타입-2 퍼지값의 순위결정

A Ranking Method for Type-2 Fuzzy Values

이승수, 이광형
한국과학기술원 전자전산학과

Seungsoo Lee and Kwang H. Lee
Department of EECS, KAIST
E-mail : {sslee, khlee}@if.kaist.ac.kr

ABSTRACT

Type-1 fuzzy value is used to show the uncertainty in a given value. But there exist many situations that it needs to be extended to type-2 fuzzy value because it is difficult to determine the crisp membership function itself. Intrinsically type-2 fuzzy values are more expressive and powerful than type-1 fuzzy values, but, at the same time, more difficult to be compared or ranked.

In this paper, a ranking method for type-2 fuzzy values is proposed. It is based on the satisfaction function which shows the possibility that one type-2 fuzzy value is greater than the other type-2 fuzzy value. Some properties of the proposed method are also analyzed.

Key words : Type-2 fuzzy set, Fuzzy comparison, Fuzzy ranking

I. Introduction

Type-1 fuzzy value is used to show the uncertainty in a given value. But there exist many situations that it needs to be extended to type-2 fuzzy value because it is difficult to determine the crisp membership function itself.

Intrinsically type-2 fuzzy values are more expressive and powerful than type-1 fuzzy values. For this advantage, there have been many researches in various fields including fuzzy control field to extend its framework from type-1 fuzzy set to type-2 fuzzy set. But this also makes it necessary to extend some operations defined on type-1 fuzzy sets.

In a lot of applications, comparison and ranking is one of the important issues. In this paper, a ranking method for type-2 fuzzy values is proposed. It is based on the satisfaction function which shows the possibility that one type-2 fuzzy value is greater than the other type-2 fuzzy value. Some properties of the proposed ranking method are also analyzed.

II. Comparison

2.1 Type-2 fuzzy value

In this paper, the term *fuzzy value* is used instead of *fuzzy number*. There are two reasons: 1) To be called as a fuzzy number, a fuzzy set should satisfy the condition that it is both convex and normalized. These two concepts, however, are quite difficult to be extended in the domain of type-2 fuzzy set. 2) Even if a fuzzy set is not a shape of a fuzzy number, it is possible to compare it with another fuzzy set if the two fuzzy sets satisfy some conditions which are more general than the conditions of fuzzy number.

Definition 1 A *type-2 fuzzy value* is defined as a type-2 fuzzy set that satisfies the following two conditions.

- A type-2 fuzzy set that is defined on a domain that has a precedence order
- A type-2 fuzzy set whose support is a finite interval

where the support of a type-2 fuzzy set is defined in [1].

The second condition is given due to a property of the proposed comparison method. It can be loosened or removed depending on the comparison method used in ranking.

2.2 Comparison of type-2 fuzzy values on a continuous domain

Comparing fuzzy values is an operation closely related to ranking fuzzy values. There are many different kinds of fuzzy comparison methods, but in the majority of case, they are only applicable to type-1 fuzzy values [4]. To rank type-2 fuzzy values, the comparison method must be defined on type-2 fuzzy values. Therefore we proposed a comparison method for type-2 fuzzy values that is an extension of a comparison method proposed by Lee et al. for type-1 fuzzy values [1][3]. In this paper, this comparison method is extended for type-2 fuzzy values on a continuous domain for more general cases.

Proposed comparison method is an approach based on the possibility theory. The difficulty in comparing two fuzzy values comes from the fact that a fuzzy value is corresponding to a range of crisp values. Depending on its actual value, a fuzzy number can be greater or lesser than the other. Because it is difficult to compare two fuzzy values directly, all the possible occurrences of two fuzzy values are compared in this approach.

If the domain of fuzzy value A and B is Δ, then the combination of actual values (x_i, y_j), $x_i, y_j \in \Delta$, will be lied within the rectangle bounded by the support of A and B as shown in Fig. 1.

Each possible combination of actual values can be categorized into one of three groups: $A > B$, $A = B$, or $A < B$. There is no vagueness because actual values are all crisp values. The possibility of a group can be calculated as the summation of the possibilities of all combinations contained within the group. This possibility is called the *satisfaction function*.

