown of groundwater elevation, saltwater intrusion in an unconfined aquifer, groundwater spring rate, and flood in the surrounding low land.

In the area, there is a sediment storage dam, which prevents the flow of sediment downstream (hereafter, this dam is named to as '*sabo* dam'), located at the upper stream of the Obaru River. The discharge measurement station S2 is located downstream of the *sabo* dam. The *sabo* dam catchment and the S2 station are selected to test the rainwater recharge model and to estimate the water budget in this paper.

2 RAINWATER RECHAGE MODEL

The rainwater recharge model is illustrated in Fig. 2. When discussing rainfall over forested areas, rainfall interception $r_{int}(t)$ needs to be considered.

The seepage coefficient a_L assigned to the vertical outlet with the height of R_0 controls the recharge rate into the groundwater. The evapotranspiration occurs from the tank while water remains in the tank. When the tank becomes empty but the amount of $EVT_1(t)$ does not yet reach the prescribed potential evapotranspiration, the additional water uptake denoted by $EVT_2(x, y, t)$ is considered in the groundwater flow equation, if the vertical distance between the ground surface and unconfined groundwater table is less than H_g^*. Eq. (1) expresses the change in the water level in the tank and eq. (2) is the recharge rate into the unconfined groundwater, respectively.

$$\frac{dh_w(t)}{dt} = \{1 - F(r)\} \cdot r(t) - q_w(t) - EVT_1(t) \tag{1}$$

$$q_w(t) = a_L\{h_w(t) - R_0\} \times Y\{h_w(t) - R_0\} \tag{2}$$

where $Y\{h_w(t)-R_0\}$ is the step function which takes 1 when $h_w(t)>R_0$, and 0 when $h_w(t)<R_0$. Since the value of $q_w(t)$ divided by effective porosity n_e is equal to the rising rate of confined groundwater elevation, the following equation is applicable:

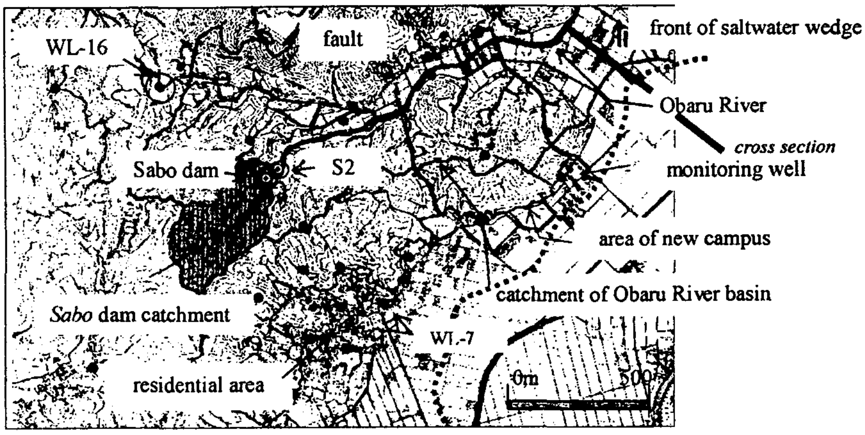

Fig. 1 Study area. Broken line the assumed present front of saltwater.

