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On Fuzzy Almost c-Continuous Mappings

퍼지 Almost c-연속사상에 관하여

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ABSTRACT

In this paper, we introduce the concept of a fuzzy almost c-continuity and investigate some of its properties.

Keywords and phrases : fuzzy almost c-continuity, fuzzy almost continuity, fuzzy c-continuity, fuzzy H-continuity, fuzzy weak continuity, fuzzy closed graph, locally fuzzy compact, fuzzy T_{2w}, fuzzy strongly closed graph, normalized fuzzy space, the graph having an upper fuzzy point.

1. Introduction and preliminaries

The study of continuity and its weaker forms constitutes an established branch of investigation in general topological spaces. Recently some researchers[1,2,7,8,16] have tried to extend these studies to the broader framework of fuzzy topological spaces. Using two notions of membership of a fuzzy point to a fuzzy set, neighborhood structure of a fuzzy point[10] and quasi-neighborhood structure of a fuzzy point[11], an investigation of fuzzy continuity, fuzzy almost continuity, fuzzy weak continuity, fuzzy c-continuity and fuzzy H-continuity has been carried out in [1,2,7,8] with almost the some degree of success as in general topological spaces.

In this paper, we extend the notion of almost c-continuity introduced by S. G. Hwang[9] to fuzzy topological spaces. Here we establish some properties of fuzzy almost c-continuous mappings. In particular, we discuss the relationship of fuzzy almost c-continuous mappings with other notions of fuzzy topological spaces such as compactness, regular openness and H-closedness.

In order to make the exposition self-contained as far as practicable, we list some definitions and results that will be used in the sequel. Let X be a non-empty(ordinary) set and let I the unit interval $[0,1]$. A *fuzzy set* A in X is a mapping from X into I. For any fuzzy set A in X the set $\{x\in X: A(x)>0\}$ is called the support of A and denoted by $S(A)$[17]. A *fuzzy point* x_λ in X is a fuzzy set in X defined by : for each $y\in X$,

$$x_\lambda(y)=\begin{cases}\lambda, & \text{if } y=x,\\ 0, & \text{if } y\neq x,\end{cases}$$

where $x\in X$ and $\lambda\in(0,1]$ are respectively called the *support* and the *value* of x_λ[11,14]. A fuzzy point x_λ is said to *belong to* a fuzzy set A in X iff $\lambda\le A(x)$[11]. A fuzzy set A in X is the union of all fuzzy points which belong to A[11]. A subfamily T of I^X is called a *fuzzy topology* on X [3] if (i) $\emptyset, X\in T$, (ii) for any $\{U_\alpha\}_{\alpha\in\Lambda}\subset T$, $\bigcup_{\alpha\in\Lambda} U_\alpha\in T$ and (iii) for any $A, B\in T$, $A\cap B\in T$. In this case, each member of T is called a *fuzzy open*)(in short, *F-open*) set in X and its complement a *fuzzy closed*(in short, *F-closed*) set in X. The pair (X,T) is called a *fuzzy topological space*(in short, *fts*). For a fts X, $FO(X)$ and $FC(X)$ denote the collection of all F-open sets and F-closed sets in X, respectively. For a fuzzy set A in a fts X, the closure $\mathrm{cl}A$ and the interior $\mathrm{int}A$ of A are defined respectively as $\mathrm{cl}A=\bigcap\{V\in I^X: A\subset V$ and $V^c\in FO(X)\}$ and $\mathrm{int}A=\bigcup\{V\in FO(X): V\subset A\}$[11].

Definition 1.1[5]. A fts X is said to be *fuzzy* T_{2w} (in short, FT_{2w}) if for any two distinct fuzzy points x_λ and y_μ in X, there exist $U, V\in FO(X)$ such that $x_\lambda\in U$, $y_\mu\in V$ and $U\odot V=\emptyset$.

Definition 1.2[2]. Let A be a fuzzy set in a fts X. Then :

(1) A is called a *fuzzy regular open set in* X if $A=\mathrm{int}(\mathrm{cl}A)$.

(2) A is called a *fuzzy regular closed set in* X if

$A = \mathrm{cl}(\mathrm{int}\,A)$.

We denote the collection of all fuzzy regular open[resp. closed] set in X as $FRO(X)$ [resp. $FRC(X)$].
It is clear that $FRO(X) \subset FO(X)$ and $FRC(X) \subset FC(X)$.