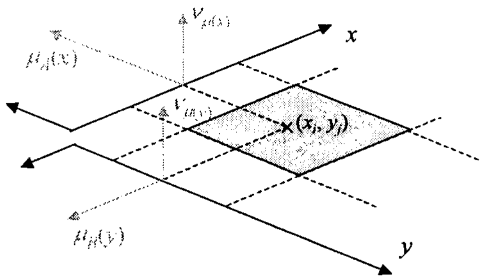

Fig. 1 Comparison of type-2 fuzzy values

Definition 2 If we denote the primary and secondary membership function of a type-2 fuzzy value as μ and ν respectively, the *satisfaction functions* $S(A > B)$, $S(A = B)$, and $S(A < B)$ for type-2 fuzzy values on a continuous domain are defined as

$$S(A > B) = \frac{\int_{-\infty}^{\infty}\int_{y}^{\infty}\int_{0}^{1}\int_{0}^{1} \mu_A(x) \otimes \nu_{\mu_A(x)}(w) \otimes \mu_B(y) \otimes \nu_{\mu_B(y)}(z)\,dwdzdxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{1}\int_{0}^{1} \mu_A(x) \otimes \nu_{\mu_A(x)}(w) \otimes \mu_B(y) \otimes \nu_{\mu_B(y)}(z)\,dwdzdxdy}$$

$$S(A = B) = 0$$

$$S(A < B) = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{y}\int_{0}^{1}\int_{0}^{1} \mu_A(x) \otimes \nu_{\mu_A(x)}(w) \otimes \mu_B(y) \otimes \nu_{\mu_B(y)}(z)\,dwdzdxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{1}\int_{0}^{1} \mu_A(x) \otimes \nu_{\mu_A(x)}(w) \otimes \mu_B(y) \otimes \nu_{\mu_B(y)}(z)\,dwdzdxdy}$$

where $\otimes$ is a t-norm operator that satisfies the following restriction.

$$\forall x, y \in [0,1], x \neq 0, y \neq 0 \rightarrow x \otimes y \neq 0 .$$

The satisfaction function for type-2 fuzzy values has the following properties.

Proposition 1 $S(A>B) + S(A=B) + S(A<B) = 1$.

Proof

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{1}\int_{0}^{1} \mu_A(x) \otimes \nu_{\mu_A(x)}(w) \otimes \mu_B(y) \otimes \nu_{\mu_B(y)}(z)\,dwdzdxdy$$
$$= \int_{-\infty}^{\infty}\int_{y}^{\infty}\int_{0}^{1}\int_{0}^{1} \mu_A(x) \otimes \nu_{\mu_A(x)}(w) \otimes \mu_B(y) \otimes \nu_{\mu_B(y)}(z)\,dwdzdxdy$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{y}\int_{0}^{1}\int_{0}^{1} \mu_A(x) \otimes \nu_{\mu_A(x)}(w) \otimes \mu_B(y) \otimes \nu_{\mu_B(y)}(z)\,dwdzdxdy .$$

$\therefore S(A>B) + S(A=B) + S(A<B) = 1$.

Proposition 2 If $\Sigma(A) \cup \Sigma(B) = \varnothing$ then $S(A > B) = 1$ or $S(A < B) = 1$, where $\Sigma(A)$ is the support of a fuzzy set A.

Proof

$\Sigma(A) \cup \Sigma(B) = \varnothing$ means that

$$\int_{-\infty}^{\infty}\int_{y}^{\infty} \mu_A(x) \otimes \mu_B(y)dxdy = 0 \quad \text{or}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y} \mu_A(x) \otimes \mu_B(y)dxdy = 0$$

$\therefore$ $S(A > B) = 1$ or $S(A < B) = 1$.

Proposition 3 If $A \equiv B$, then $S(A<B)=S(A>B)=0.5$.

Proof

$A \equiv B$ means that $\mu_A = \mu_B$, and $\nu_{\mu_A} = \nu_{\mu_B}$.

Therefore,

$$\int_{-\infty}^{\infty}\int_{y}^{\infty}\int_{0}^{1}\int_{0}^{1}\mu_A(x) \otimes \nu_{\mu_A(x)}(w) \otimes \mu_B(y) \otimes \nu_{\mu_B(y)}(z)dwdzdxdy$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{y}\int_{0}^{1}\int_{0}^{1}\mu_A(x) \otimes \nu_{\mu_A(x)}(w) \otimes \mu_B(y) \otimes \nu_{\mu_B(y)}(z)dwdzdxdy$$

$\therefore$ $S(A<B) = S(A>B) = 0.5$.

