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Observer-Based Digital Fuzzy Controller

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Abstract

This paper concerns a design methodology of the observer-based output-feedback digital controller for Takagi-Sugeno (TS) fuzzy systems using intelligent digital redesign (IDR). The term of IDR involves converting an analog fuzzy-mode-based controller into an equivalent digital one in the sense of state-matching. The considered IDR problem is viewed as convex minimization problems of the norm distances between linear operators to be matched. The stability condition is easily embedded and the separations principle is explicitly shown.

1. Introduction

The control of engineering systems often evolves a continuous-time plant controlled by feeding sampled measurements back with analog-to-digital (A/D) and digital-to-analog (D/A) devices for interfacing. Recent advancements in digital microprocessor technology have rendered considerable merit to digital control systems exhibiting inexpensiveness and flexibility in implementation of complex control algorithms. To fully enjoy the advantage of the digital technology in control engineering, various digital control techniques have been developed. Yet another efficient approach is, so-called digital redesign (DR) [2,3,5-7], to convert the well-designed analog controller into the equivalent digital one maintaining the properties of the original analogously controlled system, by which the benefits of both the analog controller and the advanced digital technology can be achieved.

It is noted that these digital redesign schemes basically work only for a class of linear systems. For that reason, it has been highly demanded to develop some intelligent digital redesign methodology for complex nonlinear systems, in which the first attempt was made by Joo *et. al.* [2]. They synergistically merged both the Takagi-Sugeno (TS) fuzzy-model-based control and the digital redesign technique for a class of nonlinear systems. Chang *et. al.* extended the intelligent digital redesign to uncertain TS fuzzy systems [3] and elaborated it [4]. However, until now, no tractable method for IDR tackling on the observer-based output-feedback TS control system has been proposed. They remain yet to be theoretically challenging issues in IDR and thereby must be fully tackled.

Motivated by the above observations, this paper aims at developing IDR for the observer-based output-feedback TS fuzzy control system. To resolve the problems above stated, we propose an alternative way-convex optimization-based IDR. It has recently been noticed that many control problems can be efficiently solved by formulating in terms of linear matrix inequalities (LMIs). Casting IDR into an LMI format is also highly desirable, since the flexibility of LMIs allows one to characterize the matching condition for the state estimation error and the stability condition of the redesigned system by LMI format, as well as the state-matching condition. The main contribution of this paper is to derive sufficient conditions of IDR in terms of LMIs. The stability condition is naturally incorporated with ease. The separation principle is also explicitly shown.

The rest of this paper is organized as follows: Section 2 briefly reviews TS fuzzy systems both continuous and discrete-time cases. In Section 3, a new IDR method is proposed for observer-based output-feedback TS fuzzy control systems. This paper concludes with Section 4.

2. Preliminaries

Consider a TS fuzzy system in which the ith rule is formulated in the following form:

R^i: If $z_1(t)$ is about Γ_1^i ... $z_n(t)$ is about Γ_1^n

$$\text{Then } \begin{aligned} \dot{x}_c(t) &= A_i x_c(t) + B_i u_c(t) \\ y_c(t) &= C x_c(t) \end{aligned} \quad (1)$$

where $x_c(t) \in R^n$ is the state vector, $u_c(t) \in R^m$ is the control input vector. The subscript 'c' means the analog control, while the subscript 'd' will denotes the digital control in the sequel. R^i denotes the ith fuzzy inference rule, $z_h(t)$ is the premise variable, Γ_h^i $i \in I_Q, h \in I_N$ is the fuzzy set of the hth premise variable in the ith fuzzy inference rule. Using the center-average defuzzification, product inference, and singleton fuzzifier, the global dynamics of this TS fuzzy system (1) is described by

$$\dot{x}_c(t) = \sum_{i=1}^{q} \theta_i(z(t))(A_i x_c(t) + B_i u_c(t)) \quad (2)$$

$$y_c(t) = C x_c(t) \quad (3)$$

in which $\omega_i(z(t)) = \prod_{h=1}^{n} \Gamma^{i_h}(z_h(t))$ $\theta_i(z(t)) = \omega_i(z(t)) / \sum_{i=1}^{q} \omega_i(z(t))$ and $\Gamma^{i_h}(z_h(t))$ is the membership value of the hth premise variable $z_h(t) \in \Gamma_h^i$