We will use the notion of fuzzy compactness in the sense of S. Gangly and S. Saha[6].

Result 1.A[6, Theorem 4.2]. Every F-closed set in a compact fts is F-compact.

Result 1.B[6, Theorem 4.6]. Let X be a fts and let $A \in I^X$. Then A is F-compact in X if and only if each F-open cover of A has a finite subcover.

Definition 1.3[13]. A fts X is said to be *normalized* if for each $x_\lambda \in F_p(X)$, there exists $U \in FO(X)$ such that $U(x) = 1$.

Definition 1.4[13]. A fts X is said to be *fuzzy locally compact*(in short, *locally F-compact*) at $x_\lambda \in F_p(X)$ if there exists a F-open set U and a F-compact set K in X such that $x_\lambda \in U \subset K$. A fts X is said to be *locally F-compact* if it is locally F-compact at each of its fuzzy points.

It is clear that every compact fts is locally compact.

Result 1.C[13, Corollary 4.2.3]. A normalized FT_{2w}-space X is locally F-compact if and only if for each $x_\lambda \in F_p(X)$ and each neighborhood V of x_λ, there exists a neighborhood U of x_λ such that $clU \subset V$ and clU is F-compact in X.

Definition 1.5[1]. A mapping $f: X \to Y$ is said to be *fuzzy almost continuous*(in short, *fal-continuous*) at $x_\lambda \in F_p(X)$ if for each $V \in FO(Y)$ such that $f(x_\lambda) \in V$, there exists a $U \in FO(X)$ such that $x_\lambda \in U$ and $f(U) \subset int(clV)$. The mapping f is said to be *fal-continuous* (on X) if it is fal-continuous at each $x_\lambda \in F_p(X)$

Result 1.D[1,Theorem 4.1; 2,Theorem 7.2]. Let $f: X \to Y$ be a mapping. Then the following are equivalent :

(1) f is fal-continuous.

(2) For each $V \in FRO(Y)$, $f^{-1}(V) \in FO(X)$.

(3) For each $F \in FRC(Y)$, $f^{-1}(F) \in FC(X)$

Definition 1.6[8]. A mapping $f: X \to Y$ is said to be *fuzzy c-continuous*(in short, *fc-continuous*) at $x_\lambda \in F_p(X)$, if for each $V \in FO(X)$ such that $f(x_\lambda) \in V$ and V^c is F-compact in Y, there exists a $U \in FO(X)$ such that $x_\lambda \in U$ and $f(U) \subset V$. The mapping f is said to be *fc-continuous*(on X) if it is fc-continuous at each $x_\lambda \in F_p(X)$.

Result 1.E[8, Theorem 2.2 and Theorem 3.3]. Let $f: X \to Y$ be a mapping. Then the following are equivalent :

(1) f is fc-continuous.

(2) For each $V \in FO(Y)$ such that V^c is F-compact in Y, $f^{-1}(V) \in FO(X)$.

(3) For each fuzzy closed compact set F in Y, $f^{-1}(F) \in FC(X)$.

Definition 1.7[7]. Let $A \in I^X$. Then A is said to be fuzzy H-closed relative to X (in short, fH-closed) if for each F-open cover $\{V_\alpha\}_{\alpha \in \Lambda}$ of A in X, there exists a finite subfamily Λ_0 of Λ such that $A \subset \bigcup_{\alpha \in \Lambda_0} (\mathrm{cl}\, V_\alpha)$. The fts X is said to be a fH-closed space if for each F-open cover $\{V_\alpha\}_{\alpha \in \Lambda}$ of X, there exists a finite subfamily Λ_0 of Λ such that $\bigcup_{\alpha \in \Lambda_0} (\mathrm{cl}\, V_\alpha) = X$.

Result 1.F[7, Lemma 2.2]. Let X be a FT_{2w}-space. If B is fH-closed in X, then $B \in FC(X)$.

Definition 1.8[7]. A mapping $f: X \to Y$ is said to be *fuzzy H-continuous* (in short, *fH-continuous*) if for each $x_\lambda \in F_p(X)$ and each $V \in FO(Y)$ such that $f(x_\lambda) \in V$ and V^c is fH-closed in Y, there exists $U \in FO(X)$ such that $x_\lambda \in U$ and $f(U) \subset V$.

Result 1.G[7, Theorem 2.4.]. Let $f: X \to Y$ be a mapping. Then the following are equivalent :

(1) f is fH-continuous.