If both A and B are type-1 fuzzy values, the following equation is satisfied for every $(\mu_A(x), \mu_B(y))$.

$$\int_0^1\int_0^1 \nu_{\mu_A(x)}(w) \otimes \nu_{\mu_B(y)}(z)dwdz = 1$$

And the satisfaction function $S(A>B)$ will be simplified as

$$S(A>B) = \frac{\int_{-\infty}^{\infty}\int_{y}^{\infty}\int_{0}^{1}\int_{0}^{1}\mu_A(x) \otimes \nu_{\mu_A(x)}(w) \otimes \mu_B(y) \otimes \nu_{\mu_B(y)}(z)dwdzdxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{1}\int_{0}^{1}\mu_A(x) \otimes \nu_{\mu_A(x)}(w) \otimes \mu_B(y) \otimes \nu_{\mu_B(y)}(z)dwdzdxdy}$$
$$= \frac{\int_{-\infty}^{\infty}\int_{y}^{\infty}\mu_A(x) \otimes \mu_B(y)dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu_A(x) \otimes \mu_B(y)dxdy}$$

which is identical to the satisfaction function defined for type-1 fuzzy values on a continuous domain [2]. The satisfaction function $S(A<B)$ can be derived in a similar way.

III. Ranking

A ranking method for type-2 fuzzy values is proposed using the comparison method proposed in 2.2.

As in the case of comparison, it is not intuitive if ranking fuzzy values produce one and only one crisp result. So we will propose a method to calculate the confidence degree of each ranking result.

Before describing the ranking method, we will introduce the concept of *preference function* of fuzzy values [3].

Definition 3 The *preference function* $R(A, B)$ for fuzzy values A and B is defined as

$$R(A,B) = S(A>B) + \frac{1}{2}S(A=B)$$

If A and B are both type-2 fuzzy values on a continuous domain, this preference function can be simplified as

$$R(A,B) = S(A>B).$$

We can consider that this preference function shows the confidence degree on the statement that 'A is greater than B.' And it satisfies the following properties.

Proposition 4 $R(A,B) + R(B,A) = 1$.

Proof

$$R(A,B) + R(B,A)$$
$$= S(A>B) + \frac{1}{2}R(A=B) + S(B<A) + \frac{1}{2}R(B=A)$$
$$= 1$$

Proposition 5 $R(A,A) = 0.5$.

Proof

$$R(A,A) = S(A>A) + \frac{1}{2}(A=A) = 0.5$$

Definitions 4 When we compare n fuzzy values, there are totally $n!$ ranking results. The *confidence degree* of ith each ranking result L_i is defined as

$$S(L_i) = \min_{j,k}(R(A_j, A_k))$$

where $A_j > A_k$ in the ranking result L_i.

Example If the preference function of four fuzzy values A_1, A_2, A_3, and A_4 are given as

R	A_1	A_2	A_3	A_4
A_1	–	0.998	1.000	1.000
A_2	0.002	–	0.183	0.681
A_3	0.000	0.817	–	0.992
A_4	0.000	0.319	0.008	–

then the confidence degree of a ranking result $A_1 > A_3 > A_2 > A_4$ can be calculated as follows.

$$S(A_1 > A_3 > A_2 > A_4)$$
$$= \min(R(A_1, A_3), R(A_1, A_2), R(A_1, A_4), R(A_3, A_2), R(A_3, A_4), R(A_2, A_4))$$
$$= \min(0.988, 1.000, 1.000, 0.817, 0.992, 0.681)$$
$$= 0.681$$

Confidence degrees of other ranking results can be calculated in the same way. The following examples show the confidence degrees of two other ranking results.

$$S(A_1 > A_3 > A_4 > A_2) = 0.319$$
$$S(A_2 > A_1 > A_3 > A_4) = 0.002$$

Assigning a confidence degree to each ranking result, final ranking result will be given as a form of a fuzzy set. For the application that require a crisp ranking result, the *representative ranking result* of each ranking result is defined as follows.

Definition 3.3 The ranking result with the largest confidence degree is called the *representative ranking result.*

In the example above, $A_1 > A_3 > A_2 > A_4$ is the representative ranking result. Furthermore, an α-cut of the result fuzzy set can be used as an alternative to get a candidate of crisp ranking result.

IV. Conclusion

A ranking method for type-2 fuzzy values is proposed in this paper. In comparison, the previous method is extended for the type-2 fuzzy values defined on a continuous domain for more general cases. And in ranking, a confidence degree is assigned to each ranking result based on the satisfaction function. The result of ranking is given as a form of fuzzy set so that it can provide much flexibility in its appliance.

The drawback of the proposed method is that there exist a few restrictions in the shape of the membership function of fuzzy value and heavy computational burden is required. This problem can be more or less evaded using discrete fuzzy values with sacrificing some precision. For more usability, the development of approximation algorithm based on heuristics is considered as a further work.

V. Reference

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