Throughout this paper, a well-constructed continuous-time observer-based fuzzy-model-based control law is assumed to be pre-designed, which will be used in redesigning the digital control law. In real control problems, we cannot always observe all the states of a system. Hence a fuzzy-model-based observer is introduced as follows:

R^i: If $z_1(t)$ is about Γ_1^i... $z_n(t)$ is about Γ_1^n

Then

$$\dot{\hat{x}}_c(t) = A_i \hat{x}_c(t) + B_i u_c(t) + L_c^i(y_c(t) - \hat{y}_c(4))$$

The defuzzified output of the observer rules is represented by

$$\dot{\hat{x}}_c(t) = \sum_{i=1}^{q} \theta_i(z(t))(A_i \hat{x}_c(t) + B_i u_c(t) + L_c^i(y_c(t) - \hat{y}_c(t))) \quad (5)$$

The controller rule is of the following form:

R^i:If $z_1(t)$ is about Γ_1^i... $z_n(t)$ is about Γ_1^n

$$\text{Then } u_c(t) = K_c^i \hat{x}_c(t) \quad (6)$$

The defuzzified output of the controller rules is given by

$$u_c(t) = \sum_{i=1}^{q} \theta_i(z(t)) K_c^i \hat{x}_c(t) \quad (7)$$

Let the estimation error $e_c(t) = x_c(t) - \hat{x}_c(t)$ then we obtain the augmented continuous-time closed-loop TS fuzzy system is

$$\dot{\chi}_c(t) = \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(z(t)) \theta_j(z(t)) \Phi_{ij} \chi_c(t) \quad (8)$$

where

$$\Phi_{ij} = \begin{bmatrix} A_i + B_i K_c^j & L_c^i C \\ 0 & A_i - L_c^i C \end{bmatrix}$$

for the pair (i, j) $I_Q \times I_Q$, where $\chi_c(t) = [\hat{x}_c(t)^T, e_c(t)^T]^T$

3. Main Results

This subsection discusses the discretization of the hybrid TS fuzzy system. Consider the digitally controlled TS fuzzy systems and the discrete TS fuzzy observer governed by

$$\dot{x}_d(t) = \sum_{i=1}^{q} \theta_i(z(t))(A_i x_d(t) + B_i u_d(t)) \quad (9)$$

$$\begin{aligned} &\hat{x}_d(kT+T) \\ &= \sum_{i=1}^{q} \theta_i(z(kT))(G_i \hat{x}_d(kT) \\ &\quad + H_i u_d(t) + L_d^i(y_d(kT) - \hat{y}_d(kT))) \end{aligned} \quad (10)$$

where $G_i = \exp(A_i T)$and $H_i = (G_i - I)A_i^{-1}B$ and $u_d(t) = u_d(kT)$ is the piecewise-constant control input vector to be determined, in the time interval $[kT, kT+T)$and $T>0$ is a sampling period. For the digital control of the continuous-time TS fuzzy system, the digital fuzzy-model-based controller is employed. Let the fuzzy rule of the digital control law for the system (9) take the following form:

R^i: If $z_1(t)$ is about Γ_1^i... $z_n(t)$ is about Γ_1^n

$$\text{Then } u_d(t) = K_d^i \hat{x}_d(kT) + J_i y_d(kT) \quad (11)$$

for $t \in [kT, kT+T)$where K_d^i and J_i is the digital control gain matrix to be redesigned for the ith rule, and the overall control law is given by

$$u_d(t) = \sum_{i=1}^{q} \theta_i(z(kT))(K_d^i \hat{x}_d(kT) + J_i y_d(kT)) \quad (12)$$

for $t \in [kT, kT+T)$

The objective is to find gain matrices for digital controller and observers in (12) and (10) from the analog gain matrices in (7) and (5), so that the closed-loop state $x_d(t)$ in (9) with (12) can closely match the closed-loop state $x_c(t)$ in (8) at all sampling time instants $t = kT$ $k \in Z^+$. Thus it is more convenient to convert the TS fuzzy system into discrete-time version for derivation of the state matching condition.