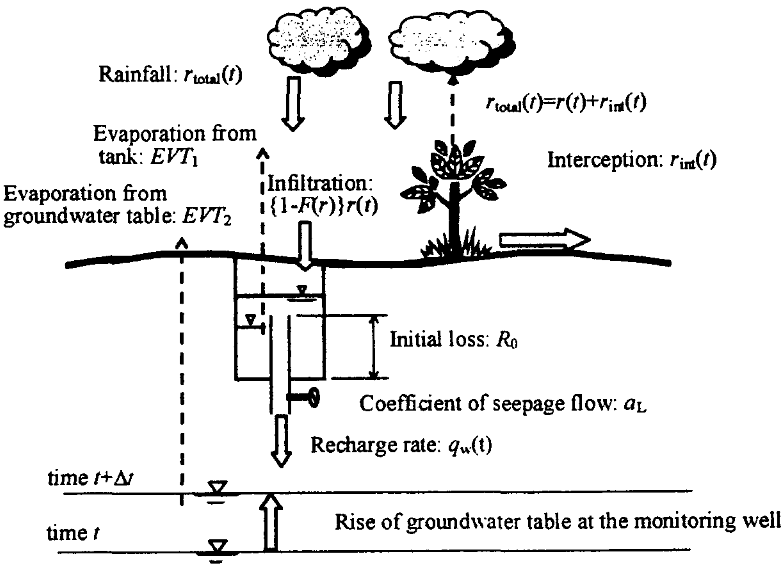

Fig. 2 Rainwater recharge model. The model separates the rainfall into the two components; one for the surface runoff, the other one for the infiltration components.

$$\frac{\partial h_f(x,y,t)}{\partial t}=\frac{q_w(t)}{n_e} \tag{3}$$

where $h_f(t)$ is the measured elevation at the monitoring well.

Eq. (4) considers that the surface runoff coefficient varies with the rainfall intensity which can reach the ground surface[1].

$$F(r)=\frac{r(t)}{r(t)+(r)_{1/2}}\cdot F_\infty \tag{4}$$

where $(r)_{1/2}$ is the value of $r(t)$ when $F(r)$ is equal to $F_\infty/2$. If F_∞ can be taken from the guideline manuals, only $(r)_{1/2}$ is the undetermined parameter in eq. (4). The parameter F_∞ is assumed to depend on only the ground surface condition. Typical values commonly used for sewage drainage planning are listed in Table 1.

Table 1 Guidelines of surface runoff coefficient by Ministry of Education, Culture, Sports, Science and Technology[2)]

Type of ground surface		Coefficient of surface runoff
Road	Permeable pavement	0.70～0.90
	Pavement	0.30～0.40
	Gravel road	0.30～0.70
Shoulder or top of slope	Fine soil	0.40～0.65
	Coarse soil	0.10～0.30
	Hard rock	0.70～0.85
	Soft rock	0.50～0.75
Grass plot of sand	Incline 0～2%	0.05～0.10
	2～7%	0.10～0.15
	7%～	0.15～0.20
Grass plot of clay	Incline 0～2%	0.13～0.17
	2～7%	0.18～0.22
	7%～	0.25～0.35
Roof		1.00
Unused bare land		0.20～0.40
Athletic field		0.40～0.80
Park with vegetation		0.10～0.25
Mountain with a gentle slope		0.30
Mountain with a steep slope		0.50
Paddy field or water		0.30～0.50
Farmland		0.10～0.30

3 QUASI THREE-DIMENSIONAL FRESH AND SALTWATER FLOW MODEL

In order to calculate the movement of fresh and salt groundwater, the following set of equations is adapted[3)] :

$$n_e \frac{\partial(h_f - h_s)}{\partial t} = -\frac{\partial\{(h_f - h_s)\cdot u_f\}}{\partial x} - \frac{\partial\{(h_f - h_s)\cdot v_f\}}{\partial y} - \sum_m Q_m(x,y,t)\delta(x-x_m)\delta(y-y_m) + q_w(x,y,t) - EVT_2(x,y,t) \tag{5}$$

$$n_e \frac{\partial h_s}{\partial t} = -\frac{\partial\{(h_s - b)\cdot u_s\}}{\partial x} - \frac{\partial\{(h_s - b)\cdot u_s\}}{\partial y} \tag{6}$$

where h_s is the elevation of the immiscible fresh-saltwater interface from the reference level; h_f is the fresh groundwater elevation; b is the height of the impermeable base; $k(x,y)$ is permeability; $Q_m(x,y,t)$ is the pumping rate of the well; (x_m, y_m) is the location of the well; $\delta(x\text{-}x_m)$ and $\delta(y\text{-}y_m)$ are the delta functions; (x,y) is the coordinate; $q_w(x,y,t)$ is the rainwater recharge rate calculated by eq. (2). It is obvious that the parameters necessary for the recharge model, n_e, R_0, a_L and $(r)_{1/2}$, vary over space.