(2) If $V \in FO(Y)$ and V^c is fH-closed in Y, then $f^{-1}(V) \in FO(X)$.

These statements are implied by

(3) If B is fH-closed in Y, then $f^{-1}(B) \in FC(X)$.

Furthermore, if Y is FT_{2w}, then all three statements are equivalent.

Let $f: X \to Y$ be a mapping. Then the subset $G(f) = \{(x, f(x)) : x \in X\}$ of the Cartesian product $X \times Y$ is called the *graph of* f.

Definition 1.9. A mapping $f: X \to Y$ is said to have a *fuzzy closed graph*(in short, *F-closed graph*) if $G(f) \in FC(X \times Y)$.

Definition 1.10[7]. Let X and Y be fts's. Then a mapping $f: X \to Y$ is said to *have a fuzzy strongly closed graph*(in short, *F-strongly closed graph*) or the graph $G(f)$ is said to be *fuzzy strongly closed*(in short, *F-strongly closed*) in $X \times Y$ if for each $(x_\lambda, y_\mu) \notin F_p(G(f))$, there exist $U \in FO(X)$ and $V \in FO(Y)$ such that $x_\lambda \in U$, $y_\mu \in V$ and $(U \times \text{cl } V) \odot G(f) = \emptyset$.

Definition 1.11[7]. Let X and Y be fts's and let $f: X \to Y$ be a mapping. Then the graph $G(f)$ of f is said to *have an upper fuzzy point in* $X \times Y$ provided that for each $(x_\lambda, y_\mu) \notin F_p(G(f))$, there exist $U \in FO(X)$ and $V \in FO(Y)$ such that $x_\lambda \in U$, $y_\mu \in V$ and if $(U \times \text{cl } V) \odot G(f) = \emptyset$, then there exists $(a, b) \in G(f)$ such that $(U \times \text{cl } V)(a, b) > \frac{1}{2}$.

Result 1.H[7, Lemma 3.3]. Let X and Y be fts's, let $f: X \to Y$ a mapping and let $G(f)$ have an upper fuzzy point in $X \times Y$. Then f has a F-strongly closed graph if and only if for each $x_\lambda \in F_p(X)$ and each $y_\mu \in F_p(Y)$ such that $y \neq f(x)$, there exist $U \in FO(X)$ and $V \in FO(Y)$ such that $x_\lambda \in U$, $y_\mu \in V$ and $f(U) \odot \text{cl } V = \emptyset$.

Result 1.I[7, Theorem 3.8]. If a mapping $f: X \to Y$ has a F-strongly closed graph, then it is fH-continuous.

Definition 1.12[1]. A mapping $f: X \to Y$ is said to be *fuzzy weakly continuous*(in short, *F-weakly continuous*) at $x_\lambda \in F_p(X)$ if for each $V \in FO(Y)$ such that $f(x_\lambda) \in V$, there exists a $U \in FO(X)$ such that $x_\lambda \in U$ and $f(U) \subset \text{cl } V$.
The mapping f is *F-weakly continuous*(on X) if it is F-weakly continuous at each $x_\lambda \in F_P(X)$.

Result 1.J[1, Theorem 5.1]. A mapping $f: X \to Y$ is F-weakly continuous if and only if for each $V \in FO(Y)$, $f^{-1}(V) \subset int(f^{-1}(cl V))$.

2. Properties of fuzzy almost c-continuous mappings

From now on, we will denote X, Y, Z as fuzzy topological space.

Definition 2.1. A mapping $f: X \to Y$ is said to be *fuzzy almost c-continuous*(in short, *falc-continuous*) at $x_\lambda \in F_p(X)$ if for each $V \in FO(Y)$ such that $f(x_\lambda) \in V$ and V^c is F-compact in Y, there exists a $U \in FO(X)$ such that $x_\lambda \in U$ and $f(U) \subset int(cl V)$. The mapping f is *falc-continuous*(on X) if it is falc-continuous at each $x_\lambda \in F_p(X)$.

Remark 2.2. All F-continuous mappings, fc-continuous mappings and fal-cont- inuous mappings are falc-continuous.

Theorem 2.3. For a $f: X \to Y$ be a mapping, the following are equivalent :

(1) f is falc-continuous.

(2) For each $V \in FRO(Y)$ such that V^c is F-compact in Y, $f^{-1}(V) \in FO(X)$.