There are several methods for discretizing a linear time-invariant (LTI) continuous-time system. Unfortunately, these discretization methods cannot be directly applied to the discretization of the continuous-time TS fuzzy system since the defuzzifed output of the TS fuzzy system is not

LTI but implicitly time-varying [1]. Moreover, it is further desired to maintain the polytopic structure of the discretized TS fuzzy system for the construction of the digital fuzzy-model-based controller. Thus we need a mathematical foundation for the discretization of the continuous-time TS fuzzy system.

Assumption 1: Assume that the firing strength of the ith rule, $\theta_i(z(t))$ is approximated by their values at time kT, that is, $\theta_i(z(t)) \approx \theta_i(z(kT))$ for $t \in [kT, kT+T)$. Consequently, the nonlinear matrices $\sum_{i=1}^{q} \theta(z(t))A_i$ and $\sum_{i=1}^{q} \theta(z(t))B_i$ can be approximated as constant matrices $\sum_{i=1}^{q} \theta(z(kT))A_i$ and $\sum_{i=1}^{q} \theta(z(kT))B_i$ respectively, over any interval $[kT, kT+T)$

If a sufficiently small sampling period T is chosen, Assumption 1 is reasonable.

Theorem 1: The pointwise dynamical behavior of the TS fuzzy system (9) can be efficiently approximated by

$$x_d(kT+T) \approx \sum_{i=1}^{q} \theta(z(kT))(G_i x_d(kT) + H_i u_d(kT)) \tag{13}$$

Proof: See the reference [1].

Remark 1: The discretized TS fuzzy system (13) contains the discretization error with the order of $O(T^2)$, which is tolerable under the choice of a sufficiently small sampling period, and vanishes as T approaches zero. Notice that the error induced in this discretization procedure is smaller than the first-order truncated Taylor series expansion of (9)

Let the error be $e_d(kT) = x_d(kT) - \widehat{x}_d(kT)$ Then the closed-loop of the augmented system the discretized version of the closed-loop system with (13), (10) and (12) is constructed to yield

$$\chi_d(kT+T) \approx \sum_{i=1}^{q}\sum_{j=1}^{q} \theta_i(z(kT))\theta_j(z(kT))\theta_k(z(kT)) \begin{bmatrix} G_i + H_iK_d^j + H_iJ_jC & L_d^iC \\ 0 & G_i - L_d^iC \end{bmatrix} \chi_d(kT) \tag{14}$$

where $\chi_d(kT)^T = [\, \widehat{x}_d(kT)^T, e_d(kT)^T]^T$

Corollary 1: The pointwise dynamical behavior of the continuous-time closed-loop TS fuzzy system (8) can also be approximately discretized as

$$\chi_c(kT+T) \approx \sum_{i=1}^{q}\sum_{j=1}^{q} \theta_i(z(kT))\theta_j(z(kT))\,\Phi_{ij}\chi_c(kT) \tag{15}$$

where

$$\Phi_{ij} = \exp\left(\begin{bmatrix} A_i + B_iK_c^j & L_c^iC \\ 0 & A_i - L_c^iC \end{bmatrix} T\right) = \begin{bmatrix} \phi_{ij}^{11} & \phi_{ij}^{12} \\ \phi_{ij}^{21} & \phi_{ij}^{22} \end{bmatrix}$$

for $(i,j) \in I_Q \times I_Q$, where $\chi_c(kT) = [\, \widehat{x}_c(kT)^T, e_c(kT)^T]^T$

Proof: It can be straightforwardly proved by Theorem 1.