4 ESTIMATION OF LOSS OF INFILTRATED WATER R_0 AND EFFECTIVE POROSITY n_e

4.1 Loss of infiltrated water

Fig. 3 is the response of the groundwater table to the accumulated rainwater of one rainfall event $\sum r(t)$. The value of R_0 can be evaluated as the quantity of the maximum accumulated rainwater that is unlikely to induce the rise of the groundwater elevation. In the case of WL-16, R_0 is approximately 9.0 mm.

4.2 Effective porosity

Fig. 4 depicts the relationships between $\sum r(t) - R_0$ and the raised groundwater elevation. The effective porosity is estimated to be 0.125.

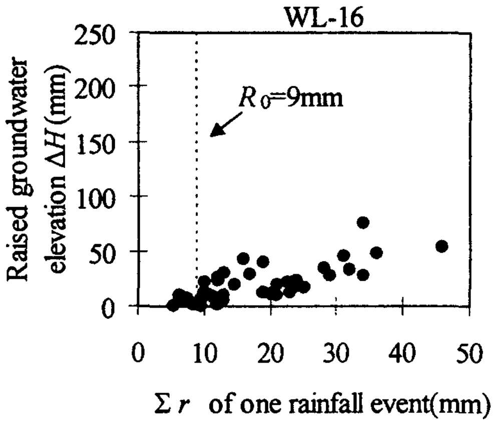

Fig. 3 Response of the groundwater table to the amount of one rainfall event.

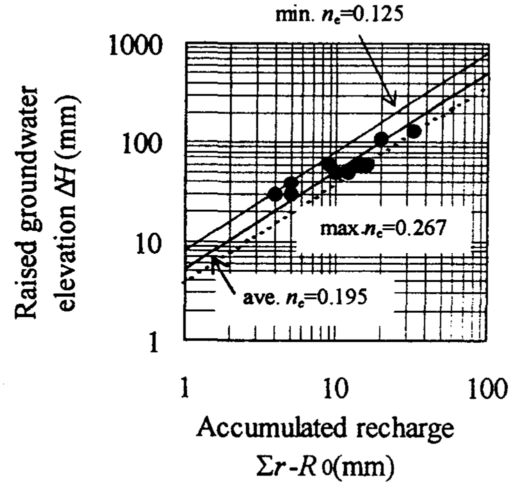

Fig. 4 Estimation of effective porosity n_e.

4.3 Parameter estimation by least square method

The values of a_L and $(r)_{1/2}$ used in the rainwater recharge model are determined by using the criterion denoted by eq. (7).

$$J = \sqrt{\sum_{t=1}^{N} \frac{\{h_{fobs}(t) - h_{fcal}(t)\}^2}{N}} \tag{7}$$

where $h_{fobs}(t)$ and $h_{fcal}(t)$ denote the observed freshwater and calculated groundwater elevation by eq. (3), respectively. The optimum a_L and $(r)_{1/2}$ are found to be 0.12 mm hour^{-1} and 6.0 mm.

5 EVAPOTRANSPIRATION AND RAINFALL INTERCEPTION

The monthly potential evapotranspiration and the rainfall interception are shown in Table 2. The annual potential evapotranspiration for the year 1997 through 1999 was 849 mm year^{-1} based on the Thornthwait method. On the other hand, the potential evapotranspiration based on the heat balance and bulk method[4)] was 860 mm year^{-1} at the nearest meteorological station located 10 km apart from the study area. The difference between them is 11 mm year^{-1}. Accordingly, there would not be a significant error in using the Thornthwait method. Since trees cover 90% of the study area, rainfall interception needs to be considered. The rainfall interception in 1999 is exemplified in the table. The hourly time series of the potential evapotranspiration was made by the monthly potential evapotranspiration neglecting the diurnal variation.

Table 2 Potential evapotranspiration by heat balance-bulk method[4)] for the period 1986 to 1990, and by the Thornthwait method for 1999. Unit in mm month^{-1}.