(3) For each $F \in FRC(Y)$ such that F is F-compact in Y, $f^{-1}(F) \in FC(X)$.

(4) For each $x_\lambda \in F_P(X)$ and each $V \in FRO(X)$ containing $f(x_\lambda)$ having F-compact complement, there exists $U \in FO(X)$ such that $x_\lambda \in U$ and $f(U) \subset V$.

(5) For each $x_\lambda \in F_p(X)$ and each $V \in FO(Y)$ containing $f(x_\lambda)$ having F-compact complement, $f^{-1}(int(clV)) \in FO(X)$

Theorem 2.4. Any restriction of a falc-continuous mapping is also falc- continuous.

Theorem 2.5. If $f: X \to Y$ is F-continuous and $g: Y \to Z$ is falc-continuous, then $g \circ f: X \to Z$ is falc-continuous.

Theorem 2.6. Let $f: X \to Y$ be F-open and surjective. If $g \circ f: X \to Z$ is falc-continuous, then $g: Y \to Z$ is falc-continuous.

Lemma 2.7. Let $f: X \to Y$ be a mapping and let $x_\lambda \in F_p(X)$. If there exists a $U \in FO(X)$ such that $x_\lambda \in U$, $U = S(U)$ and $f|_U: U \to Y$ is falc-continuous at x_λ, then f is falc-continuous at x_λ.

Theorem 2.8. Let $\{U_\alpha\}_{\alpha \in \Lambda}$ be a fuzzy open cover of X such that $U_\alpha = S(U_\alpha)$ for each $\alpha \in \Lambda$ and let $f: X \to Y$ a mapping. If $f|_{U_\alpha}: U_\alpha \to Y$ is falc-continuous for each $\alpha \in \Lambda$, then f is falc-continuous.

Theorem 2.9. Let $f: X \rightarrow Y$ be a mapping and let $X = A \cup B$, where $A, B \in FC(X)$, $A = S(A)$ and $B = S(B)$. If $f|_A$ and $f|_B$ are falc-continuous, then f is falc-continuous.

Theorem 2.10. Let $f: X \rightarrow Y$ be a mapping and let $X = A \cup B$, where $A = S(A)$ and $B = S(B)$. If both $f|_A$ and $f|_B$ are falc-continuous at $x_\lambda \in A \cap B$, there f is falc-continuous at x_λ.

Theorem 2.11. Let $f: X \rightarrow Y$ be falc-continuous. If Y is a locally compact FT_{2w}-space, then f has fuzzy closed graph.

Theorem 2.12. Let $f: X \rightarrow Y$ be a mapping and let $g: X \rightarrow X \times Y$ the graph mapping of f. If X is F-compact and g is falc-continuous, then f is falc-continuous.

3. Further results

Theorem 3.1. Let Y be a normalized locally compact FT_{2w}-space. If $f: X \rightarrow Y$ is falc-continuous and $G(f)$ has an upper fuzzy point in $X \times Y$, then $G(f)$ is F-strongly closed in $X \times Y$.

The following is the immediate result of Result 1.H and Theorem 3.1 :

Corollary 3.1. Let Y be a normalized locally compact FT_{2w}-space, let $G(f)$ have an upper fuzzy point in $X \times Y$ and let $f: X \rightarrow Y$ be a mapping. Then the following are equivalent:

(1) $G(f)$ is F-strongly closed in $X \times Y$.
(2) f is fH-continuous.
(3) f is fc-continuous.
(4) f is falc-continuous.

Theorem 3.3. Let $f: X \rightarrow Y$ be falc-continuous. If Y is a compact fts(resp. compact FT_{2w}-space), then f is fal-continuous(resp. F-continuous).

Lemma 3.4. If $f: X \rightarrow Y$ is F-weakly continuous and K is F-compact in X, then $f(K)$ is fH-closed in Y.

Theorem 3.5. Let $\{Y_\alpha\}_{\alpha \in \Lambda}$ be a family of normalized locally compact FT_{2w}-spaces and let $f_\alpha: X \rightarrow Y_\alpha$ be falc-continuous for each $\alpha \in \Lambda$. If $G(f_\alpha)$ has an upper fuzzy point in $X \times Y_\alpha$ for each $\alpha \in \Lambda$, then the mapping $f: X \rightarrow \prod_{\alpha \in \Lambda} Y_\alpha$ defined by $f(x) = (f_\alpha(x))_{\alpha \in \Lambda}$ for each $x \in X$, is fH-continuous.

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