Theorem 2: If there exist symmetric positive definite matrices Q_1, Q_2, matrices F_i, O_i, N_i, M_i with appropriate dimensions, and possibly small positive scalars γ_1 and γ_2 such that the following two generalized eigenvalue problems (GEVPs) have solutions

GEVP 1:

$$\underset{Q_1, F_i, O_i, M_i}{\text{minimize}} \quad \gamma_1 \quad \text{subject to}$$

$$\begin{bmatrix} -\gamma_1 Q_1 & \star \\ \phi_{ij}^{11}Q_1 - G_iQ_1 - H_iF_j - H_iO_jC & -\gamma_1 I \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} -\gamma_1 Q_1 & \star \\ \phi_{ij}^{12}Q_1 + H_iF_j & -\gamma_1 I \end{bmatrix} < 0 \tag{17}$$

$$\begin{bmatrix} -Q_1 & \star \\ G_iQ_1 + H_iF_j + H_iO_jC & -Q_1 \end{bmatrix} < 0 \tag{18}$$

$$CQ_1 - MC = 0 \tag{19}$$

GEVP 2:

$$\underset{Q_2, N_i}{\text{minimize}} \quad \gamma_2 \quad \text{subject to}$$

$$\begin{bmatrix} -\gamma_2 Q_2 & \star \\ (\phi_{ij}^{22})^T Q_2 - G_i^TQ_2 + C^TN^{T_i} & -\gamma_2 I \end{bmatrix} < 0 \tag{20}$$

$$\begin{bmatrix} -Q_2 & \star \\ G_i^TQ_2 - C^TN^{T_i} & -Q_2 \end{bmatrix} < 0 \tag{21}$$

then, the state $x_d(kT)$ of the discretized version (13) of the controlled system via the redesigned digital fuzzy-mode-based controller (12) closely matches the state $x_c(kT)$ of the discretized version of the analogously controlled system (15). Furthermore, the discretized system (13) is asymptotically stabilizable in the sense of Lyapunov stability criterion, where star denotes the transposed element in symmetric positions.

Proof: The proof is omitted due to lack of space.

Remark 2: It is important to address that, since the searching variables Q_1, F_i, O_i, M_i for the digital controller in GEVP 1 and Q_2, N_i for the discrete observer in GEVP 2 of Theorem 2 are not coupled, the IDR for the digital controller (12) and the discrete observer (10) can be performed independently, which indicates that the separation principle holds for IDR of the observer-based output-feedback fuzz-model-based controller.

Remark 3: The GEVP 1 in Theorem 2 is efficiently solved via semidefinite programming or LMI Control Toolbox by converting the LME (19) into

$$\begin{bmatrix} -\varepsilon I & \star \\ CQ_1 - MC & -\varepsilon I \end{bmatrix} < 0 \quad (22)$$

where $\varepsilon \cong 0 > 0$ is very small constant.

When $C = I$ and $x_0 = \widehat{x}_0$ J_i and L_d^i in Theorem 2 are redundant and the IDR problem is reduced to state-feedback case.

Corollary 2: If there exist a symmetric positive definite matrix Q_1, a matrix F_i, and a scalar $\gamma_1 > 0$ such that the following GEVP is solved

$$\underset{Q_1, F}{\textit{minimize}} \quad \gamma_1 \quad \textit{subject to}$$

$$\begin{bmatrix} -\gamma_1 Q_1 & \star \\ \phi_{ij}^{11} Q_1 - G_i F_j + H_i F_j & -\gamma_1 I \end{bmatrix} < 0 \quad (23)$$

$$\begin{bmatrix} -Q_1 & \star \\ G_i Q_1 - H_i F_j & -Q_1 \end{bmatrix} < 0 \quad (24)$$

then the digital controller becomes $u_d(t) = K_d x_d(kT)$ for all $t \in [kT, kT + T)$ where $F_i = K_d^i Q_1$ ★denotes the transposed elements in the symmetric positions, and the digital static state-feedback gain matrices K_d^i are given by

$$K_d^i = F_i Q_1^{-1} \quad (25)$$

Proof: It can be easily proven from Theorem 2.

4. Conclusion

In this paper, a new IDR has been proposed for the observer-based output-feedback fuzzy-model-based controller. The developed technique formulated the given IDR problem as constrained convex optimization problems so that the powerful and flexible numerical algorithms can be utilized. The flexibility of the LMIs enables one to incorporate the stability of the redesigned system into the IDR algorithm. The separation principle was clearly shown. The future work will be devoted to IDR for general TS fuzzy systems.

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