		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Kondo et al. for 1986 -1990	EVT_{pK}	32	31	47	64	90	102	119	127	96	71	45	36
	Average rainfall for 5 years	66	76	128	88	156	244	238	123	224	66	56	54
	Average rainfall interception for 5 years	20	17	27	24	30	34	35	31	32	21	15	19
	r_{total} of 1999	52	51	99	95	93	441	226	265	177	56	97	25
	[A] r_{intK} of 1999	**16.9**	**16.7**	**24.4**	**23.9**	**23.6**	**39.4**	**33.7**	**35.2**	**31.1**	**17.7**	**24.1**	**9.9**
Ratio	r_{intK} /EVT_{pK}	0.63	0.55	0.57	0.38	0.33	0.33	0.29	0.24	0.33	0.30	0.33	0.53
TH method	EVT_{pTH}	9.9	10.3	27.5	47.0	82.9	119.7	137.8	151.4	127.8	68.6	34.1	13.5
	[B] r_{intTH} of 1999	**6.2**	**5.7**	**15.8**	**17.6**	**27.6**	**39.9**	**40.5**	**36.9**	**42.6**	**20.3**	**11.4**	**7.1**

EVT_{pK}: Potential evapotranspiration by Kondo et al., EVT_{pTH}: Potential evapotranspiration by the Thornthwait method, r_{total}: rainfall intensity for 1999, **[A]**r_{intK} was converted from the relationship between monthly rainfall and rainfall interception. **[B]**r_{intTH} was calculated by multiplying the Ratio r_{intK} /EVT_{pK} to EVT_{pTH}. Annual potential evapotranspiration for 1986-1990 by Kondo et al. is 860 mm year^{-1} with 305 mm year^{-1} as rainfall interception in average, while annual potential evapotranspiration by the Thornthwait method is 849 mm year^{-1}.

6 THE WATER BUDGET IN THE CATCHMENT

Table 3 shows the water budget in the catchment of the *sabo* dam. The calculated average discharge for three years at the *sabo* dam ranges from 1,137 at minimum to 1,218 mm year^{-1} at maximum for the various combinations of R_0 and H_g*. The calculated discharge is larger than the observed discharge 1,022 mm year^{-1}. The difference is approximately 115 to 195 mm year^{-1} and this is approximately 11 to 19 % of the observed discharge. The authors believe this difference arises from 1) the unseizable groundwater flow rate beneath, right and left sites of the *sabo* dam in the measurement, 2) the uncertainty of the hydrogeological structure. Fig. 5 illustrates the components of the water flow. Both G_r and Sp_2 are the groundwater seepage and spring rate. *GW* is the groundwater flow at the *sabo* dam cross section. The total discharge at the *sabo* dam cross section is the summation of *Ds*, *GW*, and the spring rate Sp_1 at the left downstream of the *sabo* dam. Two cases corresponding to the rainfall interception of **[A]** and **[B]** in Table 2 were tested.

Fig. 6 also depicts the calculated and observed time series of monthly surface flow rate. Except the period from autumn in 1998 and early spring of 1999, they show a good agreement. The large difference occurred for that period is resulted from the underflow below the *sabo* dam that can not seize low flow rate as mentioned above. For the summer of 1999, we suspect there should be unrecorded water diversion for irrigation by local farmers.

7 CHECK OF GROUNDWATER LEVEL AND ELEVATION OF TWA-PHASE INTERFACE

Fig. 7 shows the calculated and observed variation of the groundwater elevation at WL-16 as depicted in Fig. 1. They show a reasonable agreement. We confirmed most of the observed groundwater elevation showed similar result. A careful examination was also conducted for the depth of the two-phase interface. Since the permeability in the low land area is sensitive to saltwater interface, small modifications are made so that the calculated interface coincides with the EC value around 20(mS/cm). In addition to this, the effect on upconing of the interface, which may be affected by the existing drainage systems, was also considered. The convex shape of the interface in Fig. 8 indicates such an effect. Therefore, it can be said that a drainage system in the low land areas induces upconing and further saltwater intrusion.

Table 3 Water budget in the catchment of the *sabo* dam for the two different rainfall interceptions. Unit of discharge in mm year^{-1}.

H_g^*		**1.5 m***			2.5 m		
R_o		9 mm	**15 mm***	20 mm	9 mm	15 mm	20 mm
Annual precipitation r_{total} =1,761 ; Observed discharge at the *sabo* dam=1,022 ; Potential evapotranspiration by the Thornthwait method 849							
Groundwater recharge $q_w(t)$	**[A]**	**1,065**	**1,002**	**960**	**1,065**	**1,002**	**960**
	[B]	**1,045**	**983**	**944**	**1,045**	**983**	**944**
Evapotranspiration with the rainfall interception $EVT_1+EVT_2+r_{int}$	**[A]**	**524**	**583**	**622**	**528**	**586**	**625**
	[B]	**542**	**601**	**639**	**547**	**604**	**641**
Direct evapotranspiration EVT_2	**[A]**	**19**	**15**	**13**	**23**	**19**	**6**
	[B]	**18**	**15**	**13**	**22**	**18**	**15**
Groundwater flow at the *sabo* dam cross section *GW*	**[A]**	**229**	**228**	**227**	**229**	**228**	**227**
	[B]	**228**	**227**	**226**	**228**	**227**	**226**
Direct surface runoff ΣQ_d 191							
Spring rate Sp_1	**[A]**	**225**	**223**	**222**	**225**	**223**	**221**
	[B]	**224**	**222**	**220**	**223**	**221**	**220**
Groundwater seepage to the river + out flow $Gr+Sp_2$	**[A]**	**573**	**542**	**522**	**567**	**537**	**518**
	[B]	**554**	**524**	**505**	**548**	**519**	**501**
Total discharge at the site of the *sabo* dam	**[A]**	**1,218**	**1,184**	**1,162**	**1,212**	**1,178**	**1,157**
	[B]	**1,197**	**1,163**	**1,142**	**1,191**	**1,158**	**1,137**

[A] in this table corresponds to the row of the rainfall interception calculated by Kondo's data in Table 1, while **[B]** for the rainfall interception by TH method in Table 2.

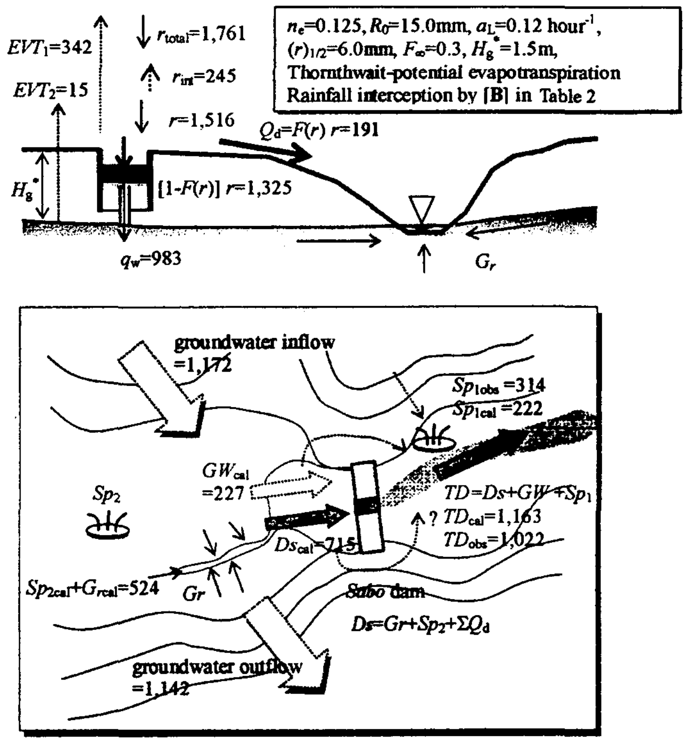

Fig. 5 Components of water flow. The values depict the three-year average annual water flux in **Table 2** corresponding to the case [B] of rainfall interception r_{int}. Unit in mm year^{-1}.

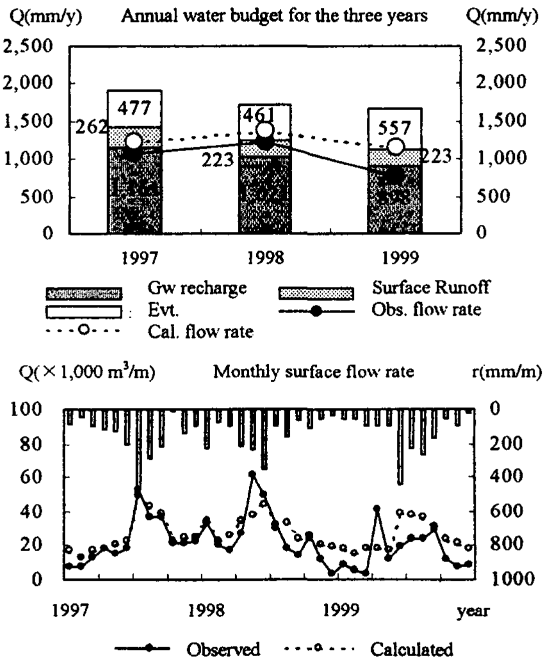

Fig. 6 Water budget calculation.

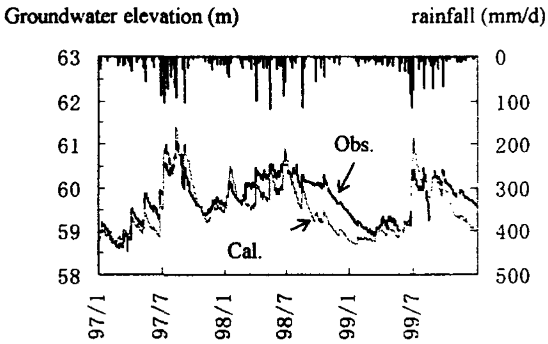

Fig. 7 Calculated and observed groundwater elevation at WL-16.

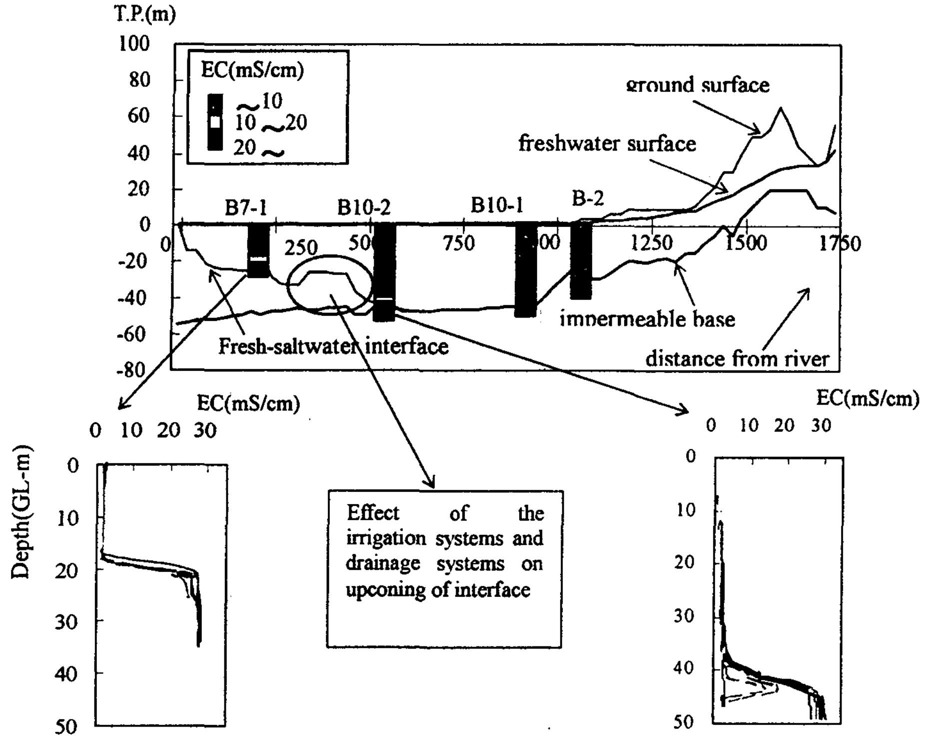

Fig. 8 Comparison of the observed and calculated depth of two-phase interface at the cross section.

CONCLUSIONS

It is shown that the present scheme is able to calculate not only the groundwater flow but also surface discharge in the study area. The parameters n_e, R_0, a_L and $(r)_{1/2}$ can be obtained easily if the record of hourly rainfall and groundwater variation is available. The parameter H_g* is also introduced in order to describe the direct evapotranspiration from the groundwater table. However, rainfall interception is shown to be important in evaluating the water budget.

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